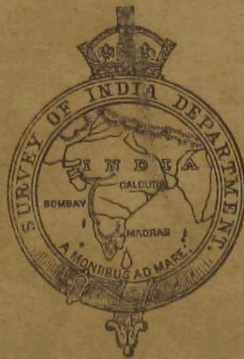


HAND-BOOK  
OF  
PROFESSIONAL INSTRUCTIONS  
FOR THE  
TRIGONOMETRICAL BRANCH  
Survey of India Department.

PREPARED BY  
COLONEL G. STRAHAN, R.E.,  
DEPUTY SURVEYOR GENERAL, TRIGONOMETRICAL BRANCH.

UNDER THE DIRECTION OF  
COLONEL H. R. THUILLIER, R.E.,  
SURVEYOR GENERAL OF INDIA.



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## P R E F A C E .



The Handbook of Professional Instructions of the Trigonometrical Branch is divided into nine parts; the first eight of these are devoted to the details of the principal operations of a geodetic survey as carried out in India and the ninth deals with various miscellaneous matters. The nine parts are as follows:—

- |      |       |                                        |
|------|-------|----------------------------------------|
| Part | I.    | The Measurement of Base-lines.         |
| ,,   | II.   | Triangulation—Principal and Secondary. |
| ,,   | III.  | Astronomical Azimuths.                 |
| ,,   | IV.   | Electro-Telegraphic Longitudes.        |
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| ,,   | VI.   | Tidal Observations.                    |
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| ,,   | IX.   | Miscellaneous.                         |

Of these the first and eighth subjects being too extensive for treatment in a handbook, are very briefly referred to; little more being given than references to such standard works as are regarded as the best authorities on these operations.

Part II is a compilation of the various instructions on triangulation issued from time to time for the guidance of the officers of the Trigonometrical Branch brought up to date; and it embodies in a form easy for reference the results of their experience since the commencement of the Trigonometrical Survey. In arranging this part of the Handbook, and especially in the chapter that treats of the computations, I am much indebted to Mr. Cole (in charge of the Computing Office) for his assistance.

Part III describing the observations and computations connected with astronomical azimuths is almost entirely due to the same officer.

For Part IV treating of Electro-Telegraphic Longitudes, I am myself solely responsible.



Part V has been contributed by Captain Burrard, R.E., and Parts VI and VII have been compiled by Mr. Eccles, the former from "A Manual for Tidal Observations" by Major Baird, R.E., and Professor G. H. Darwin's Report to the British Association of 1883; and the latter from Colonel Walker's "Memoranda on Levelling Operations".

These last three Parts necessarily describe the operations as carried on with the particular equipments now in use, and therefore are not of universal application; but it is hoped that the general principles involved have been so far explained, that any reasonably intelligent officer could, with instruments of a different design, adapt his procedure accordingly.

Part IX is in a great measure a repetition of Chapter III of the Handbook of the Topographical Branch; but as that work may not necessarily be in the possession of officers of the Trigonometrical Branch, it has been considered advisable to reprint some portion of it *mutatis mutandis* for insertion in this Handbook.

There are, and probably always will be, many differences of opinion about the relative advantages of various modes of conducting such operations as are here described, but the rules laid down have been found by experience to be the best adapted to the requirements of Indian Geodesy, and should therefore not be lightly departed from by officers of the Survey Department.

DEHRA DUN: }

July, 1891. }

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While this Handbook was in the press, a new Greenwich Ten-year Catalogue was published, giving the places of 4059 stars in Right Ascension and Declination on 1st January 1880. For this Catalogue the quantities  $e$ ,  $f$ ,  $g$ ,  $h$ ,  $l$ , and  $e'$ ,  $f'$ ,  $g'$ ,  $h'$ , and  $l'$ , have not been computed, and moreover Airy's Day Numbers are not given in the Nautical Almanac for 1891 and 1892; and presumably will not be given for subsequent dates. The method of computing star places as explained on pages 74, 106, and 178, and exemplified in page 2 of Form P. 32 for the Azimuth Computations, and in Form No. 4 of the Electro-Telegraphic computations, is therefore no longer applicable. The reduction must be made in future by means of the "Quantities for Correcting the Places of Stars" given for every day of the year in the Nautical Almanac under the heading "Apparent Places of Stars", the use of which is rendered sufficiently clear by an example worked out in the "Explanation" at the end of the Nautical Almanac. Professional Forms 32, 36 and 37 have been revised so as to suit the above mentioned alteration.



# SURVEY OF INDIA DEPARTMENT.

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## HANDBOOK OF THE TRIGONOMETRICAL BRANCH.

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### PART I.

#### The Measurement of Base-Lines.

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1. There are few scientific operations on which care and ingenuity have been more lavishly expended than on the measurement of Base-lines, though it may at first sight appear to be but a simple matter to measure the distance between two fixed points. The necessity for extreme accuracy arises from the fact that in extending a chain of triangles from a measured base, any small error in the latter is increased in the former in the exact proportion by which the length of the chain differs from that of the base-line. Many methods have been tried at various times, but one of the most trustworthy and the only one likely to be adopted in the immediate future in the Trigonometrical Branch, is the measurement by the Colby compensation bars and microscopes. A brief history of the measurement of the old Indian Base-lines will be found in Chapter V of Volume I of the "Account of the Operations of the Great Trigonometrical Survey of India", to which the reader is referred.

2. A detailed description of the Colby apparatus would be too long for insertion in this Handbook, nor is it necessary, as it may be found in many standard works, *viz.*, in Captain Yolland's "Account of the Measurement of the Lough Foyle Base, 1847", in Colonel Everest's "Measurement of the Meridional Arc of India, 1847", in Captain Clarke's "Account of the Principal Triangulation of the Ordnance Survey, 1858", in Sir Thomas Maclear's "Verification and Extension of LaCaille's Arc of Meridian, 1866", and in the 3rd Volume of the "Course of Mathematics" formerly in use at the Royal Military Academy, Woolwich. Chapter VI of Volume I of the "Account of the Operations of the Great Trigonometrical Survey of India" also contains a brief description of the apparatus and the precautions to be observed in using it, including the all-important process of comparing the bars and microscopes with the 10-foot and 6-inch standards.

3. The measurement of a base is so important and so *special* an operation that it would always be entrusted to one or more of the most experienced officers of the Department, who would necessarily have at

hand for reference some of the extensive literature on the subject; it falls therefore somewhat beyond the scope of this Handbook which is intended for the guidance of young and inexperienced officers when called upon for the first time to superintend the ordinary operations of geodesy.

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## PART II.

### Triangulation, Principal and Secondary.

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#### CHAPTER I. PRINCIPAL TRIANGULATION.

1. The principal triangulation consists of a system of chains of triangles, conforming generally in direction with meridians or parallels of latitude, and tied together by other chains at their extremities. The values of side length are derived from base-lines measured in different parts of India, and the geodetic elements of latitude, longitude and azimuth are brought up through the triangulation from a station of origin, which has been chosen at Kaliánpur.\* The latitude and azimuth were obtained there by astronomical observations, and the longitude was deduced through the triangulation from the Madras Observatory. The heights are referred to *mean sea level* as origin; for a definition of which term, *vide* Parts VI and VII of this Handbook.

2. In the course of the execution of these chains, the positions of neighbouring places of note, and of prominent points, are established by triangulation of less precise character which is called "Secondary." The points so fixed are intended for the use of detail surveyors.

3. The direction in which a series of triangles should proceed depends on the object for which it is undertaken; but it is customary in India to avoid an oblique direction, and to conform strictly to a meridional or longitudinal line.

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\* *Vide* Chapter XI, Volume II of the "Account of the Operations &c."

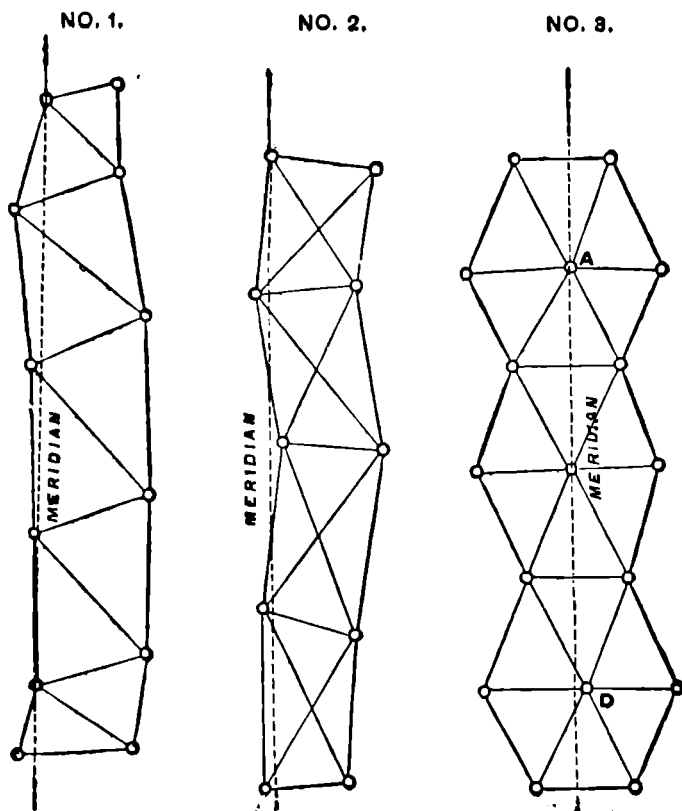
4. In geodetic operations, the determination of the elements of the earth's figure being the object in view, both directions are of equal value; for it is only by a comparison of the positions of the stations as obtained from the triangulation with those resulting from suitable astronomical observations that these elements can be arrived at. The meridional chains suffice in conjunction with astronomical determinations of latitude to determine the figure of the meridian, and the longitudinal chains combined with observations of differences of longitude (effected in the present day with extreme accuracy by means of the Electric Telegraph, *vide* Part IV of this Handbook) lead to a knowledge of the magnitude and form of the Equator, and of the circles of latitude parallel to it.

5. The initial line from which the series starts (whose length will have been determined either by direct measurement, or deduced by triangulation from a measured base) being selected, the next operation will be to choose the stations in a judicious manner in the required direction. As the azimuth of the initial line is always known approximately from the previous triangulation, there is no difficulty in determining this direction. In starting, however, from a measured base it will be necessary, unless an azimuth has been observed in connection with it, to determine the direction of the meridian by any of the methods given at pages 78 to 81 of the 3rd edition of the "Auxiliary Tables to facilitate the Calculations of the Survey of India."

6. A Plane-table reconnaissance, where possible, will be found of great assistance in laying out the stations. The initial base should be plotted on the board on any convenient scale (4 miles = 1 inch will probably be found suitable); and from either end of it, when the table has been properly set up and orientated either by compass, or by preference by the back ray to a previously fixed station, rays should be drawn to conspicuous points likely to prove suitable sites for stations in advance; from these again when visited the same process should be carried on, advantage being taken at the same time of all opportunities of fixing prominent hill peaks, trees, buildings &c., likely to be utilized in the secondary work. This reconnaissance or Plane-table triangulation is of great use in identifying points observed, as well as in selecting stations, and should on no

account be neglected. It may even in skilful hands be employed in executing a more or less good preliminary map of the districts visited, should none exist. It is assumed that the reader is familiar with the use of the Plane-table: much useful information on this point will be found in the Handbook of the Topographical Branch. The operations of selecting and constructing stations are described at some length in Chapter II of Volume II of the "Account of the Operations of the Great Trigonometrical Survey of India" to which the reader is referred.

7. A series of principal triangles may be either single or double as represented below, wherein No. 1 is a single series. A double triangulation may be arranged in several ways; for example, No. 2 is a series of successive quadrilaterals, No. 3 is a series of successive polygons. In practice it frequently happens that these varieties of figure are combined together in various ways, according to the nature of the country, and the local facilities it affords; but as each method has its peculiar advantages and disadvantages, it is proper that their relative merits should be duly considered, in order that the most judicious selection may be made which circumstances admit of.



8. The single series possesses advantages of economy of time, labor, and money, as well in the field operations as in the office computations; and it is, consequently, the most proper kind of series to be adopted when circumstances impose a strict attention to these restrictions. For instance, in a level country, like the plains of Hindustan, in which, from the absence of natural elevations, trigonometrical operations are both costly and tedious, every consideration of economy combines to recommend a single series as the most eligible arrangement. On the other hand, the advantage of accuracy will always be found greatly in favor of a double series, which, moreover, supplies a check of the most efficient kind at every stage of the work.

Relative advantages of different figures.

9. The relative advantages of quadrilaterals, polygons, and single figures, are fully entered into at pages 38, 39 and 200 of Volume II of the "Account of the Operations of the Great Trigonometrical Survey of India", a work which should be in the hands of all surveyors employed on triangulation. The conclusion there arrived at is that as a general rule a symmetrical hexagon is the best figure for triangulation in a flat country, and a symmetrical quadrilateral the best in a hilly country.

Continued.

10. In all principal series azimuths must be observed at intervals of about  $1^{\circ}$ . In order that these observed azimuths may be as free as possible from those errors which are caused by local attraction, it is desirable that no great mass of high mountains should stand either east or west of the station of observation. Local attraction acting in any direction except in the meridian will, by dislevelment of the theodolite, cause uncompensated errors in the observations for azimuth. As astronomical latitude is advantageously measured at the same stations as azimuth, it is always advisable to avoid local attraction in the direction of the meridian also, for its full effect comes in as error in the resulting latitude.

Local attraction.

11. In choosing stations it is extremely important to avoid intermediate obstacles situated on, or close to, the ray, because if the ray grazes the side of a mountain it will certainly be deflected by the vapours which arise from the sloping ground, whereby the observations will be more or less injuriously affected by lateral refraction; and if the ray grazes close over

Grazing rays.



the top of a mountain or over intervening ground or even over a village or huts, then the angles horizontal and vertical will be greatly disturbed. If circumstances, however, concur in admitting of no other choice, it will be desirable always to provide a check by introducing a polygon or quadrilateral at that part of the work, and a secondary station should also be established on the intervening hill, which may be treated as an auxiliary point, for taking vertical angles, whereby the comparative heights may be determined independently of direct observation.

12. A Principal Series should consist of triangles as large as the features of the country admit of without grazing rays, by which arrangement the number of stations, and probability of accumulation of error will be least. Considerations of economy likewise impose the same restriction, inasmuch as the additional points required for topographical purposes can be obtained at less cost by means of secondary triangles, in which accumulation of error is an evil that need not be dreaded, the limits of inaccuracy being sufficiently controlled by the principal triangulation. No general rule can be applied with respect to the magnitude of triangles, which must necessarily vary with the configuration of the surface of the country. In a mountainous region with gigantic features, (composed of lofty peaks and deep valleys) it would be as difficult as it would be injudicious to select small triangles; and in a flat country it would be absurd to struggle against natural difficulties in the vain endeavour to establish a series of great magnitude. In hilly countries, twenty to thirty miles is a convenient distance for principal stations, and such distances can usually be observed with facility, in all ordinary conditions of atmosphere, by means of heliotropes and Argand lamps. When hills are table-topped, as is generally the case where the stratification of the rocks is horizontal, it may be necessary to shorten the sides to less than twenty miles. An open country with detached peaks may frequently afford distances of thirty to forty miles with the fairest prospect of good observations: in a level country, such as Bengal, ten miles is the most favorable length of side, and the limits should be considered eight to fifteen miles.

13. The triangles selected should be symmetrical, that is, as nearly equilateral as possible, such being the best condition for ensuring accuracy when all the angles are measured; but as it is impracticable to obtain in the field

Size of triangles.

Symmetry of triangles.

triangles exactly equilateral, the rule in practice is, to admit of no angle under  $30^\circ$ , or above  $90^\circ$ . This rule applies to all the triangles which compose a series, whether principal or secondary; but, in the case of secondary triangles which do not form part of a long series, wider limits are admissible. These limits must also of necessity be extended in the case of a quadrilateral, because the four triangles of which it is composed obviously cannot be all equilateral. The best condition for that figure is when it approximates to the square, in which form no angle will be much above  $90^\circ$ , and none much less than  $45^\circ$ .

14. The stations selected should be on the highest peaks, but if these are inaccessible, it becomes necessary to adopt a lower point. Every effort should, of course, be made to reach the summit when practicable. In the case of a lower point being used, care must be taken that the view is clear in the direction of the stations in advance and that it is suitably situated for the use of topographical surveyors. This precaution has occasionally been neglected greatly to the inconvenience of detail surveyors wishing subsequently to utilize the stations for secondary work.

15. There has been at different times considerable variety in the structures built to mark the sites of the principal stations: experience has shown that one of the three classes here described will be suitable in almost all cases that can occur in Indian practice, but of course under exceptional circumstances modifications may be made where really required, but not merely at the caprice of executive officers.

*First.* Hill stations:—

Where the soil is composed of rock, a dot surrounded by a concentric circle is cut on the solid rock *in situ*, otherwise a large stone similarly marked is buried in the ground. Over the mark a small *paka* platform or pillar is built to a convenient height, and of sufficient diameter to accommodate the theodolite stand: on the summit of this pillar another mark-stone is inserted, and fixed truly vertically over the lower one. Besides the upper mark-stone it is usual to imbed in the pillar three picked, flat, heavy stones, for the tripod of the instrument to stand upon. These are called "foot stones," and they should be duly levelled, which will save trouble afterwards in adjusting the instrument.

Surrounding this pillar and separated from it by an annular space, a platform is built, on which the observer and his assistants stand. The dimensions of this platform should be at least as large as the observatory tent, and it will generally be found convenient to make it somewhat larger—12 feet  $\times$  15 feet will prove a suitable size. The annular space round the central pillar is for the purpose of isolating the instrument, and preventing any shake caused by the observer's movements; it should be filled in with loose sand, otherwise screws or other parts of the apparatus may be lost by falling into it. The distance between the two mark-stones should be recorded, but all measurements and observations should be referred to the upper mark-stone, and are so recorded in the angle books. In alluvial country where stone is scarce, it will be better to mark the circle and dot on bricks, as flat dressed stone is likely to be stolen by the inhabitants of such districts for domestic purposes.

*Secondly.* Tower stations :—

These are generally resorted to in flat countries to enable the observer to surmount trees and other obstacles to the view, and to overcome the curvature of the earth. The most approved form is a hollow rectangular tower of *paka* masonry. This is first built up to within 3 feet of its ultimate height, and upon two of the opposite walls (say north and south) are laid two stout beams, and crosswise on these near their centre four smaller ones. This platform of four small beams or battens serves as a foundation on which a perforated circular pillar of masonry to support the theodolite stand is built up to the required height. The walls of the tower are then carried up to within about 6 inches of the level of the top of the pillar when two similar beams to support the flooring for the tent and for the observer to move about on are placed on the east and west walls, and a firm railing added for the sake of the observer's safety. The dimensions of the various parts of these towers will vary according to circumstances, and it is impossible to lay down any hard and fast rule. The following formula will probably be found suitable in most cases :—

Thickness of wall at bottom in feet =  $0.2 \times h^{\frac{4}{5}}$ ,  $h$  being the height in feet.

Inside the walls should be vertical, and outside they should taper to give a thickness of about  $1\frac{1}{2}$  feet where the beams for the observer's platform

rest. The interior dimensions should be 7 feet  $\times$  11 feet, giving a space on the platform of 10 feet  $\times$  14 feet which is amply large enough for the observer and his assistants: indeed with the modern theodolites, which are considerably smaller than the old 36-inch and 24-inch instruments, less space than this might easily suffice; but the size of the observatory tent must be the final guide to the dimensions required. The perforated circular pillar on which the theodolite stands must be  $2\frac{1}{2}$  feet diameter for the new 12-inch instruments, and somewhat larger (about 3 feet) for the older pattern of 24-inch theodolites. The perforation is to enable the plumb-line to pass down through it to the mark-stone on the ground level. A second mark-stone is buried in the ground vertically below this surface one, as in the case of ordinary hill stations. It is objectionable to build solid pillars for a station exceeding 3 or 4 feet in height, as in course of time deflection is sure to occur to a greater or less extent, and so to vitiate the true centering of the instrument. Plate I shows, by sketches not drawn to scale, the general arrangement of a tower station as above described.

In Fig. 1 is shown the tower built up to the level of the cross beams supporting the isolated pillar; and also the details of the pillar itself. Fig. 2 represents the completed tower. The aperture at the bottom is to facilitate the centering of the theodolite over the mark-stone by a plumb-line. Fig. 3 shows a method of raising the theodolite boxes, and other stores, to the summit of the tower; care must be taken that the feet of the shears are bedded in holes sufficiently deep to prevent them slipping. If the tower is a low one the shears will be unnecessary, and the arrangement in Fig. 4 will probably be found more handy. It represents the observing platform extended sufficiently beyond the walls of the tower to enable the boxes to be drawn up without scraping against the brickwork. Access to the summit of the tower is gained by ladders.

*Thirdly.* Trestle stations:—

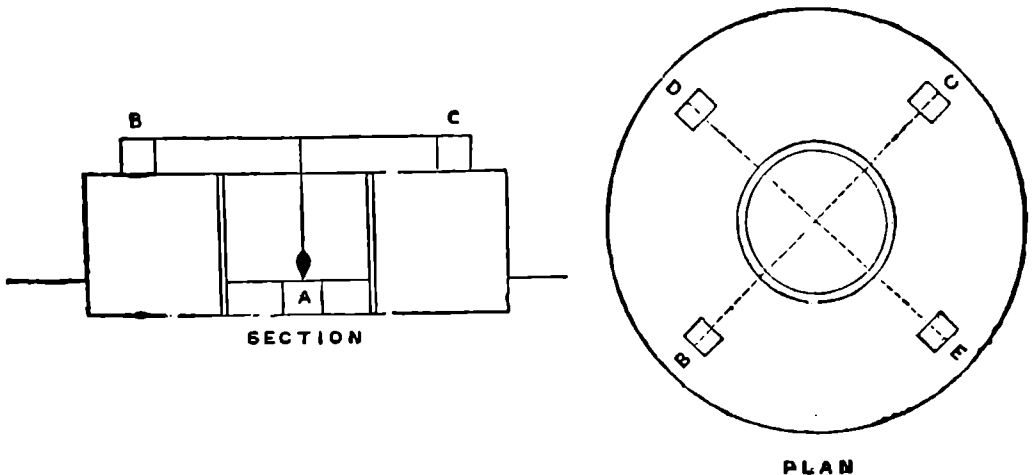
These have been found a cheap and convenient substitute for tower stations, where timber and workmen are easily obtained, and the height required is not excessive. Plate II shows the construction of a trestle station, as designed and built by Major Branfill, on the South-East Coast Series in 1874-75. It consists essentially of a braced tripod stand 17 feet high resting partly on the masonry platform, and partly on the summit of the isolated pillar. This stand is surrounded by a braced scaffolding, to which at the proper height is attached a platform for the observatory

tent, and which is carried up to a considerable height above this, for the purpose of showing the lamp or heliotrope to other stations. In Major Branfill's case palmyra trees which had been cut down during the process of ray clearing were used, and found to answer well. It appears from his report that lashings only were used for joining the various parts of the structure, the cost of the whole being only about Rs. 30. No difficulty would be experienced in raising the theodolite boxes on to a platform of this description.

16. The simplest practical method of determining the height to which a tower must be raised is to erect on the site of the station a temporary scaffolding of bamboo or other suitable material, sufficiently high to bring clearly into view at the time of minimum refraction the lowest of all the neighbouring signals that have to be observed. It may be useful to remember that if  $h$  is the height of a tower, the ray from it will graze the earth's surface at a distance  $d$  approximately, such that  $d = \sqrt{\frac{3h}{2}}$ ,  $d$  being measured in miles and  $h$  in feet.

17. The method of adjusting mark-stones is as follows:—Let  $A$  be the mark at the foundation, either engraved on the rock, or on a heavy embedded stone. Let the external part of the platform be built up to the intended height of

Adjustment of mark-stones.



the next mark, and place upon it four heavy stones, arranged in a quadrilateral figure, in such wise that threads  $BC$  and  $DE$  stretched diagonally across may intersect near the centre. Adjust these threads to correspond with a plumb-line suspended over the mark  $A$ , and when the coincidence is complete, mark the four exterior stones, by pencil lines, or lines scratched with a knife. Arrangements must now be made for

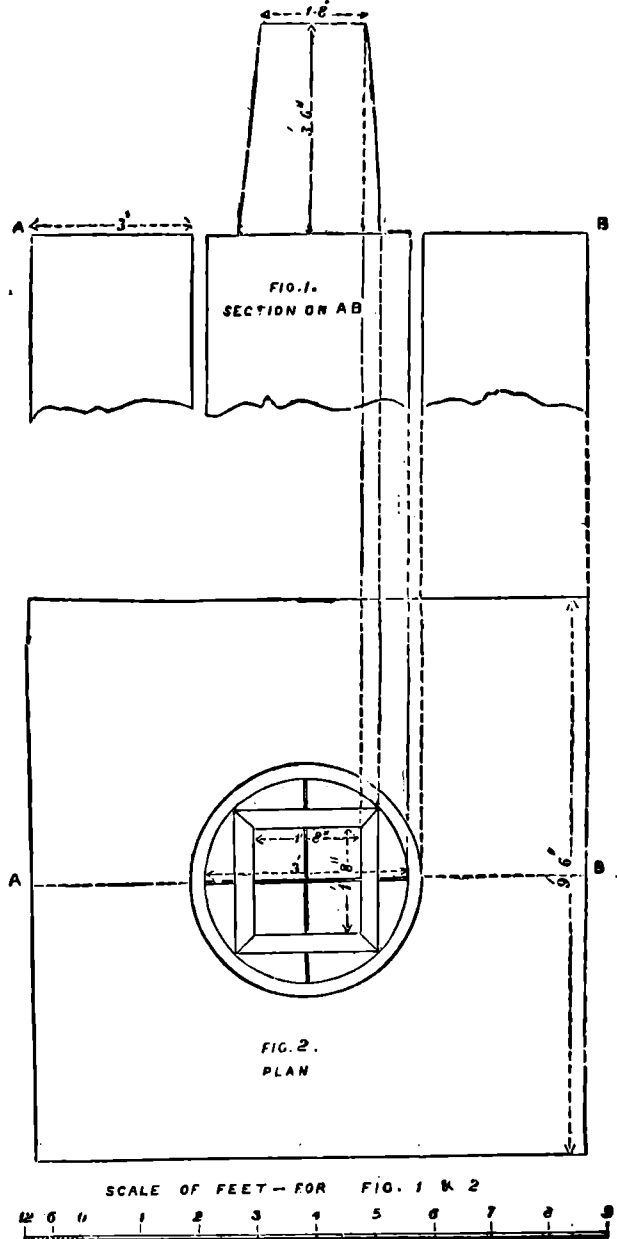
protecting these stones, either by covering them over, or appointing a man to watch each, while the pillar in the centre is being built up nearly to the level of *BC*, at which height the next mark-stone should be fixed and adjusted, to correspond with the cross threads before adverted to.

18. At principal stations as soon as the observations are completed the mark-stone must be protected and concealed

Construction of protecting pillar.

from view by a masonry pillar, which should be built in such a manner that future surveyors may use it instead of a

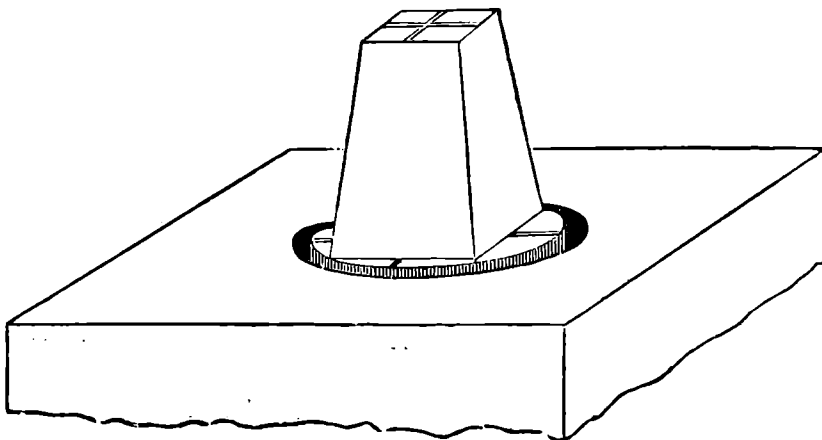
stand to rest their theodolites on, and centre the instruments with almost as great accuracy as if they were to set them up visibly over the mark. Grooves must first be cut on the surface of the existing isolated pillar in straight lines from the circumference to the edge of the mark-stone, and in such directions that if they were carried over the stone, they would intersect each other at right angles on the mark. Then a rectangular masonry pillar, of which each side should equal seven-tenths of the diameter of the circular pillar, must be raised on the latter pillar to a height of three feet six inches, tapering upwards, so as to make the summit twenty inches square. The sides of the rectangular pillar will be parallel or perpendicular to



the grooves on the circular pillar, the extremities of which latter will be left visible for reference. The accompanying figures 1, 2 and 3 exhibit all these details very clearly. Lines to fix the exact position of the mark on the upper surface of the pillar may be drawn by means of a simple square bar of iron or wood about four feet long, fitted with a plummet at either end, both suspended from the same face of the bar. The plummets must, by moving the bar, be brought first over one pair of grooves and then over the other, lines being drawn in both cases along the edge of the bar used as a ruler. In the case of tower stations it will be sufficient to pile some loose stones or rubbish over the mark-stone and then to brick up the aperture at the foot of the tower.

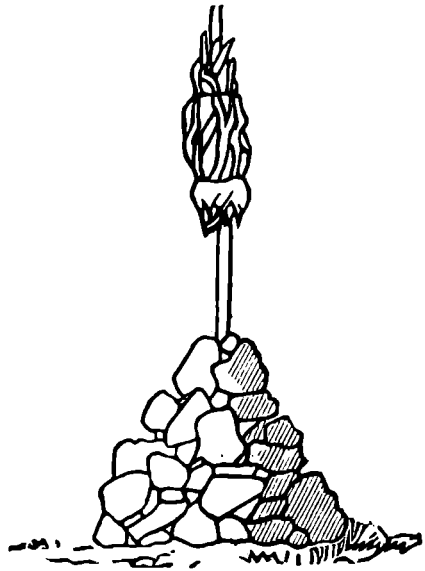
19. Grooves should be cut into the surface of the rectangular pillar as in that of the circular. The masonry should be of the best burnt brick and mortar procurable; the exterior surfaces of the bricks to be neatly cut and finished, and unplastered. Figure 3 represents a completed pillar. A pyramidal pile, seven feet square at base and six feet high, either of earthwork

FIG. 3.



and stones or *kacha* bricks cemented and plastered with mud, should be built over the pillar for its protection, and to prevent water from getting into the annulus between it and the platform. If the station

is likely to be subsequently required for secondary work it is usual to set up over the protecting pillar a pile of stones supporting a pole with brushwood tied on to it as in the annexed diagram. The pile of stones should be about 5 or 6 feet high and the pole about as much more. The station must then be made over to the care of the senior Native official of the village in whose lands it is situated. For this purpose, a document must be drawn up in duplicate in accordance with the form provided for the purpose, *viz.*, O.I.Δ, the original being sent to this office for record, while the duplicate is left in the hands of the person to whom the charge of the station is committed. The authorized form is as follows:—



(1) The station named \_\_\_\_\_ of the Great Trigonometrical  
 Survey of India, situated in latitude \_\_\_\_\_ longitude \_\_\_\_\_ on the  
 lands of the village \_\_\_\_\_ in Pargana \_\_\_\_\_ District \_\_\_\_\_ is  
 hereby placed in the charge of\*  
 and will remain under his charge and that of his successors until further  
 orders.

(2)† He is authorised to prevent all persons from injuring the station, but he will permit any person who may have occasion to visit it for Survey purposes, to remove the covering pile of stones or earthwork, and refer to the station mark, which is indicated by the intersection of the cross lines on the surface of the masonry pillar. When such a person has completed his observations, he will be required to erect a fresh pile of earth and stones, 7 feet square at base, and 6 feet high over the pillar.

(3) Any person wilfully injuring the station, or neglecting to comply with the terms of this document, will be liable to be prosecuted by the

\* Here give name and official rank of individual in question.

† In the case of tower stations para. (2) should be scored out.



District Officer, to whom a report must be made without delay by the official in charge of the station.

Given under the orders of the Government of  
dated 30th October, 1865.

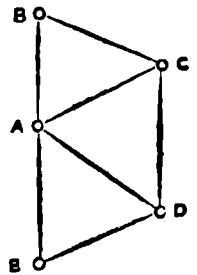
Date of Transaction

Signature of Survey Officer

Signature of Native Official

20. The selection of stations is merely preparatory to the final observations, and is called carrying on an Approximate Series. It is usual to choose a few triangles in advance, and then detach a party to continue the Approximate Series, and prepare the stations ahead, while the final observations are progressing in rear. This division of labour is of great advantage in accelerating progress, and, if judiciously arranged, with regard to local circumstances, and the organization of the party, is productive of the best results.

21. The angles at a station are taken thus: supposing the observer to be at  $A$ , and the signals at  $B, C, D, E$  all visible, the instrument is carefully levelled and adjusted, and so fixed that some station,  $B$  for instance, reads  $0^\circ$ , or zero in microscope  $A$ :  $B$  is then called the zero station. Suppose the telescope to be brought up from the left hand of  $B$ , and turned gently, so that  $B$  may enter the field of view, and come near the centre wire, but not pass over it; the instrument must then be clamped, and the bisection of  $B$  completed, by using the slow motion screw. All the micrometers, or verniers, are now read, and the assistant records the readings in a fair legible hand in the Angle Book. The observer should then look again into the telescope to see that  $B$  remains bisected. If found correct the telescope is to be carefully unclamped and moved towards  $C$ , care being taken not to overshoot it. The clamping, bisecting, and reading is done as before, and similarly also for  $D$  and  $E$ . A complete set of observations is thus obtained at zero  $0^\circ$  by a continuous motion from left to right. Now, after overshooting the station  $E$  the



telescope is brought back by a continuous motion from right to left, to each station in succession, and the readings recorded. This will give a second set at zero  $0^\circ$ . A third set, or, as it is commonly called, round, is then taken in a similar way, and the three rounds, if accordant will suffice for that zero; if the discrepancies in the values of any angle exceed  $2''$  another measure of that angle must be taken. It is by no means a matter of indifference (except at the centres of polygons) which station is selected as the zero station. The left and right hands should be taken alternately at the successive stations for reasons given at pages XII to XVII of the Introduction to the Great Indus Series, Volume III.

22. When the signals to be observed lie all round the station observed at, as at the centre of a polygon, the observer must be careful to complete each swing by intersecting the same signal as he commenced with. Thus if he begins with *B* in going round from left to right he will end with *B*, and again beginning with *B*, having first overshoot it, he will swing round from right to left and end on *B*.

23. Now turn the telescope through  $180^\circ$  in a vertical plane, and round  $180^\circ$  in azimuth, so that if the face of the vertical circle were previously to the left hand, it will now be to the right hand; *B* will then read  $180^\circ$ , and this is called zero  $180^\circ$  *FR*, the former position being zero  $0^\circ$  *FL*, *i.e.*, face left. Proceed as before, and take three sets of observations, the motion of the instrument being in one set continuous from left to right, in the next from right to left, and in the third from left to right.

24. Having thus obtained six sets of observations for one position of the instrument a "change of zero" must be made, *i.e.*, the horizontal limb must be turned round through some definite arc, thus altering the readings of all the stations by that amount. In this position also six sets are to be secured when a further "change of zero" and another six sets are to be taken, as far as is desirable. The angle through which the limb is shifted at each change of zero and also the number of changes vary with the different classes of instrument. The following table gives the details for the three kinds of Theodolite commonly employed. Full information on this subject is given in Volume II, Chapter IV, to which the reader is referred.

Zero changes.

Five Microscope Theodolite:—

$$\frac{0^{\circ} 0'}{180^{\circ} 0'}, \frac{79^{\circ} 12'}{259^{\circ} 12'}, \frac{158^{\circ} 24'}{338^{\circ} 24'}, \frac{237^{\circ} 36'}{57^{\circ} 36'} \text{ and } \frac{316^{\circ} 48'}{136^{\circ} 48'}$$

Three Microscope Theodolite:—

$$\frac{0^{\circ} 0'}{180^{\circ} 0'}, \frac{70^{\circ} 1'}{250^{\circ} 1'}, \frac{140^{\circ} 2'}{320^{\circ} 2'}, \frac{210^{\circ} 3'}{30^{\circ} 3'}, \frac{280^{\circ} 4'}{100^{\circ} 4'} \text{ and } \frac{350^{\circ} 5'}{170^{\circ} 5'}$$

New 12-inch Theodolites with two microscopes:—

$$\frac{0^{\circ} 0'}{180^{\circ} 0'}, \frac{210^{\circ} 1'}{30^{\circ} 1'}, \frac{60^{\circ} 2'}{240^{\circ} 2'}, \frac{270^{\circ} 3'}{90^{\circ} 3'}, \frac{120^{\circ} 4'}{300^{\circ} 4'} \text{ and } \frac{330^{\circ} 5'}{150^{\circ} 5'}$$

25. Having described in a general manner the method of observing the angles in a Geodetic Survey, it is necessary now to remark upon the chief precautions necessary to be taken, in order to ensure the greatest degree of accuracy, which the means at our disposal admit of.

Precautions.

26. The observer ought to be skilful, scrupulous, and patient and should cultivate, by incessant practice, the power of vision, and delicacy in handling instruments. The instrument should be kept in good order, and handled with the greatest care. Separate instructions on this head are given below.

Continued.

27. The stability of the foundation of the pillar and due isolation of the instrument are essential elements of accuracy, as already remarked in para. 15. The instrument should also be carefully centred over the station mark, and made truly level, for effecting both of which purposes appropriate apparatus is attached to every instrument.

Continued.

28. The operation of levelling is thus performed. Let the level on the body of the instrument be placed parallel to a line joining two foot-screws, and, by means of these, bring the bubble to float in the centre of its tube. Then turn the instrument round  $180^{\circ}$  in azimuth, and if the bubble continues to float in the centre, that diameter of the limb which is parallel to the line joining the two foot-screws must be truly level; otherwise half the difference of the reading at each end of the bubble is the error,

Levelling.

which ought to be corrected by the foot-screws, and the other half difference of the reading is the error of the level itself. The latter error need not be corrected at all, if it amounts only to a few divisions, because it can always be allowed for in performing the adjustment. For instance, if the right hand end of the bubble reads  $a$  divisions, and the left hand end reads  $b$  divisions, in one position of the instrument, and after a semi-revolution in azimuth the same ends of the bubble read  $a'$  and  $b'$ , then  $\frac{a + a'}{2}$  and  $\frac{b + b'}{2}$  are the readings which the bubble ought to have when the instrument is truly level. Now turn the instrument round  $90^\circ$  in azimuth, and, by means of the third foot-screw, bring the bubble to read  $\frac{a + a'}{2}$ ,  $\frac{b + b'}{2}$ . The diameter at right angles to the former one will now be approximately levelled, and if this operation be repeated two or three times, the adjustment will be perfected. The process should always conclude with the third foot-screw. If the error of the level be very large, that is to say, if the reading of one end of the bubble differs considerably from the reading of the other, it can be rectified by the small capstan-headed screws attached to the level for that purpose. The proper time for effecting the practical correction is when the instrument is nearly level; but if the error be small, it is better, as before stated, to leave it alone, and allow for it in levelling, because the materials of the instrument have a tendency to settle into a position of equilibrium, in which they will remain steady, but, if frequently disturbed, the screws will in time work loose, and no confidence can then be placed in the permanence of the adjustments.

29. The apparatus attached to the level for the purpose of adjusting it in due relation to the axis of the instrument, is very different in various instruments; for, in some, there may be a single screw at one end for raising or depressing that extremity of the level; in other instruments there are two antagonizing screws at one end of the level, and one must be released before the other is tightened.

30. In most modern instruments, however, there are three screws at each end, of which the central of each set is a drawing screw, and the two external ones are pushing screws. In this case, if one end of the level requires

*Continued.*

lowering, release the central screw, and tighten the external ones. On the contrary, for raising that end of the level, release the external, and tighten the drawing screws. It is a general rule, in working antagonizing screws, that is to say, screws that produce opposite results, that one should be released before the other is tightened, otherwise the screws are liable to be bent, or the threads broken, and the efficiency of these delicate instruments to be completely destroyed.

31. The criterion of the instrument being level, is, that the reading of each end of the level remains constant during a complete rotation in azimuth. The accuracy of the adjustment should therefore always be tested before final angles are taken, by reading the level in a position parallel to two foot-screws, and then successively at  $90^\circ$ ,  $180^\circ$  and  $270^\circ$  in azimuth. A difference of about one division may be considered of no account in taking terrestrial angles, but the most scrupulous care must be taken in levelling the body of the instrument when stars, or other elevated objects, are observed.

32. The process of levelling is liable to disturb the centering of the instrument over the station mark, which should therefore be duly looked to. Instruments are centered either by means of a plummet, or by a look-down telescope.

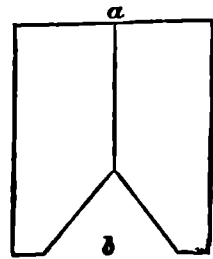
33. The point of suspension of the plummet may, perhaps, not coincide precisely with the centre of the axis; therefore, if the plummet revolves with the instrument when the latter is turned round in azimuth, the centre will not agree after a semi-revolution, and must be corrected for half the difference. If the point of suspension of the plummet is not attached to the axis of the instrument so as to revolve with it there is no method of correcting the eccentricity.

34. When a look-down telescope is employed for centering, the instrument must first be duly levelled, the eyepiece of the look-down must be adjusted to distinct vision of the wires, and the object-glass adjusted to distinct vision of the station mark, so as to be free of parallax. Now, by means of the centering screws, move the instrument till the station mark is duly bisected by the cross wires. Then turn the instrument

round  $180^\circ$  in azimuth, and if the station mark is seen on one side of the wires, half the difference is the error of centering, the remainder being the error of collimation of the wires. Adjust the wires accordingly, and also the centering, after which relevel, and repeat the process, which will generally suffice to perfect the adjustment. The advantage of a look-down telescope over the plummet is two-fold;—first, it is not disturbed by currents of wind; secondly, the adjustment is more minutely performed, by reason of the magnifying powers of the telescope. This latter advantage is, however, more apparent than real, as it is of no use to centre the theodolite more accurately than the signals observed.

35. The body of the instrument being duly centred and levelled, the next care of an accurate observer is to render the transit axis truly horizontal. The telescope of every theodolite, with its attached vertical circle, is supported on a horizontal axis, the ends of which, called “pivots,” rest in angular supports, termed **V**s. The imaginary line, joining the centre of the pivots, is the axis on which the telescope rotates, and, in order that the latter may describe true verticals, the axis must be truly horizontal. This adjustment is effected by raising or lowering one of the **V**s by the appropriate screw, placed beneath the **V** for that purpose,—the amount of adjustment being regulated by the indications of a riding or striding level, which must be placed upon the pivots during the process of levelling them.

36. The riding level is furnished with two feet, which are cut into the shape of notches, or inverted **V**s, and there is either a single screw or two antagonizing screws at one end of the tube, whereby to adjust the bubble to parallelism with the line of bearing of the feet. There is also generally attached to one end of the riding level a little cross level, by means of which the inverted feet **V**s are made to rest always with the same points on the pivots, and any slight discrepancy in the verticality of the feet **V**s is thereby prevented from having any effect. If the figure in the margin represent an inverted foot **V**, and *a b* be the line bisecting the angle of the **V**, then it is clear that if *a b* be truly vertical, each side of the **V** will rest



equally on the pivot; but if the similar line  $a' b'$  of the other foot be not truly parallel to  $a b$ , then, on a slight declination of the line  $a b$ , the level will rest on the pivots by three points only, instead of four, and the bubble will, by reason of the obliquity of the  $\mathbf{V}$ , run up towards one end. The remedy for this evil is to render the two feet of the level truly parallel with respect to the imaginary lines bisecting the inverted  $\mathbf{V}$ s. For this purpose release the screws which attach the feet to the tube of the level, and gently turn the feet until, by successive trials, it is found that the bubbles remain stationary, even when the level is inclined a little on one side or the other of true verticality, as respects the bisecting line of the  $\mathbf{V}$ s. The proper bearing on the pivots will thus be maintained, and the riding level will give true indications, even without the aid of a cross level.

37. To level the transit axis—Uncover the pivots by removing the clips which retain the pivots in their  $\mathbf{V}$ s. In-

*Continued.*

roduce the riding level gently between the radii of the vertical circle, and place it upon the pivots. Adjust it to the cross level, if it have one, and read off the ends of the bubble. In order to distinguish one end of the bubble from the other, it is usual to call the end next the little cross level  $L$ , and the opposite end  $O$ ; but if there be no cross level, it is as well to mark the letters  $L$  and  $O$  on the tube, as characteristic marks. It is also usual to call the divided face of the vertical circle  $F$ , and the other face  $P$ . Now, suppose, in the first instance, that the  $L$  end of the level is at  $F$ , and the readings of the bubble in that position are  $l$  and  $o$ , gently take off the riding level, and reversing it, end for end, replace it on the pivots, so that  $L$ , which before was at  $F$ , will now be at  $P$ ; read off the bubble, and let the readings be  $l'$  and  $o'$ , then half the difference between the reading of the same ends in the two positions will be the inclination of the transit axis, to be adjusted by the proper screw under one of the  $\mathbf{V}$ s, whereby that  $\mathbf{V}$  may be raised or lowered. If  $\frac{l - l'}{2}$  be greater

or less than  $\frac{o' - o}{2}$  the difference is occasioned by unequal expansion of the level, and the mean must be adopted as the true error. The other half difference of the readings is the error in the level itself, which, if it amount to any considerable quantity, may be practically corrected by the appropriate screws attached to the level.

38. It is inadvisable to alter the inclination of the transit axis for an error amounting only to four or five seconds, because frequent tampering with the screws will in time loosen them, and destroy the permanency of the adjustments, the stability of which is a most important circumstance in making observations, because it is of no avail to render an instrument free from error at the commencement of a set of observations if the adjustments do not continue permanent throughout.

39. The value of the divisions of the scale of all the levels employed should be determined either at the Mathematical Instrument Office, Calcutta, or at the Trigonometrical Branch Office, Dehra Dún, by an instrument called a bubble tester, before the party takes the field; but if for any reason this is impracticable, it may be ascertained by affixing the level to the frame of the vertical circle, or making it ride parallel to the telescope, and then taking the readings of the microscopes in two positions of the bubble, whence comparing the number of divisions of the level scale run over by the bubble with the corresponding angular motion of the vertical circle, as measured by the microscopes, the value of one division of the level scale will be obtained by simple proportion, as shown in the following example:—

| Temperature. | VERTICAL MICROSCOPES. |      |            | Angular Difference. | LEVEL.    |          |              |          |          | Computed value of one division of the Level Scale. |
|--------------|-----------------------|------|------------|---------------------|-----------|----------|--------------|----------|----------|----------------------------------------------------|
|              | D.                    | E.   | Mean.      |                     | Readings. |          | Differences. |          |          |                                                    |
|              |                       |      |            |                     | L.        | O.       | L.           | O.       | Mean.    |                                                    |
|              | 0' "                  | "    | 0' "       | "                   | <i>d</i>  | <i>d</i> | <i>d</i>     | <i>d</i> | <i>d</i> | "                                                  |
|              | 7 25 47'0             | 41'0 | 7 25 44'00 | ...                 | 85'6      | 48'4     | ...          | ...      | ...      | ...                                                |
|              | 7 26 20'5             | 14'1 | 7 26 17'30 | 33'30               | 50'9      | 83'0     | 34'7         | 34'6     | 34'65    | 0'9610                                             |
|              | 7 26 12'1             | 6'0  | 7 26 9'05  | 8'25                | 59'9      | 73'9     | 9'0          | 9'1      | 9'05     | 0'9116                                             |
|              | 7 25 62'1             | 56'2 | 7 25 59'15 | 9'90                | 70'1      | 63'8     | 10'2         | 10'1     | 10'15    | 0'9755                                             |
|              | 7 25 52'2             | 45'0 | 7 25 48'60 | 10'55               | 81'0      | 53'1     | 10'9         | 10'7     | 10'80    | 0'9769                                             |
|              | 7 25 44'2             | 37'3 | 7 25 40'75 | 7'85                | 89'9      | 44'3     | 8'9          | 8'8      | 8'85     | 0'8870                                             |
|              | 7 26 11'1             | 4'9  | 7 26 8'00  | 27'25               | 61'2      | 73'0     | 28'7         | 28'7     | 28'70    | 0'9495                                             |

5'6615

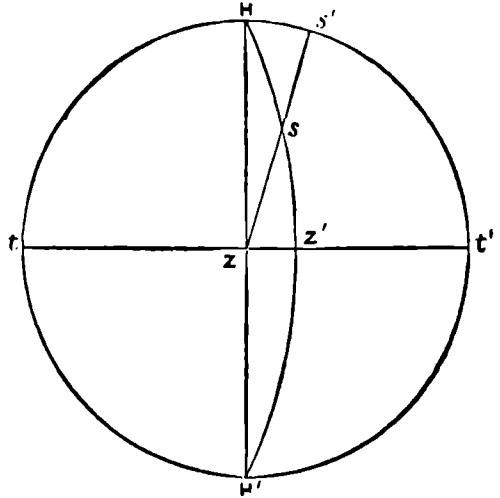
Mean value of one division of the level scale, ... .. 0'9436



40. If in the figure in the margin, which represents an orthographic projection on

Effects of dislevelment.

the plane of the horizon,  $Z$  be the true zenith, and  $t t'$  be the direction of the transit axis, then, when that axis is level, the telescope will rotate in the vertical plane  $H Z H'$  at right angles to  $t t'$ , and when the telescope is elevated  $90^\circ$  it will point to  $Z$ , the true zenith of the place. But if the transit axis be inclined to the plane of the horizon, the telescope will decline from the point  $Z$  by an angle  $Z Z'$  equal to



the amount of the dislevelment of the axis. The telescope will now rotate in an oblique plane  $H Z' H'$ , and if  $S$  be an elevated object, the telescope will refer its position to  $H$  on the horizon, instead of to  $S'$ , which is the true horizontal point appertaining to  $S$ , as determined by the true vertical  $Z S S'$  passing through  $Z$  and  $S$ . The angle  $H Z S'$ , or corresponding arc  $H S'$  is the error in azimuth occasioned by the dislevelment of the transit axis, and in the spherical triangle  $S Z Z'$  right angled at  $Z'$ , we have  $\cos S Z Z' = \tan Z Z' \cot Z S$ , or as the arcs  $Z Z'$  and  $H S'$  are both small, amounting in fact to a few seconds, we have

$$H S' = Z Z' \times \tan \text{altitude.}$$

41. This formula clearly shows that when the altitude is  $0^\circ$ , an inclination of the transit axis produces no error

Continued.

in the azimuth readings; and also the error in azimuth must be very small when an object is little elevated, as is the case with terrestrial objects. It is also evident that the error lies in the direction in which the pivot is higher, so that if the left hand pivot be higher, the error will be *minus*, and corresponding correction *plus*; but, on reversing the face of the instrument, the pivots change positions with respect to the observer, and the errors will have an opposite sign; consequently, if the mean be taken between an observation face left, and another face right, the influence of an inclination in the axis will be entirely eliminated, and this will also happen with regard to depressed objects, in which the signs of the errors will be opposite

to those in the case of elevated objects. Hence, if angles be taken between an elevated and a depressed object, those on a single face will be burdened with the sum of the errors due to the depression and elevation, but the mean of the angles between face left and face right will be free from error.

42. The next adjustments to be undertaken are those of the telescope itself, which are four in number, and all have reference to the wires, or ocular lines, situated near the eye end. These adjustments are,—

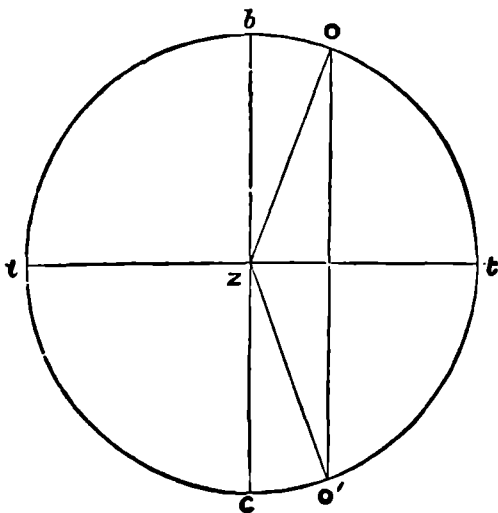
- 1st—Distinct vision of the wires,
- 2nd—Freedom from parallax,
- 3rd—Collimation in azimuth,
- 4th—Verticality of the vertical wires, and horizontality of the horizontal wire.

43. The only adjustment of a personal nature is that for distinct vision of the wires, which varies with the focus of the observer's eye, and the eye-piece must therefore be pushed in or drawn out of its cell, until distinct vision is obtained. It is advisable to direct the telescope to the sky, or hold a piece of paper obliquely in front of, and a short distance from, the telescope, so that the wires being projected on a blank field, may be viewed by the eye, undistracted by other objects. The wires will be truly in the focus of the eye-piece when they appear sharply defined, and all the little specks of dust on them are seen clear and distinct.

44. Now direct the telescope upon a distant object, a heliotrope for instance, which bisect with the vertical wire. Move the eye gently to one side, and if the object still appears bisected there is no parallax, and the wires are truly in the focus of the object-glass. If, however, on moving the eye to one side, the image of the object appears to move with the eye, then the focus of the telescope lies beyond the wires, and the object-glass and wires must be approximated to each other: this is called far parallax. If, on the other hand, the image appears to move in a contrary direction to the eye, the imperfection is called near parallax, and may be rectified by augmenting the distance between the object-glass and the wires: this adjustment is effected in some instruments by moving

the object-glass, in others by moving the tube which contains the wires. Small instruments usually have a pinion and rack to regulate the adjustment of the focus, but this is not the case with the larger class of instruments.

45. The line of collimation in a telescope may be defined to be the line joining the centre of the wires and the centre of the object-glass. In a theodolite or alt-azimuth instrument, the telescope rotates on the transit axis, and the line of collimation in a telescope thus mounted should be at right angles to the transit axis, otherwise, it will not describe a great circle, when the transit axis rotates. For, let  $tt$  and  $oz$  be the directions of the transit axis and line of collimation respectively; and suppose the line  $oz$  to rotate round the axis  $tt$  previously made truly level, it is clear that it will not pass through the zenith at all, but after a semi-revolution the point  $o$  will arrive at  $o'$ . The line of collimation will in fact describe a cone, the base of which will be a small circle parallel to the vertical circle, which intersects the transit axis at right angles.



46. Having defined what the line of collimation is, the next thing to be considered is, the practical method of adjusting it. There are several ways in which this adjustment may be effected, and of these the following is the one usually adopted in the field with moderate-sized instruments.

47. Direct the telescope to a distant fixed object which should be nearly on the observer's level, and bisect it with the wire. Then lift the telescope out of the  $\mathbf{Y}$ s, and replace it with reversed pivots, so that the right hand pivot may now be at the left hand. This must be done with delicacy, taking care not to shake the instrument, as the success of the observation depends

on the  $V$ s remaining unmoved. If the line of collimation be true, the object will appear bisected in the second position of the telescope, otherwise it will be seen to one side of the wires, and half the apparent is the real error to be corrected by the appropriate screws at the eye-end of the telescope. Two or three repetitions will suffice to perfect this adjustment, and when it has once been effected, it should not again be tampered with unless the error of collimation amounts to more than 3" or 4".

48. It is very clear that, by pursuing the process described in the foregoing paragraph, the amount and direction of collimation can easily be measured on the limb of the theodolite, and may be used as an element of reduction, instead of practical adjustment. All that is necessary is, after intersecting the distant fixed object, to read off the microscopes, then lifting out the telescope, and replacing it with reversed pivots, again bisect the object, and read off the microscopes. One half the difference of the readings will be the error of collimation, and its sign may be known from the following consideration:—Suppose the instrument, in the first instance, to have stood face left, with the pivots in their usual  $V$ s, and suppose the readings in that position to have been in excess of those obtained after reversal, then the error of collimation is + as respects all observations on face left, and – as respects those on face right. Consequently, in applying the reductions dependent on this error, the corrections will have opposite signs, *viz.*, – for face left, and + for face right.

49. If the distant fixed object, by means of which the collimation has been determined, is situated above or below the horizon, the collimation error thus obtained must be reduced by multiplying it by the cosine of the elevation or depression of the object observed. On the other hand, the amount of the collimation error at the horizon being known, the azimuthal error at any altitude may be obtained by multiplying by the secant of the altitude.

50. This clearly shews that an error of collimation produces its least effect on azimuthal readings, when the altitude of the observed object is nothing. It is moreover evident that the system of observing with the face of the instrument alternately left and right must necessarily eliminate the effect of the error, because it lies in contrary directions on the two faces.

51. The permanence of the direction of the line of collimation is a most important consideration, as any change during the course of the observations will prevent the errors cancelling each other. It is found in practice that the line of collimation is liable to a slight fluctuation of a few seconds, on account of the unequal expansion of the metal composing the telescope, but the errors incidental to this fluctuation have a tendency to compensate during a long series of observations.\* In observing stars for azimuth, the greater their elevation, the greater is the error produced in the resulting azimuth by unadjusted collimation.

Effects of erroneous collimation.

52. The previously described method of adjusting for collimation is that which is usually practised with moderate-sized instruments; but it was found inapplicable to the large theodolites. The difference of the readings on opposite faces, as obtained by observing an object alternately face left and face right, gives double the error of collimation, and such is the accuracy with which these instruments are graduated, that its amount and direction can always be inferred with great confidence from the regular observations, and may easily be corrected, when required, by the appropriate screws at the eye-end of the telescope. This method, however, does not appear to be altogether satisfactory for instruments with three microscopes or verniers, because the microscopes, on reversing the face, will stand  $60^\circ$  apart from their previous positions, thus embracing the greatest change in zero which the instrument is capable of, and introducing the full amount of uncertainty due to graduation. Another objection to this method in instruments which are not provided with a full vertical circle, is the extreme difficulty of reversing the pivots of a large heavy instrument without some jar or shake, the occurrence of which there is no means of determining, and which, if it exists, vitiates the whole observation.

Inconvenience in applying the above method.

53. Gauss's method of collimating is free from the foregoing objections, being equally independent of errors of graduation, and of the practical difficulty of reversing pivots. The principle of it is as follows:—Suppose two telescopes each fitted with a cross of spider lines at the solar focus in

Gauss's method of collimating.

\* As a precaution against unequal expansion, it is proper to clothe the telescope and axes with woollen cloth, and to protect the instrument from cold currents of air, &c., and from the prejudicial effects of solar rays.

the ordinary way, be placed approximately in the same straight line, object-glass to object-glass. An observer looking through one telescope will then see, by a well known optical principle, the cross in the other as well as in his own, and by a slight motion of either telescope the two crosses may be made to coincide. The lines of collimation of the two telescopes are then exactly parallel (if not coincident) and if produced indefinitely will cut the celestial sphere at the two opposite poles of a great circle. Now suppose the telescope whose collimation is to be tested to be placed somewhere between these two, sufficient room being left for this purpose, with its line of collimation parallel to theirs, it is evident that if its cross coincides with that of one of the collimators in one position it will, if properly collimated, coincide with that of the other after it has been rotated  $180^\circ$  round the transit axis.

54. The practical execution of this method of collimating consists

*Continued.*

in placing two small theodolites, one on each side of the instrument to be collimated, so that all three instruments shall be nearly in the same straight line.\* To effect this, set the telescope of the centre instrument level, and place a theodolite so that its wires may be seen on looking through the centre instrument. Now turn the centre telescope  $180^\circ$  in a vertical plane, and place the other theodolite so that its wires may also be seen. Then remove the centre telescope, by lifting it out of the Vs, whereby the two external telescopes may be adjusted upon each other, so that their wires shall mutually intersect; after this they are not to be disturbed. Replace the centre telescope, and directing it upon one of the small theodolites, intersect the image of its wires and then turn over  $180^\circ$  in a vertical plane, so that the telescope may now point to the other theodolite; and, if the wires of the latter are found to be also intersected, the line of collimation is truly at right angles to the axis of rotation, otherwise one half the apparent is the true deviation to be adjusted, or measured by the microscopes, as the case requires. It is obvious that this method, with a little modification, may be made applicable to collimation in altitude†.

\* A few tenths of an inch are not important because it is the parallelism of the rays, and not the exact centering of the object-glasses, upon which the method depends.

† For example: By means of the vertical tangent screw intersect the wires of one of the auxiliary theodolites, turn round  $180^\circ$  in azimuth, and the telescope will point to the other auxiliary theodolite. One half the apparent deviation in altitude will be the error of collimation to be adjusted by the screws acting on the wires. After which the micrometers should be adjusted to zero. This method is seldom employed, as with an altitude and azimuth circle, the index error can easily be corrected by means of observations taken on both faces as will be explained below.

55. The distance between the telescopes is necessarily limited by the fineness of the wires and the magnifying powers in the auxiliary theodolites, the wires of which are not likely to be visible at a greater distance than ten or twelve feet; consequently, they must be placed about five feet from the telescope to be collimated, or at any distance greater or smaller, which on trial may be found suitable. To render the wires of the latter distinct, an assistant should reflect light into its eye-piece, and for this purpose a slip of paper held obliquely will answer very well. The wires are seen more plainly when plain glass is substituted for the eye-piece of the telescope.

56. The fourth adjustment in the telescope referred to in para. 42 consists in rendering the wires in the telescope perfectly vertical and horizontal. The systems of wires used in the focus of telescopes are very various, a common form consists of a single vertical crossed by a horizontal wire, which is the best for the principal triangulation, as the objects observed are generally luminous points. For observing flag-staves, spires, and other perpendicular objects, a St. Andrew's cross is employed in connection with one horizontal wire. As the intersection of three lines, however, occupies a large undefined space, it is preferable that the horizontal wire should not pass through the intersection of the oblique lines, but form a small triangle therewith. In this case horizontal angles should be observed to the centre of the cross and vertical angles on the horizontal wire. A system of wires consisting of five, seven, or more vertical wires and one horizontal wire (or occasionally two very close ones) is usually applied to astronomical circles. These wires are generally fixed at a proper angle on a perforated plate, which admits of a small play for the purpose of adjustment. To verify the horizontal wire all that is necessary is to set the telescope after levelling it on a distant steady defined object near the horizon, and move the instrument in azimuth, when the object should appear to move along the wire from one extremity of the field to the other. To adjust the vertical wire all that is necessary is to move the telescope in altitude, so that a distant object may appear to traverse or run along the vertical wire. If the object does not remain intersected at all parts of the field, the error ought to be rectified by moving the wire-plate in the appropriate direction. The relative position of the wires, with respect to each, other having been fixed by the maker, it is only possible to

adjust one wire by means of the wire-plate, but if there be a micrometer at the eye-end, its wire can generally be adjusted separately.

57. The wires should be stretched tightly, for if they be slack

Method of wiring diaphragms. their intersection will not remain permanent during the revolution of the telescope on its axis. As wires are liable to be broken, to become slack by damp, or uneven by accumulated dust, it is necessary that every surveyor should be able to replace them when required. The best substances to apply to large instruments are spider lines, and to small ones, the fibres of raw silk. To procure spider lines, prepare some card frames, as shewn in the margin. Next look out in the garden, or among trees and bushes, for some healthy spiders, and take one up on the edge of the card frame. Then gently shake the frame, to detach the spider, which will hang therefrom, as shewn in fig. 1. Wind up the fibre, so that the turns may be rather widely apart, *vide* fig. 2. When the end of the card is reached, make a notch (*a*), in which insert the end of the thread, and then cut off the rest with a pair of scissors. If more cards are wanted, do not let the spider fall to the ground, where it would be covered with dust, but lay hold of the line at *b*, and place it in the notch of another card, which wind up as before. The cards should afterwards be placed between leaves of clean paper, to preserve them from dust. Having obtained the fibres, they may be fixed as follows:—Take out the wire-plate, remove the old wires, and clean the varnish from the engraved cuts. This must not be done by scraping with a knife, but by using spirits of wine and warm water, wiping the old varnish off, until the cuts appear quite clean. Now take a card, examine the fibre with a magnifying glass, and select a clean uniform piece. Take two little balls of wax, which attach to the extremities of the selected fibre, and cut off the remainder. If one of

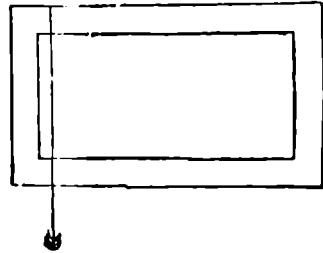


FIG. 1.

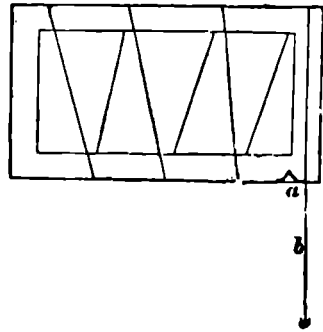


FIG. 2.

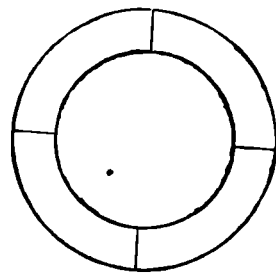


FIG. 3.



the balls of wax be held in the hand, and the other be allowed to hang freely, the fibre will become straight and untwined. Now take a camel's hair brush, dipped in clean water, and rub the fibre gently, for the purpose of cleaning and damping it. Put the wire-plate upon a little block of wood, *vide* fig. 5, and place the fibre upon it, taking care to examine with a magnifying glass that the wire falls into the proper cuts. The two little wax balls hanging on either side serve to stretch the thread, and keep it in its place. The cross wire is prepared in a precisely similar manner, and as many more as may be required. Then, taking care that they are all truly adjusted in their respective lines or cuts, let a drop of varnish fall upon each cut, and put a tumbler over the apparatus, to protect it from dust. In twenty-four hours the varnish will have set, the ends of the



FIG. 4.

fibres may then be cut off, and the wire-plate carefully replaced in the telescope. This last is the most difficult part of the undertaking, because it requires very delicate handling to replace a wire-plate without breaking a wire, the method of proceeding may however be acquired by patience and practice. The best varnish to use is copal, but sealing wax dissolved in spirits of wine, friar's balsam, or laudanum, will answer. Fibres of raw silk are most suitable to small instruments, and these can easily be applied by following precisely the same rules. The previous damping of the fibres ensures their becoming tight and well stretched when dry. The best time to apply fresh wires is during the rains, when there is no dust flying about, and the atmosphere is rather damp. All the instruments should be carefully examined at that time, and those which require it should be fitted with fresh fibres. Freedom from dust also renders this season the most favorable for cleaning the axis and other working parts of the instrument which require to move glibly.

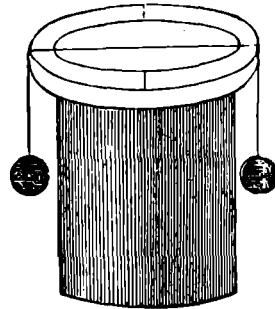


FIG. 5.

58. The next subject for consideration is the azimuthal microscopes, upon the proper adjustment of which the value of the azimuthal observations greatly depends.

Adjustment of microscopes.

59. When a circle is graduated into spaces of 5' each, the rough reading of the degrees and fractional parts of the degree, as far as to the last 5' space, is performed by an index having a single stroke upon a piece of metal, which is adjustable, and generally fixed on the clamp-plate. The remaining minutes and seconds are shewn by the micrometer microscopes, of which there are generally three or five attached to the body of the instrument in old theodolites and two in the modern ones. These are firmly affixed to radiating arms, and are placed at equal intervals round the circle. The reading microscope, as now constructed, is a species of compound microscope consisting of three lenses, one of which is the object lens, and the other two form a positive eye-piece; the amplifying lens being omitted, as the field is not required to be extensive, and the measure is made near its centre. When firmly fixed and truly adjusted it is capable of sub-dividing the minute into seconds and fractions of a second with great accuracy and facility. It admits of the divisions on the limb being well illuminated, and does not injure them by friction, as happens with the vernier, over which it possesses all the advantages of convenience, combined with high magnifying power and micrometrical nicety of measurement. Its value, however, is entirely dependent on its being kept in proper working order and true adjustment, for which reason a knowledge of the principles of its construction is essential to an observer who aspires to great precision.

60. The arc usually measured by a microscope is a divided space of the value of 5', to be sub-divided into 300" by five revolutions of the micrometer-screw, which therefore has its circular head graduated and numbered from zero by 10", 20", 30", &c., up to 60", every fifth stroke being longer than those of the other graduations, and terminating with a lozenge. Now, in order that the screw may be competent to measure a 5' space without excess or defect, it is clear that the image of the latter must be magnified so as to occupy precisely the length of five revolutions of the screw. Suppose that an object (the divided limb of the circle for instance) is placed exactly at the solar focus of the object lens, the rays passing through the lens become parallel, and the image will be formed at an infinite distance; but if it be placed beyond solar focus, an image is formed within the tube, and the two points where the divided limb and its image are situated, are called conjugate foci, the latter of which recedes upward as the other approaches the object lens. If we call the distance of the limb from the lens  $f$ , and

the distance of its image from the same lens  $F$ , the length of the image will exceed that of the object in the ratio of  $F$  to  $f$ , or  $\frac{F}{f}$  will represent the magnified state of the image. Hence it is manifest that the expression  $\frac{F}{f}$  will have an increased value if we either augment  $F$  or diminish  $f$ . This is the fundamental principle to be attended to in adjusting the *runs*, *viz.*:—If the object-glass be protruded, it will approach nearer to the limb, whereby the size of the image will be increased and will require more traverse of the micrometer-screw of the microscope to measure it, and this will also be the case if the whole microscope be made to descend towards the limb.

61. Before describing the method of performing the adjustments, it may be necessary farther to premise that the microscope is in correct adjustment when the following conditions are strictly fulfilled, *viz.*, when the image of the divided limb and the micrometer wires are so distinctly visible together that no parallax can be detected by varying the position of the eye, in which state of good vision, five revolutions of the screw must exactly measure one of the 5' spaces of the limb, or ten revolutions measure two 5' spaces.

62. To obtain distinct vision of the wires, place a slip of white paper on the limb, and illuminate it by means of the reflector, the wires will thus be projected on a blank field, in which state the eye is better able to judge of their distinctness. Now slide the eye-piece in its cell, until the wires appear sharply defined. This is a personal adjustment, and varies with the focal length of the observer's eye.

63. If, on removing the paper, the limb is not visible, the microscope must be moved bodily up or down,\* till the limb is seen distinctly. The means of effecting this differ in the various classes of instruments in use, and cannot therefore be described in detail here; but no difficulty in manipulation is likely to arise. Having rendered the lines on

\* It can easily be found out whether the microscope requires to be lowered or raised to produce distinct vision, because, if, on thrusting in the eye-piece, distinct vision is produced, there is far parallax, and microscope must be lowered, and *vice versa*. After this experiment the eye-piece must be carefully restored to the position adapted to distinct vision of the wires.

the limb visible simultaneously with the wires, bring one of the divisions to the centre of the field, and intersect it with the micrometer wire. Now move the eye to the left or right, and if the intersection remain steady, there is no parallax; but if the object appears to shift its place with respect to the wires, then notice in which direction it moves and correct the parallax according to the instructions given in para. 44.

64. The microscope having thus been freed from parallax, it will be necessary next to place a division of the limb,  $0^{\circ}$  for example, exactly under the centre of the microscope, and then to measure with the micrometer the number of revolutions and divisions between  $359^{\circ} 55'$  and  $0^{\circ} 5'$ . If due care be taken to note the way in which the graduation on the micrometer head runs from 0 to  $10''$ ,  $20''$ ,  $30''$ , it will be found that one cut or stroke on the limb will be arrived at by revolving the micrometer in the same direction with the numbering of the seconds on the head, while the other stroke will be reached by a motion of the micrometer contrary to the order of graduation.\* Subtract the reading of the latter from the reading of the former, and the difference will be the number of minutes and seconds measured by the micrometer. If this fall short of  $10'$ , the magnifying power is too small, and the object-glass must be protruded, in order to augment the image; but if the measured space exceed  $10'$ , the object-glass must be screwed upwards to diminish the image. Now, in the former case, protruding the object-glass will throw the image higher up the microscope, and produce near parallax, to correct which the whole microscope must again be bodily moved farther from the limb, and this again will diminish the run, so that the first correction should be a little over done to counteract the subsequent adjustment for parallax, and the whole process must be repeated two or three times, until the two conditions, *viz.*, correctness of run and freedom from parallax, are very closely satisfied.

65. It must, however, be remembered that the divisions of the limb are themselves affected by error of graduation, to eliminate the effect of which it is desirable to take a mean of several  $5'$  spaces. Moreover, the plane of

\* Note that the final intersection of a division on the limb with the micrometer should always be made by screwing up against the spring, whether the motion of the microscope as a whole is with or against the order of graduation.

the circle may not be exactly at right angles to the axis of rotation, in which case all parts of the circle will not be at the same distance from the object lens, and there will be a difference of run occasioned by the circle approximating nearer to, or receding farther from, the microscope. It is also to be recollected that the excellence of the observations depends, not on a single microscope, but on the joint operation of all, so that although each may be near the truth, still the sum of their errors may be considerable; whereas, if the errors neutralised each other, the magnitude of the individual errors would not be of so much importance. To produce, therefore, the greatest degree of accuracy, the adjustment must be perfected with due regard to the foregoing considerations.

66. In general, with an instrument that has been previously in use,\*  
 Final correction of one microme- the adjustments will be found already near the  
 ter. truth, and all that is required is occasionally an  
 alteration of the run of a single microscope, so as to compensate the  
 combined errors of the others. Now as the value of one micrometer  
 has only the weight of one-third in producing the mean, so the cor-  
 rection to be applied to an individual microscope must amount to three  
 times the sum of the errors of all the microscopes. For instance, if the  
 mean value of three micrometers amount to  $10' + 2''$  for a  $10'$  space,  
 then  $3 \times 2'' = 6''$  is the correction to be applied to a single microscope  
 to reduce the sum of the whole to  $10'$ , and this correction is much more  
 easily applied than a smaller one to each.

67. The rule to be adopted is therefore as follows:—Set the instru-  
 Rule for obtaining the mean value of all the runs. ment to  $0^\circ$  by the index, and a whole degree  
 stroke will then fall nearly under the zero of  
 each microscope. Now, as the micrometer in observing is limited to  
 less than a  $5'$  range on each side of zero, it is unnecessary to go beyond  
 that extent on either side. It is also immaterial to read the whole degree  
 stroke in the centre, because the sum of the  $5'$  spaces will be given at  
 once by the reading of the extremes; consequently, it will suffice to  
 measure the  $10'$  spaces under micrometers *A*, *B*, *C*, respectively, repeat-  
 ing each measure at least once, for the sake of precision, and to avoid  
 mistakes. Now turn the instrument round  $180^\circ$  in azimuth, and measure

\* Unless the microscopes have been dismantled, or taken off for the purpose of being cleaned, or of allowing the axis to be cleaned, or for any other similar reason.

the 10' spaces as before. The mean value of each microscope will thus be obtained from the mean of 10' spaces at opposite diameters of the limb, and therefore in the case of three-microscope instruments the mean value of the runs of all the microscopes will be got from six 10' spaces at every 60° of the limb apart, or from twelve independent 5' spaces, which should suffice. Now, if the mean error of the microscopes amounts to more than 1" in 10', it will be advisable to correct that microscope, which is most erroneous, by a quantity equal to three times the combined error.\* In performing this adjustment the rules for avoiding parallax (as given in para. 63) ought to be strictly adhered to.

68. The following example will render this subject quite clear, and it is only necessary to add that the runs should be taken at every principal station, before beginning the measurement of the angles, and duly recorded in the angle book, according to the form subjoined.

EXAMPLE.

*Observations to determine the runs of the Micrometers belonging to Harris' 15-inch Theodolite in use with the Karára Meridional Series, 19th April, 1845.*

| Reading of the Index.      | HORIZONTAL MICROSCOPES. |                  |              |                 |                  |              |                 |                  |              | REMARKS.                                                                                                                                                                       |
|----------------------------|-------------------------|------------------|--------------|-----------------|------------------|--------------|-----------------|------------------|--------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
|                            | A.                      |                  |              | B.              |                  |              | C.              |                  |              |                                                                                                                                                                                |
|                            | Lowest Reading.         | Highest Reading. | Differ-ence. | Lowest Reading. | Highest Reading. | Differ-ence. | Lowest Reading. | Highest Reading. | Differ-ence. |                                                                                                                                                                                |
| o                          | "                       | "                | ' "          | "               | "                | ' "          | "               | "                | ' "          | The circle is divided into 5' spaces, and the runs were taken from 5' below to 5' above zero, the reading of the centre division being omitted as of no effect upon the means. |
| o                          | 32'6                    | 24'6             | 9 52'0       | 30'6            | 31'1             | 9 60'5       | 42'5            | 45'8             | 9 63'3       |                                                                                                                                                                                |
| ..                         | 32'7                    | 25'0             | 9 52'3       | 30'9            | 30'8             | 9 59'9       | 42'3            | 46'0             | 9 63'7       |                                                                                                                                                                                |
| 180                        | 30'2                    | 27'9             | 9 57'7       | 34'0            | 36'1             | 9 62'1       | 25'2            | 22'5             | 9 57'3       |                                                                                                                                                                                |
| ..                         | 30'0                    | 28'1             | 9 58'2       | 34'3            | 35'9             | 9 61'6       | 25'4            | 22'2             | 9 56'8       |                                                                                                                                                                                |
| Mean of each,              |                         | 9 55'05          | ...          | ...             | 9 61'03          | ...          | ...             | 9 60'28          |              |                                                                                                                                                                                |
| Mean of three Microscopes, |                         | ...              | ...          | ...             | ...              | ...          | ...             | 9 58'79          |              |                                                                                                                                                                                |
| Error on 10' run,          |                         | ...              | ...          | ...             | ...              | ...          | ...             | 0 1'21           |              |                                                                                                                                                                                |

\* This correction, if the amount be only a few seconds, will not require the object-glass to be disturbed, because it will generally suffice to raise or lower the whole microscope, until the required value of the runs is correctly obtained. The rationale of this proceeding is that the microscope is more sensitive, as regards the value of the run, than it is with respect to parallax, or distinct vision. On account of the spherical aberration of the lenses, the exact position of the point of distinct vision is undefined to a small extent, and within this small limit the run may be sensibly varied without producing any appreciable parallax. This peculiarity will perhaps be rendered clearer by applying numerical values to the expression  $\frac{F}{f}$  which represents the magnified state of the image. Let  $\frac{F}{f} = \frac{4}{1}$ , and let the microscope be raised 0.01 of an inch, which will make  $f = 1.01$ . This will reduce the image in the focus in the proportion of 4 to 3.96, in which ratio the run will also be reduced, consequently as 4 to 3.96 so is 3.96 to 297". Therefore the descent or ascent of such a microscope will produce a variation of 3" for 0.01 of an inch perpendicular rise, while the latter quantity would hardly be perceptible in the shape of parallax.

PRINCIPAL TRIANGULATION.

[PART II.]

EXAMPLE—(Continued).

Adjusted *A* Microscope to read about 3''<sup>6</sup> more, after which the runs were as follows :—

| Reading of the Index.      | HORIZONTAL MICROSCOPES. |                  |                    |                 |                  |                    |                 |                  |                    | REMARKS. |
|----------------------------|-------------------------|------------------|--------------------|-----------------|------------------|--------------------|-----------------|------------------|--------------------|----------|
|                            | A.                      |                  |                    | B.              |                  |                    | C.              |                  |                    |          |
|                            | Lowest Reading.         | Highest Reading. | Difference.        | Lowest Reading. | Highest Reading. | Difference.        | Lowest Reading. | Highest Reading. | Difference.        |          |
| °                          | "                       | "                | ' "                | "               | "                | ' "                | "               | "                | ' "                |          |
| °                          | 20 <sup>5</sup>         | 16 <sup>4</sup>  | 9 55 <sup>9</sup>  | 40 <sup>5</sup> | 40 <sup>9</sup>  | 9 60 <sup>4</sup>  | 52 <sup>4</sup> | 55 <sup>9</sup>  | 9 63 <sup>5</sup>  |          |
| °                          | 20 <sup>6</sup>         | 16 <sup>3</sup>  | 9 55 <sup>7</sup>  | 40 <sup>8</sup> | 40 <sup>9</sup>  | 9 60 <sup>1</sup>  | 52 <sup>5</sup> | 55 <sup>8</sup>  | 9 63 <sup>3</sup>  |          |
| 180                        | 18 <sup>1</sup>         | 19 <sup>6</sup>  | 9 61 <sup>7</sup>  | 44 <sup>2</sup> | 46 <sup>0</sup>  | 9 61 <sup>8</sup>  | 35 <sup>3</sup> | 32 <sup>4</sup>  | 9 56 <sup>9</sup>  |          |
| °                          | 18 <sup>0</sup>         | 19 <sup>8</sup>  | 9 61 <sup>8</sup>  | 44 <sup>8</sup> | 46 <sup>5</sup>  | 9 61 <sup>7</sup>  | 35 <sup>3</sup> | 32 <sup>3</sup>  | 9 57 <sup>0</sup>  |          |
| Mean of each,              |                         |                  | 9 58 <sup>73</sup> | ...             | ...              | 9 61 <sup>00</sup> | ...             | ...              | 9 60 <sup>18</sup> |          |
| Mean of three Microscopes, |                         |                  | ...                | ...             | ...              | ...                | ...             | ...              | 9 59 <sup>97</sup> |          |
| Error on 10' run,          |                         |                  | ...                | ...             | ...              | ...                | ...             | ...              | 0 <sup>03</sup>    |          |
| Error on 5' run,           |                         |                  | ...                | ...             | ...              | ...                | ...             | ...              | 0 <sup>015</sup>   |          |

69. The vertical microscopes are similarly adjusted, and their runs should be taken at every station, and duly inserted in the angle book according to the

Runs of vertical microscopes.

following form :—

| Reading of the Index.    | VERTICAL MICROSCOPES. |                  |                    |                 |                  |                    | REMARKS.                                                        |
|--------------------------|-----------------------|------------------|--------------------|-----------------|------------------|--------------------|-----------------------------------------------------------------|
|                          | A.                    |                  |                    | B.              |                  |                    |                                                                 |
|                          | Lowest Reading.       | Highest Reading. | Difference.        | Lowest Reading. | Highest Reading. | Difference.        |                                                                 |
| °                        | "                     | "                | ' "                | "               | "                | ' "                |                                                                 |
| F. R. °                  | 27 <sup>5</sup>       | 23 <sup>0</sup>  | 19 55 <sup>5</sup> | 32 <sup>0</sup> | 22 <sup>1</sup>  | 19 50 <sup>1</sup> | The vertical circle of this instrument is divided to every 10'. |
| °                        | 28 <sup>0</sup>       | 23 <sup>9</sup>  | 19 55 <sup>9</sup> | 31 <sup>1</sup> | 23 <sup>0</sup>  | 19 51 <sup>9</sup> |                                                                 |
| F. L. °                  | 24 <sup>2</sup>       | 18 <sup>2</sup>  | 19 54 <sup>0</sup> | 34 <sup>0</sup> | 35 <sup>5</sup>  | 19 61 <sup>5</sup> |                                                                 |
| °                        | 24 <sup>7</sup>       | 17 <sup>7</sup>  | 19 53 <sup>0</sup> | 34 <sup>1</sup> | 35 <sup>5</sup>  | 19 61 <sup>4</sup> |                                                                 |
| Mean of each,            |                       |                  | ...                | ...             | ...              | 19 56 <sup>2</sup> |                                                                 |
| Mean of two Microscopes, |                       |                  | ...                | ...             | ...              | 19 55 <sup>4</sup> |                                                                 |

Adjusted *A* Microscope to read about 9'' more after which the runs were as follows :—

|                          |                 |                 |                    |                 |                 |                    |  |
|--------------------------|-----------------|-----------------|--------------------|-----------------|-----------------|--------------------|--|
| °                        | "               | "               | ' "                | "               | "               | ' "                |  |
| F. R. °                  | 15 <sup>2</sup> | 19 <sup>7</sup> | 19 64 <sup>5</sup> | 20 <sup>2</sup> | 11 <sup>7</sup> | 19 51 <sup>5</sup> |  |
| °                        | 15 <sup>6</sup> | 19 <sup>8</sup> | 19 64 <sup>2</sup> | 19 <sup>8</sup> | 11 <sup>5</sup> | 19 51 <sup>7</sup> |  |
| F. L. °                  | 12 <sup>8</sup> | 15 <sup>6</sup> | 19 62 <sup>8</sup> | 21 <sup>0</sup> | 22 <sup>3</sup> | 19 61 <sup>3</sup> |  |
| °                        | 12 <sup>5</sup> | 15 <sup>1</sup> | 19 62 <sup>6</sup> | 21 <sup>4</sup> | 22 <sup>5</sup> | 19 61 <sup>1</sup> |  |
| Mean of each,            |                 |                 | ...                | ...             | ...             | 19 56 <sup>4</sup> |  |
| Mean of two Microscopes, |                 |                 | ...                | ...             | ...             | 19 60 <sup>0</sup> |  |
| Error on 20' run,        |                 |                 | ...                | ...             | ...             | 0 <sup>0</sup>     |  |

70. The foregoing operations are very liable to disturb the lateral adjustment of the microscope, as this is regulated by the three screws, which serve to move the microscope bodily a short space, so as to bring it over its proper division on the limb. After having, by means of these lateral screws, brought the microscope nearly over its proper division, the remaining minute portion of lateral adjustment can be readily effected by moving the comb, for which purpose there is a screw at the reverse end of the micrometer box. If the comb be moved or adjusted in the slightest degree, care must be taken to set the zero of the micrometer head to correspond with the zero of the comb. For this purpose take the milled nut of the micrometer head in one hand, and the graduated head in the other hand, then turn the latter round (taking care not to turn the micrometer screw also), and when zero on the head corresponds with the index on the micrometer, the adjustment is complete. The graduated head only retains its place by friction, therefore it is necessary to be careful in working the micrometer not to lay hold of the graduated head, but to use the milled nut which is given for that special purpose.

71. The microscopes should be fixed  $120^\circ$  apart in a three microscope theodolite, and should coincide with a whole degree stroke when the index is at  $0^\circ$ ; but on account of the errors of graduation, it is advisable, in order that the minutes appertaining to microscope *A* may also answer for *B* and *C*, that the latter should be set to read  $30''$  or  $40''$  in excess, whereby the minute given by *A* will be common to *B* and *C*, provided care be taken to add  $60''$  to the readings of the latter when they fall into the next minute above. Suppose *B* and *C* are set to read  $30''$  and  $40''$  more than *A*, then if the reading of *A* is  $0^\circ 3' 50''$ , that of *B* will be recorded as  $80''$ , and that of *C* as  $90''$ , which number of seconds are referable to  $0^\circ 3'$ . If this precaution be not taken the minutes, and sometimes the degrees, may require to be separately registered for each microscope, because, on account of errors of graduation, readings would frequently be obtained, as shewn in the following example, which would prove inconvenient for registry:—

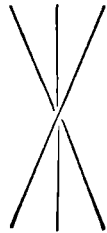
| <i>A</i>          | <i>B</i>             | <i>C</i>             |
|-------------------|----------------------|----------------------|
| $0^\circ 0' 10''$ | $359^\circ 59' 50''$ | $359^\circ 59' 55''$ |
| 62 11 5           | 10 45                | 10 55                |



72. The vertical microscopes must, besides being set diametrically opposite to each other, be freed from index error; that is to say, when the telescope is pointed to an object in the horizon the microscopes must be at zero, or if the telescope be pointed to an object of known elevation the microscopes must be set to give that elevation. The true elevation of an object may be obtained by observations on both faces, the mean of which when corrected for level gives the required elevation.

Adjustment of vertical microscopes.

73. There remain yet to remark a few other points which should be attended to. For instance, it is clear that the micrometers should act in a tangential direction to the circle; therefore, if during the foregoing adjustments the direction of the microscope should by chance be changed, all that is necessary is to turn the microscope gently round in the collar until the micrometer is brought to act in the proper direction, when the collar-screws should be tightened, to render its position permanent. If the wires form a cross, and the divisions on the limb are lines, then the tangential direction ought to be correctly attained, when the angles of the cross wires are truly bisected by the divisions, as shewn in the margin. Other instruments may be differently constructed, but the peculiarities of such cases will suggest the method of placing the micrometer in a tangential direction.



Final tests.

74. It is usual also at this time to take care that each microscope is set over the same part of the division lines, that is to say, the microscopes should be equidistant from the centre of the circle.

Continued.

75. It has been mentioned in a footnote to para. 64, that the intersection of a division should always be made by screwing against the spring. This is done in order that the shoulder of the micrometer head may be made to press steadily on its proper bearing. When the micrometer head is released it would leave its bearing, and the screw would not be retracted but for the action of the spring, upon the efficiency of which the uniformity of the motion of the micrometer during the releasing process entirely depends. It has, however, been found in practice that, owing

Avoidance of lost motion.

to irregularities of friction, or other causes, the spring does not act upon the screw uniformly and instantaneously. The head may be released by a small quantity before motion takes place. This is called lost motion, and occurs in all screws when the action is suddenly reversed. No reliance can therefore be placed on the action of a micrometer, unless the spring and screw are acting in opposition, whereby the bearing of the female screw is preserved, and uniformity and steadiness of action is maintained in immediate obedience to the rotation of the micrometer head.

76. It is not to be supposed, from what has just been remarked, that the micrometer is never to be turned to the left, that is to say, released; because it is clear that, if constantly turned one way, the wire would soon arrive at the extremity of the field. The screwing up motion is indispensable only for the purpose of finishing an intersection. The rule is, to select that line or division which appears to fall nearest to zero in the micrometer, whereby the excursions of the wire are limited within a range of 5'. If the screw requires to be released, in order that the wire may reach the line which it is intended to intersect, then all that is necessary is to overshoot that line, by carrying the wire about a minute beyond it, and then to reverse the motion. Complete the intersection by the screwing up rotation.

77. The graduation of an instrument consists of very fine engraved lines; and it is obvious that if the light falls obliquely on the graduation, one side of the lines will be illumined, and the other will cast a shadow. The apparent centre of the line, will in this case not coincide with the true centre, and if the graduation be coarse, the discrepancy produced may amount to two or three seconds. The obvious precaution to be taken is to throw light perpendicularly on the graduation, taking care that the field is equally illumined, and that both sides of the division lines are seen equally distinct, and sharply defined. Care should be taken in illuminating the field of the telescope at night to avoid any side light upon the wires, and not to alter the illumination between observations of two adjacent signals.

78. No good observations can be expected if currents of air are allowed to impinge on the instrument, or if it is subjected to the influence of solar rays, or

any other causes calculated to disturb the equilibrium of the expansible materials of which an instrument is composed. Independent of this evil, it is also to be remarked that, if the wind is allowed to blow on the telescope, it frequently produces a lateral deflection to the amount of a few seconds. To guard the instrument from disturbing causes of this nature, it is sheltered by a tent, called an observatory tent, the observations being taken through small windows protected by movable *pardahs*.

79. It is not, however, sufficient to be skilful and scrupulous in the manual and optical processes of observing; equal care and attention must be bestowed on the registry of the data in the angle books, otherwise the care and skill bestowed on the observations will be wasted.

Care in recording.

80. The observer should give out the degrees and minutes in an audible voice, and the recorder should repeat them while he is writing them down: unless this precaution be taken, errors may arise from peculiar pronunciation or defective hearing. Similarly, the readings of the micrometer heads should be given out distinctly to the nearest tenth of a second, and duly repeated by the writer. The figuring on the micrometer heads is seldom neatly executed, and a liability sometimes exists to read 30" for 50", and *vice versa*, which mistake in one microscope of a three-microscope theodolite would affect an angle by nearly 7". Errors of this and similar kinds can only be avoided by strict attention. The recorder should examine, when the observations are repeated, whether any reading is discrepant, and should give due notice to the observer of such an occurrence. The relative readings of the microscopes will generally serve to detect any error to the amount of a minute; in the reading of *B* and *C*, for instance, if these be set to read always in excess of *A*, it will be necessary occasionally to add 60" to their indications. The recorder should always take out the angles while the observations are in progress, to enable the observer to note any errors at the time and to repeat a measure, if necessary, according to the precept laid down in para. 21. Some observers take no heed of the results while they are engaged in observing, but make out the angles afterwards, a practice which cannot be too strongly deprecated.

Duties of recorder.

81. The book in which observations are recorded is called an angle book. Two of these books are kept on each series, one of which is used in the observatory and called the "Original Angle Book," the other is a transcript of the former and is termed the "Duplicate Angle Book." These should be paged, indexed and signed at the foot of each page. The original is retained in the Dehra Office and the duplicate ultimately sent to the Calcutta Office.

82. The angle book should on no account be suffered to fall into arrears. The original should be examined by two computers, and attested by their signatures, as well as by that of the observer. The duplicate should be compared with the original by two persons, and attested by their signatures. It is a standing rule, that, in order to avoid mistakes, all computations and comparisons should be performed independently by two persons, who should each sign both copies; and, unless such precautions have been observed, the results are considered untrustworthy as final work. Observations should never be recorded on loose scraps of paper, or in pencil in rough books. The original record is always the most valuable document, and the labour of copying, besides being expensive, causes delay. It is usual to add up all the angles observed on any one zero together, and divide them by the number, by which means as many sets of means are obtained as there are zeros, each mean being considered as an integral observation. It is a fixed rule never to reject an observation, unless there be some obvious error in it, an explanation of which should be inserted in the column of remarks: the circumstance of its differing, however widely, from the mean is not a sufficient cause for its rejection.

83. Having now remarked upon all the minutiae to be attended to in the observatory\*, it still remains to inculcate the necessity of the signals being carefully adjusted over the station marks.† To insure this condition being fulfilled, the men who work the lamps and heliotropes should be trained to perform their parts with skill and fidelity.

84. Notwithstanding all the precautions which may be taken in making the observations, and recording them with fidelity and precision, it will be found that

\* It may here be remarked that in observing azimuthal angles, the vertical clamp and tangent-screw should never be used, as it is liable to deflect the telescope laterally. The telescope should be raised or lowered to the proper altitude of the object by the action of the hand only, the vertical clamp being left quite free. Similarly, in observing vertical angles it is unnecessary to clamp the azimuth circle; the vertical clamp and tangent-screw should alone be employed.

† A condition as indispensable to accuracy as the centering of the instrument itself.

every mean angle is nevertheless burdened with small residual errors, which it is impossible entirely to eliminate, and which must therefore be dispersed in the subsequent computation, agreeably to the rules which will hereafter be given. These errors arise partly from imperfections in the instrument, and in the observer, and some part of them may also be attributed to unavoidable sources; such, for instance, as the displacement by refraction to which an observed object is liable on account of the rays of light passing through a medium of very variable density and temperature.

85. There is reason to believe that refraction takes place chiefly in the vertical plane; but owing to the smoke and various vapours perpetually rising from the earth probably not having an uniform density at a given height above the surface, the ray in its passage through these must unquestionably be to some extent, liable to lateral as well as vertical refraction. We see, in fact, both by night and day that this cause is perpetually in operation; for the small disc of an argand lamp, which is only twelve inches in diameter, and in a clear, settled atmosphere, is reduced to a luminous point, swells out sometimes for several nights in succession into a broad ill-defined blaze, subtending occasionally two minutes of the horizon, and vibrating more like a sheet of fire than an object intended for accurate intersections, whilst by day the visible disc of the heliotrope, though only two inches in diameter, is even wilder and more straggling.

86. The only method of overcoming these sources of irregularity is to await a favourable state of the atmosphere, and be prepared to profit by it when it offers itself. Such occasions occur for day observation almost every sunny day for a shorter or longer period, between half-past three P.M. and sunset. The lamps are also frequently well adapted for observation from sunset to past midnight, especially in the early part of the season, but at other times they are steadiest from after midnight till about sunrise.

87. In hilly countries, where the stations are placed on lofty peaks, with deep intermediate valleys, objects are less disturbed by atmospheric causes than under other circumstances. Very good observations may then be obtained for

Uncertainty of refraction.

Time for observing horizontal angles.

Precautions against lateral refraction.

an hour or two after sunrise ; but, in general, however promising the apparent state of the atmosphere may be at sunrise, it can rarely be depended on. It is at this period of the day that *mirage* is most conspicuous. Two or three heliotropes are sometimes seen piled one above the other, or placed side by side, sometimes uniting together to form columns or pillars of fire subtending several minutes, although the rays of light pass through an aperture limited to two inches in diameter. This kind of lateral refraction is, however, too conspicuous to be overlooked ; but there is another insidious sort which ought carefully to be guarded against, because at the time it is in operation the visible disc of the lamp or heliotrope appears at first sight small, well-defined, and steady ; if, however, it be carefully bisected by the wire, and then watched, it will be found after the lapse of a few minutes gradually to diverge to one side, and after a shorter or longer time will again move off slowly in another direction. This treacherous state of the atmosphere is very frequently in operation, and the only remedy is to watch carefully, and adjust the position of the telescope so that the object may appear to diverge equally on each side of the wire. With this precaution good observations can be made, but each intersection occupies several minutes of time, and the observations proceed very slowly.

88. The accordance, or otherwise, of angles taken on the same zero, enables an observer to judge of the accuracy of his intersections, and the average errors of the triangles will show how far care and precaution have succeeded in eliminating the imperfections of the instrument, and other sources of error. Even with the best theodolites an observer may be satisfied with his work, if his triangular error does not average over one second.

Test of good work.

89. The formulæ from which the relative heights of the stations of a Trigonometrical Survey are deduced are explained in the third Edition of the Auxiliary Tables, page 24 *et seq.*, and more fully in Chapter XIII, Volume II of the "Account of the Operations of the Great Trigonometrical Survey of India," and it will be unnecessary to enter here upon this part of the subject. The procedure in the field is limited to the careful measurement, 1st, of the vertical angles, with the corresponding level readings, and 2nd, of the heights of the telescope of the theodolite and of the signal above the platforms whose relative heights are required.

Deduction of heights of stations.

90. It is generally assumed that the refraction at both stations is the same; but, in order to justify this assumption as far as possible, the back or reciprocal vertical angles ought to be taken at one and the same instant.\* But as this is not always possible, the next best method is to be careful to observe them about the same hour from noon, for then the errors caused by terrestrial refraction are more likely to compensate each other. The best period of the day for observing verticals is the time of minimum refraction, which usually occurs from about 2-45 to 3-45 P.M. Long experience has proved that reciprocal vertical observations, taken at that period of the day, notwithstanding the unsteadiness of the heliotropes, give the most accordant results, and may therefore be confidently relied on. If the stations are not visible at the time of minimum refraction, the precaution should be taken of observing them simultaneously, by means of two instruments and two observers.

91. Four sets of observations, two with the face each way, are sufficient at each station, except in extensive plains, where a second and third day's measures are desirable. All stations that are visible should be observed, as verificatory results may thus be secured. Vertical observations taken with the face each way, that is to say, with the face of the vertical circle in one instance to the left hand, and in the next to the right, are called collimated observations, because the mean of the two faces gives a vertical angle free from error of collimation, which is not the case with observations on a single face.

92. The method of observing is as follows:—The instrument should be duly levelled and adjusted previous to the time of minimum refraction; and as soon as signals are steady, or nearly so,† direct the telescope to one of the principal stations, and having clamped the vertical circle, bisect the signal with the horizontal wire, and note the hour and the minute at which the observation was made. Now note the readings of the two ends of the level that is attached to the vertical microscopes,‡ calling

\* This was done on the Great Indian Arc, in crossing the plains of Hindostan, because the vertical angles were there subject to the influence of extraordinary refraction and *mirage*, on account of the rays grazing along the surface of the ground.

† Heliotropes are seldom perfectly steady till 4 or 4½ P.M., but at that period they are beginning to rise at the rate of a second per minute. Although they may appear a little agitated at 3 P.M., still the refraction is then least, and the observations give the best results. In fact, any error arising from the agitated aspect of the heliotropes will be much less injurious than refraction, and indeed may be counteracted in a great measure by multiplying the observations, because the agitation of the disc does not produce a constant error in one direction, as happens in the case of uncertain refraction.

‡ Theodolites differ in the manner of attaching this level but its indications must always be read whatever its construction.

them "eye end" and "object end" respectively, and read off the microscopes of the vertical circle, according as the vertical angle may be an elevation or a depression, and let the readings be duly recorded in the angle book; release the clamp; turn over the telescope  $180^\circ$  in a vertical plane, and the instrument round  $180^\circ$  in azimuth; bisect the same station again, and record the readings of the level and microscopes as before, together with the hour and minute of time at which the bisection was made. The mean of the two observations will give one collimated vertical angle. Repeat the last observation as before, and then reverse the instrument, and take another observation, which will complete the second collimated vertical angle. If these two be accordant they will suffice, otherwise a third, and even a fourth, ought to be taken. Now direct the instrument to another station, and take the vertical angles in the same manner, and in that way proceed until all the surrounding stations have been observed; but if the signals begin to rise, which will be ascertained from the depressions diminishing, or altitudes increasing, it will be necessary to desist for that day, and reject any observation which may have been made after the time of minimum.

93. On arrival at a station which has been observed from one in rear, the back or reciprocal observations  
Reciprocal observations. must be taken within a minute or two of the corresponding times. If any atmospheric change should appear to take place intermediately to the observations of the reciprocal vertical angle, the equality between the refractions, which the nature of the problem demands, is likely to be destroyed; and therefore too long a space of time ought not, when it can be avoided, to be allowed to elapse before the reciprocal observations are made.

94. It must be borne in mind that if the instrument gives altitudes  
Different methods of recording elevation and depression. on one face, it will give zenith distances on the other, and the graduation on the micrometer head will, in this case, be according to the order of the figuring; consequently, the system of reading altitudes on one face, and zenith distances on the other, is, perhaps, the safest course to pursue. Occasionally, however, altitudes are read on both faces, by taking the seconds from the micrometer head in an inverse order, thus  $50''$  is read  $10''$ ,  $40''$  is read  $20''$ , and so on, which gives altitude instead of zenith distances. This was the method followed on the Great Arc, and several



other Series, and with care and experience it may be practised with facility, and without liability to mistakes. With regard to depressions it may also be remarked that the face of the instrument which gives altitudes according to the direct order of graduation, will give nadir distances in the same direct order for all objects below the horizon; and on the opposite face which gives zenith distances according to direct order of graduation, depressions may be read off in direct order. Instead of nadir distances, however, depressions may be read off at once by supposing the divisions on the micrometer head reversed, *viz.*,  $50'' = 10''$ ,  $40'' = 20''$ , &c., as in the case of altitudes.

95. Those instruments which give altitudes and nadir distances in direct order on one face, and zenith distances and depressions on the other, are numbered in four quadrants from  $0^\circ$  to  $90^\circ$ , following each other round the circle; that is to say,  $0^\circ$ ,  $10^\circ$ ,  $20^\circ$ , &c., to  $90^\circ$ ,  $10^\circ$ ,  $20^\circ$ , &c., to  $90^\circ$ ,  $10^\circ$ ,  $20^\circ$ , &c., to  $90^\circ$ ,  $10^\circ$ ,  $20^\circ$ , &c., to  $90^\circ$ ; the last  $90$  being the zero of the first quadrant. Other instruments however are divided from  $0^\circ$  at the eye and object end to  $90^\circ$  both ways. These figures, as far as the circle is concerned, give altitudes and depressions on both faces; but if the ordering of the graduation on the micrometers on one face is direct for elevations it will be reverse for depressions, and *vice versa* on the opposite face.

96. On account of the variety of graduation which obtains in different instruments, no fixed rule can be laid down, and the peculiarities of each must therefore be studied carefully by the observer.

97. The following rules obtain in all instruments:—For altitudes the micrometer should be moved to the division apparently above the zero of the micrometer at the eye end, and to the division below zero of the microscope at the object end. It is, of course, exactly *vice versa* with respect to depressions; that is to say, the divisions which appear below zero at the eye end, and above zero at the object end, should be chosen. Now, if on revolving the micrometer head, so that the wires shall proceed from zero in the microscope to the divisions above indicated, if it be remarked that the graduation on the micrometer head follows in the natural order of numbering, then the reading must be taken direct, otherwise it must

be made reverse. After studying each instrument a rule can generally be framed like the following which applies to Troughton and Simms' 36-inch theodolite, and probably to others by the same makers, *viz.* :—

$$\begin{array}{l} \text{F. L.} \left\{ \begin{array}{l} \text{Elevations read direct.} \\ \text{Depressions read reverse.} \end{array} \right. \\ \text{F. R.} \left\{ \begin{array}{l} \text{Elevations read reverse.} \\ \text{Depressions read direct.} \end{array} \right. \end{array}$$

A rule such as this, having once been carefully deduced for the particular instrument in use, will prevent subsequent mistakes.

98. The same precautions which are necessary in adjusting and reading the azimuthal microscopes are also indispensable in using vertical microscopes. They should be freed from parallax, and adjusted for runs; they should also act in the direction of a tangent to a circle, and all measurements should be made with that motion of the screw which draws against the spring; moreover, they ought to be set truly horizontal, with respect to the axis of azimuth motion and the division of the vertical circle, as they will then be free from index error, or nearly so.

99. Although the mean of two observations with the face each way gives a result independent of collimation and level error, still, when the former is large much inconvenience is felt. All that is necessary to free the instrument from this error, or rather to render that error very small, is to take the altitude or depression of any fixed terrestrial object on both faces, and deduce the mean, which will be the true vertical angle. Then, while the telescope remains firmly clamped, with the object truly bisected on its horizontal wire, move the microscopes by their appropriate screws, till they give the true reading of the vertical angle, as deduced from both faces. In all instruments some method of moving the microscopes through a small space laterally is provided, varying according to the ideas of different makers; by means of this adjustment the whole microscope must be moved, until the cross wires coincide with the proper division on the limb. This adjustment, however, will not generally allow of the microscope being set much nearer than a minute, and the remaining part must be effected by altering the graduated head of the micrometer, and then setting the comb to correspond therewith, as described in para. 70, when describing the adjustments of the azimuthal microscopes.

## PRINCIPAL TRIANGULATION.

[PART II.]

100. In para. 89 it has been remarked that the height of the instrument, and of the signal above their respective platforms must be recorded. The former should be carefully measured by the observer, and the latter by the men in charge of the heliotropes and lamps, so that on arrival at each station, the heights of the signals observed may be duly verified, and noted in the form subjoined.

| EYE STATIONS. |                     |              | OBJECT STATIONS. |             |         |                   |
|---------------|---------------------|--------------|------------------|-------------|---------|-------------------|
| NAME.         | Height of           |              | NAME.            | Height of   |         | By whom Measured. |
|               | Marks above Ground. | Instruments. |                  | Heliotrope. | Lamp.   |                   |
|               | Feet.               | Inches.      |                  | Inches.     | Inches. |                   |
| Imlia ...     | 0'0                 | 69'75        | Asrofpur ...     | 26'         | ...     | Ramdeen.          |
|               | 8                   |              | Thana ...        | 30'         | ...     | Jeri.             |
|               | 24                  |              | Ragaopur ...     | 82          | ...     | Bikari.           |
|               |                     |              | Samnadeo ...     | 30          | ...     | Manbod.           |

101. Hitherto we have only considered the case of stations at which the back, or reciprocal, vertical angle can readily be observed; but it is clear that if the amount of refraction were known, the relative heights could be deduced from verticals taken at one extremity of a ray: this method is however not considered sufficiently rigorous for determining the heights of principal stations, though it is largely used in secondary work and will be more fully explained in the next Chapter which describes that class of operations.

102. It has been mentioned in para. 56, that luminous signals are always employed in principal triangulation. These are of two kinds, *viz.*, heliotropes used for reflecting the sun's rays by day, and lamps with argand burners, to consume either oil or kerosene, for night use. A heliotrope (called also heliostat and heliograph) consists of a plane circular mirror, so mounted on a small tripod as to be movable on two axes, one vertical and the other horizontal, whose directions pass accurately through the centre of the mirror, at which point a small hole about  $\frac{1}{10}$  of an inch diameter, is drilled through the glass. Each heliotrope is accompanied by a small

sight-vane, consisting either of an aperture in a board with two cross wires spanning it; or of a leaden weight (or even a lump of clay) of 2 or 3 lbs., into which is inserted vertically a straw or very thin slip of wood, on which a piece of thread is tied loosely, so as to be movable up and down at pleasure. The latter very simple contrivance has been used for many years in the Topographical Branch, and found to be perfectly serviceable, and of a construction so simple and easily improvised anywhere, as to leave nothing to be desired on that score. In principal triangulation the sight-vane for the heliotrope is attached to the top of the lamp box as shown at *A* in Plate III. Heliotropes are made of various sizes,—8 inches diameter will perhaps be found the most convenient, being very portable, and yet quite capable of being used as a signal at 60 or 70 miles. For distances less than 20 miles it will be found advisable to reduce the light by means of diaphragms. A two-inch or three-inch aperture suffices for 15 miles, if the heliotrope be carefully aligned. It is convenient for the observer to carry with him a heliotrope of larger size, say 12 inches diameter, for the purpose of flashing to the signallers, when he wishes them to show their own heliotropes.

103. The process of aligning a heliotrope, or so placing it that the sun's rays are reflected in any desired direction, is very simple, and soon learned by ordinary *khalásís*. It is carried out as follows:—The sight-vane is carefully centered over the station mark, and the heliotrope then placed at a convenient distance from it, (about two feet is suitable) and facing the point on which it is to be aligned, in such a manner that the *khalási* can see both the sight-vane and the distant mark, through the hole at the centre of the heliotrope. By moving the heliotrope slightly, the distant mark, the cross wires of the sight-vane, and the hole, are then placed in the same straight line. The mirror should now be turned towards the sun by means of its motion on the two axes, (the tripod remaining fixed) in such a direction that the image of the hole, which is a small black speck in the middle of the bright reflection of the mirror, falls on the cross wires of the sight-vane. An observer at the distant mark will then see the sun reflected in the mirror.

104. If the heliotrope is situated nearly in a line between the observer and a low sun, it is found rather difficult to adjust it accurately; in such cases the difficulty may be got over by the use of a second heliotrope, which is so

*Continued.*

placed as almost to face the sun, and reflect its rays on the first one, which can under these circumstances be made nearly to face the observer. A little ingenuity will enable the observer to arrange a code of signals for conveying a few simple orders to the heliotropers; it is unnecessary to specify such in detail, as each triangulator will probably prefer his own; but there is one that is almost universally used in the Department for calling in a heliotroper when no longer required, and that is, the display of two heliotropes simultaneously about 50 yards apart.

105. In Plate III are given drawings of a lamp as now used for a night signal. Formerly country oil used to be burned, but in the present day kerosene is found more convenient. The part of the lamp that has to be centered over the mark-stone is the middle of the glass disc in the front of the lamp box. Two nicks  $G G$ , one at the top and one at the bottom of the box, are so made that the line joining them bisects this glass disc. When properly adjusted the flame of the lamp should be very nearly central with regard to this glass, and the reflector so placed that its axis may coincide with the ray to the observer's station. This adjustment can only be done by estimation, but the construction of the lamp is such that, when the box itself is properly aligned by means of the sights on its upper surface, the positions of the flame and reflector are very closely correct.

106. The approximate adjustment of the alignment of the lamp is performed by means of the eye-hole  $B$  and a pin  $C$ . The apparatus stands on three legs  $T$ , two only of which are seen in the drawing. The plummet  $P$  hangs from the top of the heliotrope sight-vane, and when correctly placed its line should pass accurately through the cross wires of the sight-vane of the heliotrope, through the two nicks on the lamp box, and through the station mark. The heliotrope  $H$  stands on a board  $D$  which slides in a groove, and is removable for convenience of packing. The boards  $E E$ , which are screwed on to the top of the box, are intended to receive a second heliotrope for double reflection as described in para. 104. The four small battens  $F F F F$  are grooved to receive a sliding panel to protect the glass, when the lamp is not in use.

107. The last point upon which it is necessary to dwell in the instructions for principal triangulation, is the urgent necessity for a full, clear, and intelli-

Adjustment of lamp.

Continued.

Description of stations.

gible description of each of the principal stations of observation. There is no reason why the trigonometrical and astronomical stations of the Survey of India should not, with ordinary care, last for centuries; but, in order to be of real service at any future time, the sites should be named and described in such a manner, that they can be readily identified by any reasonably intelligent person. It is not pretended that this can always easily be done, especially in the thinly-inhabited, forest-clad districts met with in many parts of India and Burma, but the observer must ever bear in mind that it is a point of the highest importance, and strenuous endeavours must be made to secure it.

108. *Firstly* as to the name. When the station is situated on a hill with a well-known local name, it will be best to

*Continued.*

retain that name. In other cases the name of the nearest village, or of the village on whose lands it is situated, must be given to it. *Secondly* as to the description. This must embrace an account of the details of its construction, the height and size of the platform and pillar, the number and positions of the mark-stones embedded in them, also the materials of which they are built, and whence they were brought. If on a hill, the best means of ascending it should be specified; and the village, if any, from which the ascent should be made, or supplies obtained, should also be entered in the description. The approximate distances and bearings of several neighbouring villages, temples, hills, rivers or other permanent objects should be stated, and any other information given which would simplify the identification of the site subsequently, not omitting the names of the province, district, thána, tahsíl, pargana or other local sub-division in which the station is situated. The description, which should be written at the time of visiting the station, must be in a clear and legible hand in both the original and duplicate angle books; names must be hand-printed, or at the least written in a large round hand, due care being taken to ensure orthography in the transliteration of native words. An authorized list of the correct spelling of proper names can be obtained for most districts, and should be in the possession of the party for ready reference.

109. When any stations of previous triangulation, which have been already described and closed according to the

*Stations revisited.*

directions given in paras. 18 and 19, are visited, the condition in which they are found must be stated, in order to obviate any uncertainty whether the original site has been recovered or not; the

measures taken for subsequent protection should also be described, or any alterations that may have been made in the construction of the station. Occasions have arisen in which some doubt has been experienced as to which of two closely contiguous stations has been occupied. The observer should be alive to such contingencies and invariably notice in his description of stations any such possible cause of confusion.

## CHAPTER II. SECONDARY TRIANGULATION.

1. The object of secondary triangulation is purely topographical.

*Intersected points.*

The principal series provides too few points for the detail surveyors, and the secondary work supplements the principal by breaking down its large triangles into smaller ones, and fixing in addition a great number of minor points by rays taken to them from two or more stations. These minor points are generally known in the Department as *intersected points*, and are not as a rule visited by the triangulator, who contents himself with fixing them by the intersection of two or more rays, whence the name of *intersected points*.

2. The secondary triangulation may be laid out either in chains or a network, or a combination of the two, as may

*Chains or networks.*

seem most suitable for the particular piece of country under survey. Where any large river with a broad valley intersects the district, a series may with advantage be carried along it, with stations on either bank, the only disadvantage being the necessity for frequent crossing and recrossing. This can, however, be partly obviated by the judicious co-operation of two observers, one on either side of the river. Most commonly, however, a network is the more convenient plan of proceeding.

3. The rigorous principles laid down in Chapter I may be considerably relaxed in the case of secondary work.

*Vernier theodolites.*

In the first place vernier theodolites of the better class have been generally considered sufficiently good for the measurement of the angles. They are considerably lighter than the microscope theodolites, and simpler in construction and better adapted for the use of Assistant Surveyors, to whom the secondary triangulation is generally entrusted.

4. The adjustments of vernier theodolites are, for the most part, the same as those of microscope instruments, differing only in the case of the verniers themselves: these require no adjustment, being immovably fixed at the proper distances apart by the maker. The descriptions given in paras. 27-56 of the previous Chapter apply to both classes of instruments; and any observer, who has mastered them will have no difficulty in applying his knowledge to the simpler form of theodolite.

5. The adjustment of the vertical verniers needs a few remarks. The true vertical angle to any distant well-defined point must be determined by observations on both faces, corrected for level error, as described in para. 99, (the correction for level may be avoided by bringing the bubble to the middle of its run for both observations by means of the clips, before reading the verniers). The verniers must now be set to read as nearly as possible the value of this vertical angle, and the telescope made to intersect the distant object *by the clips only*. It will now be found, probably, that the bubble of the level attached to the verniers is displaced from the centre of its run; it has only to be brought back by its own adjusting screws, when the process will be complete. It will never, except by pure chance, happen that both verniers can be made to read the true vertical angle; in practice there is always some discrepancy between them. The best plan is to make the mean of their readings equal to the true vertical angle, by which their combined index error will be reduced to the lowest quantity possible; but it will be always safer to observe vertical angles on both faces, when great accuracy is aimed at.

6. It is very desirable, though not indispensable, that all the three angles of every triangle should be observed. One of the most important reasons for observing all the angles, especially in secondary work, is to avoid gross errors, such as  $10^\circ$ , resulting from an erroneous reading of the figures on the limb: in the case of a simple chain such an error might escape detection, and lead to endless trouble and uncertainty if it occurred in a triangle of which two angles only were measured. In the case of a network such a fault could be localized almost with certainty, and therefore the measurement of the three angles is not of such vital importance; still, however, it renders the work more complete and



satisfactory, and should not be neglected except on good grounds. In fixing intersected points it is advantageous, when practicable, to observe them from more than two stations. If observed from one only, it is obvious that they cannot be localized at all; if from two, they are fixed by one triangle only, and there is no certainty that the same point has been intersected from both stations; whereas if observed from three or more stations, the data are sufficient to give two or more values of one of the rays, and therefore to furnish a test of the correctness of the identification, by the agreement or otherwise of the several values.

7. The number of zeros may be considerably reduced on comparison with those required for principal work. Two zeros and two changes of face should suffice even for fairly large triangles: a twelve-inch or fourteen-inch theodolite is generally preferred for this class of work. Luminous signals should be employed for any rays exceeding 7 or 8 miles in length; for shorter ones the pole and brush, such as is shown in the wood-cut in para. 19, Chapter I, is suitable in hilly country, if it stands up against the sky-line. In flat country, or where the back-ground, against which the signal is likely to be projected, is nearly the same colour as itself, a white-washed *gharra* may be placed on the top of the pole, or the pole and brush may be altogether superseded by a small bell tent.

8. The care in centering the theodolite and the heliotropes should not be relaxed, as the menials employed will in all probability become careless about this process, if they observe that the officer in charge is not always equally scrupulous about its accuracy; and it should be remembered that, as the signallers are under no sort of supervision when on detached duty, it is doubly necessary that they should acquire a *fixed habit* of always centering with the greatest attainable nicety, whether really requisite or not for the particular operation in hand. Heliotropes when used for distances of 4 or 5 miles should be reduced by stops to about 2 inches diameter; if the light thus diminished is still too dazzling, an effectual way of subduing it is by holding a single fold of linen or cambric a few inches in front of the object-glass of the telescope.

9. Stations for secondary triangulation should be so selected as to obtain as far as possible an uninterrupted view on all sides. If the station is situated on a

Number of zeros.

Centering the theodolite and signals.

Selection of stations.

forest-clad hill top, it will be found a very convenient plan to leave one large tree standing as close as possible to it, after clearing the jungle away; such a mark is frequently found of great use to plane-table surveyors. The same remark applies to hill tops used for intersected points. The platforms for the more important stations must be built in much the same fashion as the principal stations, with an isolated central pillar and a mark-stone, unless the mark can be cut on the rock *in situ*. Masonry tower stations are never likely to be required, and should not be built without special sanction.

10. In the column of remarks in the angle book a brief description of each intersected point, when necessary for identification, should be given; such as the particular corner or minaret of a building observed, a single tree, or pile of stones on a hill top &c., &c., and if such points are lettered or numbered, great care must be taken that each point is always distinguished by the same number, when observed from two or more stations. The horizontal angles between the stations and intersected points may either be taken on one zero without change of face, in which case the readings should be checked by the recorder calling out the setting of the limb, and the observer noting, after the limb has been put to that setting, whether the point is duly intersected: or they may be taken on one zero with one change of face, in which case the above check is unnecessary, as the recorder can see at a glance whether the two readings differ by  $180^\circ$  as they ought. The latter method is certainly the better, the only objection being that it takes up slightly more room in the angle book, and adds a few more figures to the computations. The means of horizontal angles between secondary stations should be retained to one place of decimals of seconds, and between stations and intersected points to the nearest second only. In observing objects such as large trees, rounded hill tops &c., which are not capable of accurate intersection, it is unnecessary to record the reading of more than one vernier on each face.

11. Vertical angles should in all cases be taken on both faces, otherwise the error of collimation, which may be considerable, comes in purely as error in the vertical angle observed. Vertical angles to *stations*, as distinguished from *intersected points*, should be taken twice on each face, *viz.*, once with a motion of the telescope from below upwards, and once with reversed

motion, *i.e.*, from above downwards. The observation of vertical angles should be limited to about one hour on either side of minimum refraction, at least where any accurate determination of height differences is aimed at. In observing heights of buildings or trees or hill tops &c., it is a matter of considerable importance to specify clearly in the angle book the point selected, and also to give the height of the instrument and signal above the station platform.

12. In secondary triangulation, as in principal, it is desirable to observe reciprocal vertical angles, at very nearly the same hour of the day at both stations; but in the former class of work the rule must necessarily be relaxed in many cases, notably in that of intersected points. It was stated in para. 101, Chapter I, that if the amount of refraction were known the relative heights could be deduced from verticals taken at one extremity of a ray. It is usual to express the length of the ray in seconds of arc, and the constant of terrestrial refraction as a fraction of it, so that if  $r$  be the total amount of refraction,  $k$  the constant, and  $C$  the contained arc,  $r = kC$ , whence  $k = \frac{r}{C}$ : and, if we assume an equality between the refractions at the extremity of a ray, we can obtain the value of  $k$  from every pair of reciprocal observations at every station. The mean of all the values of  $k$  at any one station may, therefore be adopted as the constant of refraction at that point, and may, without risk, be applied to correct a single vertical angle to a secondary point, when its reciprocal observation is wanting. Let  $D$  be the observed depression of that secondary or intersected point, and  $C$  the contained arc due to its distance, then  $D + kC = \Delta$  will be the true depression cleared of refraction. Let the unobserved reciprocal vertical be  $D^1$  or  $-E^1$ , its corresponding value in *vacuo* being  $\Delta^1$ , then as  $C = \Delta + \Delta^1$  therefore  $\Delta^1 = C - \Delta$ , and the subtended angle  $S = \frac{1}{2}(D - D^1) = \Delta - \frac{1}{2}C = D + kC - \frac{1}{2}C$  or for an elevation  $S = E - kC + \frac{1}{2}C$ . The use of Table XIV, 3rd Edition, Auxiliary Tables, enables the computer to obtain the subtended angle, without the intermediate step of computing the unobserved vertical angle.

13. The relative heights of intersected points thus obtained are quite good enough for the purpose required, if the precaution is taken of making the observa-

Co-efficient of refraction.

tions of the vertical angles at about the same interval from noon, as the principal verticals from which the value of  $k$  is derived. The value of  $k$  averages about  $\cdot 066$  in hilly country where the rays are quite free from all obstructions, and pass near no intervening objects: in flat sandy countries like Rajputana and parts of the Punjab it is very irregular, even being occasionally negative. For an interesting discussion of this question the reader is referred to Appendix No. 3, Volume II, and to the Preface to Synoptical Volume VII, where a table of the values of the co-efficient, found in actual practice, is given.

14. It is seldom necessary to employ any very elaborate precautions for the protection of secondary stations. A rough pile of earth or stones is generally sufficient to protect them from all but wilful injury. This cannot easily be guarded against, except by placing them under the care of an official in the same way as principal stations, a course which the comparatively small importance of secondary work rarely warrants; all stations should, however, be carefully described in the angle book, and the direction and distance of neighbouring villages or other permanent marks noted for subsequent identification.

Protection of secondary stations.

15. Somewhat akin to the secondary triangulation, as defined in paras. 1 and 2 of this Chapter, is the system known in the Trigonometrical Branch as *Ray tracing*. It was first instituted as a means of ascertaining the exact direction of a distant station, which happened to be concealed by intervening jungle, in order to regulate the clearing of the line. The method is thoroughly explained in Chapters II and VII, Part IV of Thuillier and Smyth's Manual of Surveying, a work which is assumed to be as a matter of course in the possession of every field party, and need not therefore be given in detail here.

Ray tracing.

16. The reader should consult the "Handbook of the Topographical Branch" for further hints on the management of secondary triangulation, as this class of work is more intimately connected with that Branch than with the Trigonometrical, which concerns itself chiefly with the more rigorous methods of principal triangulation for geodetic purposes.

Management.

## CHAPTER III. COMPUTATION OF PRINCIPAL TRIANGULATION.

1. The usual abstracts of angles having been made in the angle book, the primary value of each angle resulting therefrom, is the arithmetical mean of the zero means, and is generally symbolized by  $M$ . When the number of measures on each zero is not the same, a small correction has to be calculated to apply to  $M$  to obtain what is called the *corrected or concluded angle*  $C$ . If the number of measures on each zero are equal  $M$  and  $C$  are the same. Now no measured quantities are ever absolutely correct, though some may possess greater reliability, or, as we may say, greater *weight* than others; and it is usual in refined observations, when a result has been derived from several independent measures, to calculate its *weight* relatively to other results similarly obtained, by means of the accordance of the several measures.\* In triangulation the weights thus obtained are made use of in the subsequent calculations, as will be explained hereafter. The weights of angles are calculated in a form designed for the purpose, *P. 8, Computation of Weights of Observed Angles*. (See the example at the end of this chapter). To employ this form enter in the first column the number of observations on the first zero, and in the second column the seconds of observation successively; then do the same for the second zero, and so on. In the third column enter the zero means, and in the fourth the differences of the several observations from the zero means to which they appertain, regardless of sign; each of these quantities is symbolized by (*e.o.*). In the fifth column enter the squares of the quantities (*e.o.*). In the sixth column enter the differences between the zero means and their general mean  $M$ , with the signs + or - as the case may be; these quantities are symbolized by (*e.g.*) Square these quantities and enter them in the next column.

Now put

$$e^2 = \frac{\text{Sum of } (e.o.)^2}{\text{Number of measures} - 1};$$

$$c^2 = \frac{\text{Sum of } (e.g.)^2}{\text{Number of zeros} - 1}.$$

\* Besides the term *weight* for denoting the reliability of any result deduced from observations there are other terms, viz., *error of mean square* or briefly *e.m.s.*, and *probable error* or *p.e.* The relation between these is denoted as follows:—

$$\sqrt{\text{weight}} = \frac{1}{e.m.s.} = \frac{1}{.6745 \times p.e.}$$

The *error of mean square* may be defined as the square root of the mean of the squares of the apparent errors of observation. By Sir G. Airy, Chauvenet and German writers this is called the *mean error*.

Next divide  $e^2$  successively by the number of observations on each zero and adding  $c^2$  to each quotient take their reciprocals: these added together give the weight  $w$ , and hence the weight reciprocal  $\frac{1}{w}$  or  $u$ .\* Only two places of decimals are retained throughout the calculation.

2. The correction to the arithmetical mean  $M$ , to produce the concluded angle  $C$ , is next obtained as follows:—  
 Corrected or concluded angle. Look out the smallest of the reciprocals from which  $w$  is derived, and subtract it from each of the others, entering the differences in the column provided, and in the next column enter the corresponding values of (*e.g.*) with their proper signs. Sum the several products and divide by  $w$ ; the result is the correction required.

3. The next step in the calculation of principal triangulation is that of the spherical excess of each triangle.  
 Spherical excess. This does not require that the angles and sides should be employed to any degree of nicety. A preliminary calculation of the sides of the triangles is made in form *P. 14, Computation of Secondary Triangles*. The angles are entered to the nearest second, and the amount by which their sum differs from  $180^\circ$  is divided equally among them, thus giving angles for computation. In filling in *P. 14*, the names in each triangle should be so arranged that the first two stations include the known side, and the third is opposite to it. In the column for *log sines* it is convenient to enter the *cosecant* of the third angle instead of the *log sine*. Vertically over the *log sines*, figure for figure, is entered the *log* of the known side. Now by adding together the *log side*, the first *log sine* and the *log cosecant* we obtain the *log value* of the side opposite the first station, and in like manner we find the other side. Proceeding in this manner preliminary values of all the sides may be obtained; now enter Form *P. 9, Computation of Spherical Excess*, and employing the mean latitude for the triangulation for entering Table III of the Auxiliary Tables, perform the calculation as indicated in the form; the result is the spherical excess. Five place logs are sufficient in Form *P. 9*, and the spherical excess is taken out to three places. For any further information and an example, see page 12 of the Auxiliary Tables, Third Edition.

\* This is the (*e.m.s.*)<sup>2</sup>

4. The triangulation of India consists of chains formed of single triangles, and groups of triangles constituting quadrilaterals, polygons and compound figures; (vide Part II, Chapter 1). Now if the angular measurements were absolutely correct it would follow that (1) every triangle would have the sum of its angles exactly equal to  $180^\circ + \text{the spherical excess}$ ; (2) at every point where angles have been measured completely round the horizon, the sum would equal exactly  $360^\circ$ ; and (3) the ratio of any two sides of the triangulation would be constant, whatever succession of triangles is employed to arrive at it. But the observed angles fulfil none of these conditions, except by mere chance, and the next stage in the calculations is to find the necessary corrections to the angles to satisfy them while maintaining another condition, *viz.*, that these corrections shall be mathematically the most probable. This is done by what is called the "Method of Least Squares." It will not be necessary here to enter into any explanation of the theory of the process, but only to explain the working of it which is departmentally known as "grinding the figure."

*Firstly.* As regards single triangles; it is obvious that they can only fulfil the first condition. Add the three angles together and subtract  $180^\circ + \text{the spherical excess}$ ; the remainder, which will be either *plus* or *minus*, will be the triangular error or  $e$ . Now if we call the errors of the three angles respectively  $x_1$ ,  $x_2$  and  $x_3$ , and the corresponding reciprocal weights  $u_1$ ,  $u_2$ , and  $u_3$

$$x_1 = \frac{u_1 e}{u_1 + u_2 + u_3}, \quad x_2 = \frac{u_2 e}{u_1 + u_2 + u_3}, \quad x_3 = \frac{u_3 e}{u_1 + u_2 + u_3}.$$

These are the most probable *errors* of the angles, and to employ them their signs must be changed and the quantities applied to the respective angles.

*Secondly.* Quadrilaterals:—A quadrilateral consists of three triangles\* connecting four points (*vide* page 4) and has eight observed angles, two at each point. For obtaining the errors of the angles a form is provided numbered *P. 10*. The conditions to be fulfilled are the first and third. For an example see the end of this chapter.

\* There are apparently four triangles, but only three are independent.

At the top of the form is a typical figure for general use, with the angles numbered, and it is intended that the names of the stations of any figure under reduction should be written in against the station numbers, I, II, &c.; while in the right-hand upper corner are to be entered the observed angles and their reciprocal weights. Now carrying the eye to the left, provision will be found for calculating the three triangular errors  $e_1$ ,  $e_2$  and  $e_3$ , and in the same line the error  $e_4$  or the error of the side ratio

$$\frac{I - IV}{I - II}, \quad \frac{I - II}{I - III}, \quad \frac{I - III}{I - IV}.$$

or  $\frac{\sin (X_2 + X_3)}{\sin X_1}, \quad \frac{\sin X_6}{\sin X_3}, \quad \frac{\sin X_8}{\sin (X_6 + X_7)}.$

This ratio, if the angles were errorless, should obviously equal unity, *i.e.* its logarithm should equal 0. In filling in this part of the form, one-third of the spherical excess of the triangle must be deducted from each angle, and at the time of taking out the log sines, the changes for 1" of arc should be entered in the columns headed *s*. 7-place Log Tables are used in the calculation, but an 8th place is found by interpolation and employed. The resulting error will be in the 8th place of decimals, and for convenience is multiplied by 10,000,000, the quantities *s* being treated in the same way. Next enter Table I and fill in the reciprocal weights  $u_1$ ,  $u_2$ , &c., at the head of each column; also in the line D fill in, above the *s*'s, their values as given in the calculation of side error; the proper signs will be given by means of the signs in front of the symbols below.

5. The filling in of Tables II and III requires no explanation; it is only necessary to remark that where + signs are printed in the form, the quantities are positive, but where - signs occur, the computer must judge for himself whether they should be turned into + or not. This rule holds good for all the forms used in the Calculating Section of the Trigonometrical Branch Office. The quantities in Table III, when summed horizontally, form the co-efficients of  $f_1$ ,  $f_2$  &c., in Table IV. In this table the *e*'s are now for the first time brought into use and the sign = is to be understood as preceding each. Having filled in the first four lines of Table IV we have four equations which have to be solved by rule as below, the rule having been devised to save labour:—Multiply the first equation through by the



co-efficient of  $f_2$  and divide by the co-efficient of  $f_1$  and record it in the first line below, omitting the resulting co-efficient of  $f_1$ . The next line will be obtained by multiplying the first equation through by the co-efficient of  $f_3$  and dividing by the co-efficient of  $f_1$ ; but as the co-efficient of  $f_3$  is 0, the resulting quantities will be 0, and they have been printed so in the form. The third line is obtained by multiplying the first equation through by the co-efficient of  $f_4$  and dividing by that of  $f_1$ . The co-efficients need not be entered where no provision is made for them. Now add the co-efficients in the first line of the new group to those of equation 2 and record below; then add the co-efficients in the second line of the new group to those of equation 3 and record next and treat the fourth equation similarly. We have now three equations involving three unknown quantities,  $f_2, f_3$  and  $f_4$ . Going through a similar process to that already described, we come to two equations involving two unknown quantities,  $f_3$  and  $f_4$ , and lastly we reduce the equations to one involving one unknown quantity,  $f_4$ , and dividing out by its co-efficient we have the value of  $f_4$ . This brings us to the bottom of the form. Carrying the eye up to the right the remainder of Table IV is found. Taking the last line but one, fill it up from the first equation in the group involving two unknown quantities,  $f_3$  and  $f_4$ , transferring the term involving  $f_4$  over to the other side. Then substitute for  $f_4$ , add the two terms and divide by the co-efficient of  $f_3$ , and this gives  $f_3$ . To find  $f_2$ , employ the first of the group of equations involving the three unknown quantities  $f_2, f_3$  and  $f_4$  and enter the third line of the last portion of Table IV, and similarly for  $f_1$ . Now enter Table V and multiply the quantities in Table I by the values of  $f_1, f_2, \&c.$ , as indicated. The resulting values obtained from this table are the most probable *errors* of the angles. These are usually computed to four places of decimals of seconds and retained to three places in subsequent calculations. To ascertain that no mistakes have been made the errors are first entered in the test columns of the triangular and side error calculations. In the first case the sums should equal  $e_1, e_2$  and  $e_3$  respectively, and in the last putting

$$s_1 x_1 + s_2 x_3 + s_{6,7} (x_6 + x_7) = S'_i$$

and

$$s_{2,3} (x_2 + x_3) + s_6 x_6 + s_8 x_8 = S'_r.$$

$$S'_r - S'_i = e_4.$$

If these tests are not satisfied some mistake in the calculations has been made.

*Thirdly.* Polygons.—Polygons are formed by a number of triangles with their apexes all meeting at a point, which the triangles completely surround (*vide* page 4). The angles are subject to all three conditions specified in para. 4, page 60. The errors are computed in the form *P. 11.* which has been constructed for a polygon of six sides, but may be employed for figures of a greater or less number of sides with a little adaptation. The principle is the same as for the quadrilateral, and the form needs no explanation beyond what has been given for the quadrilateral.

*Fourthly.* Complex Figures.—If more complex figures have to be treated than quadrilaterals or polygons, reference to Chapter XVI of Volume II of the *Account of the Operations of the Great Trigonometrical Survey of India*, is recommended. In such cases special care must be taken to obtain all the equations and to avoid any redundant ones. This may always be done by building up the figure and forming the equations at the same time. The employment of the minimum number of rays required to fix all the points will, if the angles are all observed, give as many triangular equations as there are points less two. Every ray afterwards introduced gives an equation for each angle observed in fixing it, either triangular, side or central, and these should be settled on before introducing another ray.

6. After the angular errors have all been found, the form for the calculation of Principal Triangles, *P. 12*, is entered. The order in which the stations are recorded is the same as described under "Spherical Excess," para. 3, page 59. The observed angles are next entered to two places of decimals. The spherical excess of each triangle is divided equally among the three angles and entered with a — sign to three places of decimals. The angular errors with their signs changed are entered in the column for 'Correction' to three places, and the seconds of the angles for computation (which are plane angles) are the algebraical sum of the three columns. The sum of the three plane angles should exactly =  $180^\circ$ . The rest of the calculation is much the same as already described, except that although 7-place logs are used, an eighth place is always derived by interpolation and employed.

Triangles.

## COMPUTATION OF SECONDARY TRIANGULATION.

[PART II.]

7. The calculation of latitude, longitude and azimuth is fully explained in the Auxiliary Tables, in the explanation of Table V. The form employed is numbered *P. 16*.

8. The form for computing heights is *P. 19*. In it are first recorded the astronomical date and mean time of observation, then the name of the stations, next the number of observations from which the vertical angle is derived and then the vertical angles at each station; the letter *I* in this column being intended to be changed into an *E* or *D* according as the angle is an elevation or depression. The filling up of the columns numbered 4, 5, 6 and 7 is clearly explained in the form itself. The calculation next proceeds in the lower part of the form, and is first carried as far as height of  $B - A$ , and the result is entered above in column 8: then by the addition of the known height of *A*, the height of *B* in column 9 is obtained. The mean value of *B* in column 10 is from two, three or more determinations from surrounding stations.

9. The remainder of the calculation deals with refraction. It is computed in the last four lines at the bottom of the form, and is equal to  $\frac{1}{2}C - R + \beta$ . The factor of refraction or  $\frac{r}{C}$  is obtained from columns 11 and 12, and a mean value should be used for the determination of the heights of any intersected points which have been observed from the principal station to which the value appertains. In secondary calculation refraction is less needed, and the calculation is sometimes omitted, the co-efficient of refraction being assumed to be 0.07. For further particulars see the Auxiliary Tables and the explanations of Tables IX to XV, especially of Table XIII.

## COMPUTATION OF SECONDARY TRIANGULATION.

10. Secondary triangles are of different characters and are treated accordingly. Some are executed in chains to supply the place of principal triangulation, the instruments used being the largest class of vernier theodolites, *viz.*, 10-, 12- and 14-inch. These triangles may be called 1st class. It is not customary to *grind* the figures formed by them; but in computing them spherical excess must first be found and divided equally among

the angles which are kept to one place of decimals of seconds, and then any triangular error that remains must be also equally distributed; after this the sides may be calculated in Form *P. 13*. It may be noted that the same angle for computation is obtained whether the spherical excess be computed or not; but the spherical angle, in contradistinction to the plane angle, is subsequently required in calculating latitude, longitude and azimuth.

11. Secondary triangles obtained with smaller instruments do not have their spherical excess worked out; but any discrepancy between the sum of their angles and  $180^\circ$  is equally divided between the angles, which are retained to the nearest second only. Form *P. 14*. Such triangles are employed to form small net-works based on well established points, and the omission of spherical excess in the subsequent calculations of latitude, longitude and azimuth is of no practical importance.

12. There is a third class of triangles fixing intersected points, that is, points which are not visited, but are observed from the extremities of a known base. In these triangles only two angles are measured, and the third angle is found by taking the difference of the sum of the two observed angles from  $180^\circ$ . Form *P. 15*.

13. If the triangles are large and of such a character that the spherical excess has had to be computed, the form for principal latitudes and longitudes should be used, *viz.*, *P. 16*. For smaller triangles and for intersected points at a greater distance than 8 miles, *P. 17* may be employed, in which the third and fourth terms of the expressions for latitude, longitude and azimuth are omitted. For intersected points at distances not exceeding 8 miles, *P. 18* should be used, in which only the first terms of latitude, longitude and azimuth are computed.

14. The calculation of heights for secondary work is in the same form as for principal heights, *viz.*, *P. 19*; but it is not usual to determine the value of refraction, a factor,  $0.07$ , as mentioned on the preceding page, being generally adopted for heights where only one vertical has been observed. In the

## MISCELLANEOUS COMPUTATIONS.

[PART II.]

calculation of height under these circumstances, sufficient instruction will be found in the Auxiliary Tables.

15. When all the calculations have been effected which have been previously described, an abstract of all the data obtained is made in Form *P. 24*.

Abstract.

## MISCELLANEOUS COMPUTATIONS.

16. Besides the calculations already described there are one or two which have to be resorted to from time to time. These are:—Computation of a triangle by two sides and the included angle, in cases where a side is required as a base for further calculation. Form *P. 25* has been provided for the purpose.

A triangle from two sides and the included angle.

In this form a subsidiary angle  $x$  is first computed by the formula

$$\log \sin C + \log a + \text{colog } b = \log \tan x,$$

and then  $A$  is computed from the formula

$$\log \sin C + \log \sin x + \log \text{cosec } (C - x) = \log \tan A;^*$$

then  $B = 180^\circ - (A + C);$

and the third side  $c$  is obtained from the formula

$$\log \text{cosec } A + \log \sin C + \log a = \log c.$$

\* The formula is derived as follows:— $a, b$  and  $C$  are given.

$$\begin{aligned} \tan A &= \tan \left( \frac{A+B}{2} + \frac{A-B}{2} \right) \\ &= \frac{\tan \frac{A+B}{2} + \tan \frac{A-B}{2}}{1 - \tan \frac{A+B}{2} \tan \frac{A-B}{2}} = \frac{\cot \frac{C}{2} + \tan \frac{A-B}{2}}{1 - \cot \frac{C}{2} \tan \frac{A-B}{2}} = \frac{\cot \frac{C}{2} + \frac{a-b}{a+b} \cot \frac{C}{2}}{1 - \cot \frac{C}{2} \cdot \frac{a-b}{a+b} \cot \frac{C}{2}} \\ &= \frac{2a}{a+b \tan \frac{C}{2} - a-b \cot \frac{C}{2}} = \frac{2a}{a \left( \tan \frac{C}{2} - \cot \frac{C}{2} \right) + b \left( \tan \frac{C}{2} + \cot \frac{C}{2} \right)} = \frac{a \sin C}{b - a \cos C}. \end{aligned}$$

Put  $\frac{a}{b} \sin C = \tan x;$  then  $\tan A = \frac{\tan x}{1 - \tan x \cot C} = \frac{\sin x \sin C}{\sin (C-x)}.$

17. Another useful calculation is the interpolation of a position from which three known points have been observed. Form *P. 26* is provided for this. A station from observations to three known points. If *A*, *B* and *C* are the three known points (see diagrams at bottom of form), and *S* the station to be interpolated, three cases occur, (1) when *S* is outside the triangle *ABC* and *C* is beyond the line *AB*; (2) when *S* is within the triangle *ABC*; (3) when *S* is without the triangle *ABC* and *C* is within the side *AB*. In the first case *S* may fall on, or very nearly on, the arc of the circle circumscribing the triangle *ABC*, which happens when the angles *ASB* and *ACB* are equal, or nearly equal, to  $180^\circ$ : in this case the problem is impossible.

Let the angles  $ASC = \alpha$ , and  $BSC = \beta$ ,

and let the angles  $CAS = x$ , and  $CBS = y$ ;

then

$$x + y = 360^\circ - (\alpha + \beta + C).$$

Now compute an angle  $\phi$  from the formula

$$\log \sin \alpha + \log \operatorname{cosec} \beta + \log a + \operatorname{colog} b = \tan \phi,$$

and compute  $\frac{x - y}{2}$  from the formula

$$\log \tan (\phi - 45^\circ) \log \tan \frac{x + y}{2} = \log \tan \frac{x - y}{2};$$

then  $x$  and  $y$  are the sum and difference of  $\frac{x + y}{2}$  and  $\frac{x - y}{2}$ .

Next compute the angles *ACS* and *BCS*, the angle *ACS* being  $= 180^\circ - x - \alpha$ , and the angle *BCS*  $= 180^\circ - y - \beta$ .

18. The next calculations—those of azimuth and distance between

Distances and azimuths of points of which the latitudes and longitudes are known.

points of which the latitudes and longitudes are known—are fully explained in the Auxiliary

Tables, pages 22, 23 and 71, 72.



PLATE I.

Fig. 1.

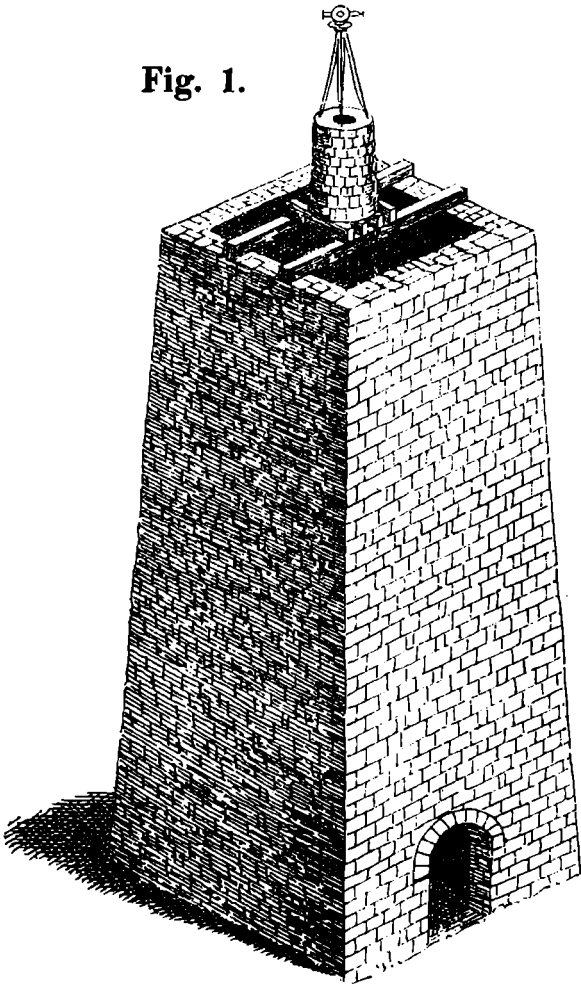


Fig. 2.

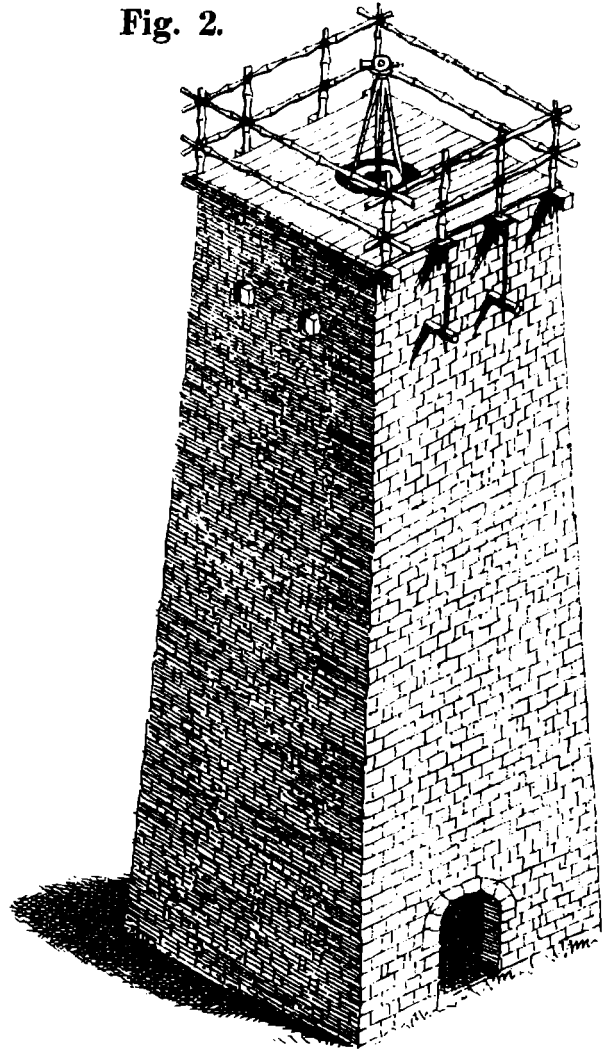


Fig. 3.

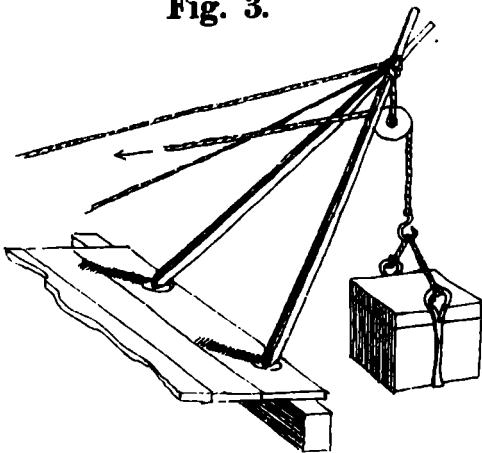
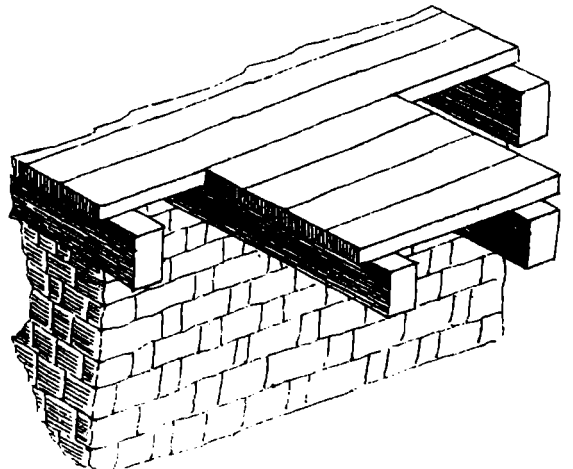


Fig. 4.







# PLATE II.

## REFERENCE.

- A Masonry pillar and annulus, section and plan.
- B Closing pillar in section.
- C Wooden braced stand, elevation.
- c'c' Masonry support for ditto, in plan.
- D Observatory tent, } in elevation and plan.
- E Theodolite box, } in elevation and plan.
- FF Palmyra platform, }
- G Lamp and Heliotrope

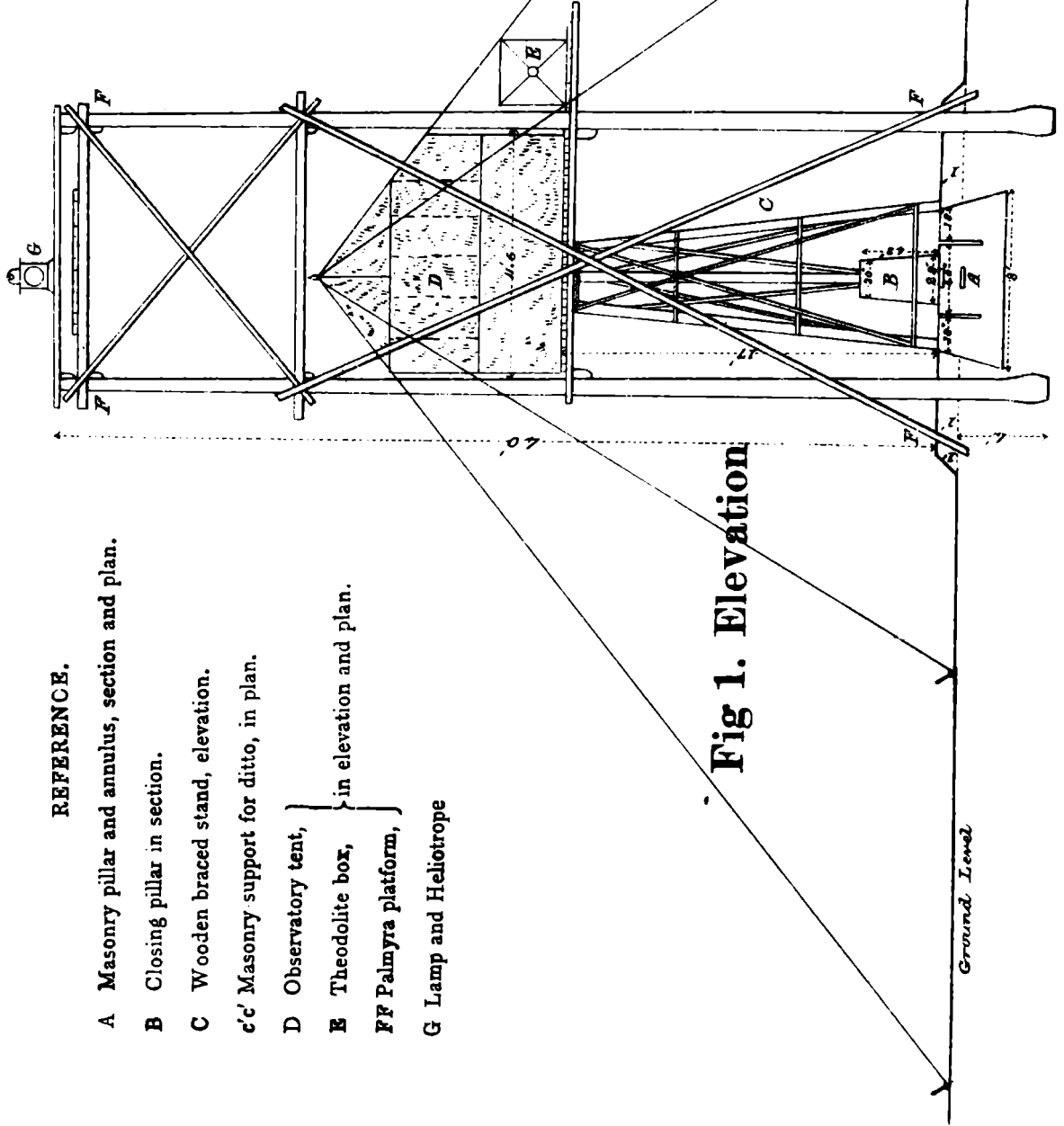


Fig 1. Elevation

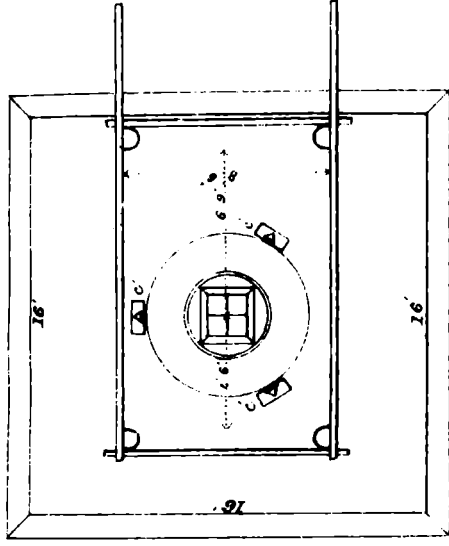
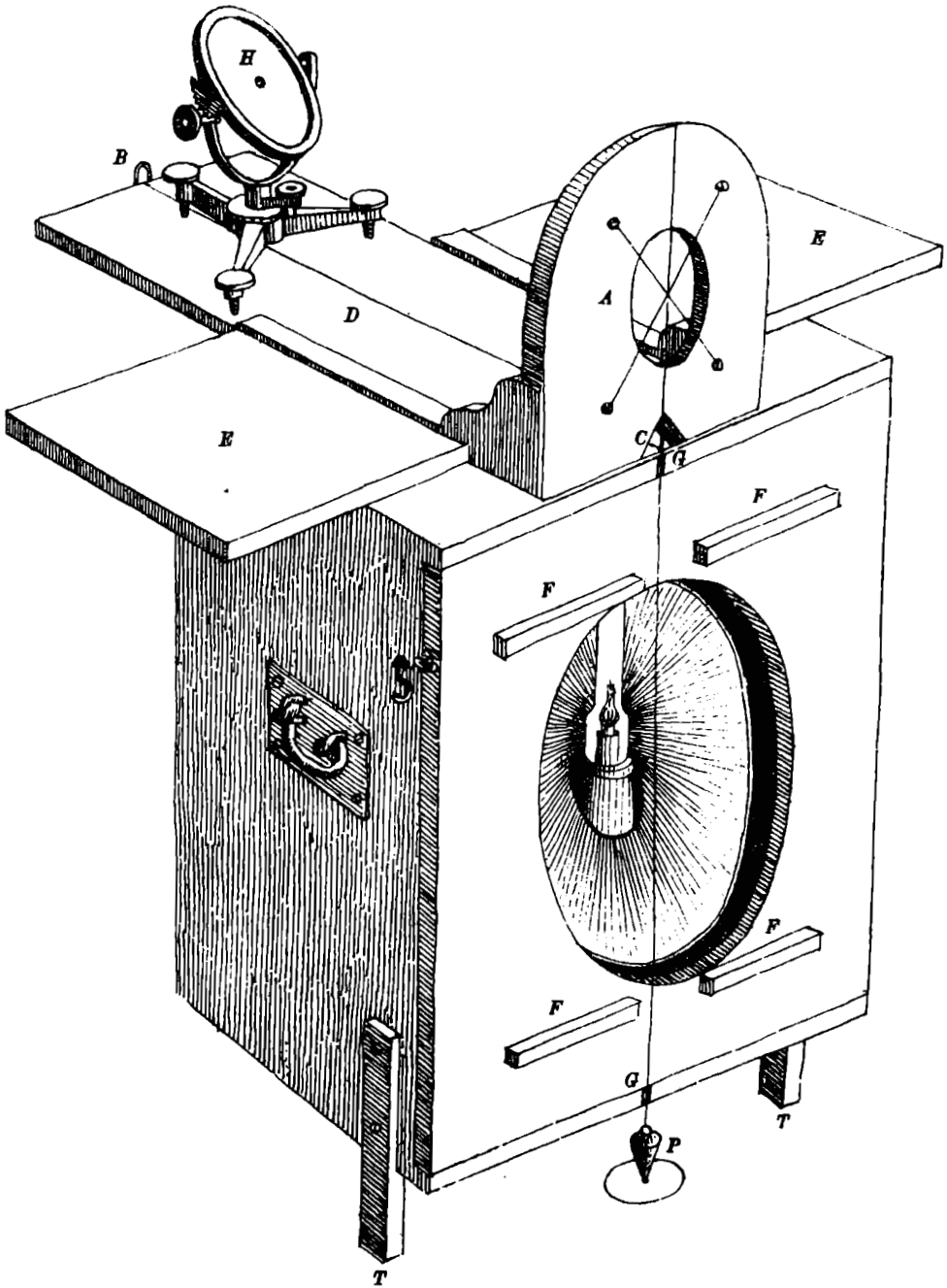


Fig 2. Plan



# PLATE III



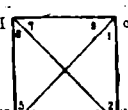

*Photoincographed at the Office of the Trigonometrical Branch, Survey of India, Dehra Dún, October 1889*







Reduction of Borta-Bonik Quadrilateral by the Method of Minimum Squares.

| <p><math>X_1</math> denotes Concluded Angle.<br/> <math>x_1</math> " most Probable Error of <math>X_1</math>.<br/> <math>w_1</math> " Reciprocal Weight "<br/> <math>s_1</math> " change of Log Sin <math>X_1</math> for 1".<br/> <math>S_{w_1}</math> " " " <math>\{X_{w_1} + X_1\}</math> for 1".</p> | <p>Explanation of Symbols.<br/> <math>e</math> denotes Spherical Excess.<br/> <math>c</math> " right hand member, or Error, of equation.<br/> <math>f</math> " Factor (indeterminate).</p> | <p><u>Borta H.S.</u> or III  or IV<br/> <u>Dhaura H.S.</u><br/> <u>Sunda H.S.</u> or I  or II</p> | <p>Statement of Data.</p> <table border="1" style="width:100%; border-collapse: collapse;"> <thead> <tr> <th>No.</th> <th>X</th> <th>w</th> <th>No.</th> <th>X</th> <th>w</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>68-33-0.17</td> <td>0.05</td> <td>5</td> <td>37-19-9.51</td> <td>0.07</td> </tr> <tr> <td>2</td> <td>42-1-19.78</td> <td>0.05</td> <td>6</td> <td>73-15-11.27</td> <td>0.09</td> </tr> <tr> <td>3</td> <td>38-58-35.75</td> <td>0.06</td> <td>7</td> <td>32-57-39.54</td> <td>0.07</td> </tr> <tr> <td>4</td> <td>30-27-8.60</td> <td>0.06</td> <td>8</td> <td>36-28-3.67</td> <td>0.06</td> </tr> </tbody> </table> | No.         | X    | w | No. | X | w | 1 | 68-33-0.17 | 0.05 | 5 | 37-19-9.51 | 0.07 | 2 | 42-1-19.78 | 0.05 | 6 | 73-15-11.27 | 0.09 | 3 | 38-58-35.75 | 0.06 | 7 | 32-57-39.54 | 0.07 | 4 | 30-27-8.60 | 0.06 | 8 | 36-28-3.67 | 0.06 |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------|------|---|-----|---|---|---|------------|------|---|------------|------|---|------------|------|---|-------------|------|---|-------------|------|---|-------------|------|---|------------|------|---|------------|------|
| No.                                                                                                                                                                                                                                                                                                     | X                                                                                                                                                                                          | w                                                                                                                                                                                                                                                                     | No.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            | X           | w    |   |     |   |   |   |            |      |   |            |      |   |            |      |   |             |      |   |             |      |   |             |      |   |            |      |   |            |      |
| 1                                                                                                                                                                                                                                                                                                       | 68-33-0.17                                                                                                                                                                                 | 0.05                                                                                                                                                                                                                                                                  | 5                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | 37-19-9.51  | 0.07 |   |     |   |   |   |            |      |   |            |      |   |            |      |   |             |      |   |             |      |   |             |      |   |            |      |   |            |      |
| 2                                                                                                                                                                                                                                                                                                       | 42-1-19.78                                                                                                                                                                                 | 0.05                                                                                                                                                                                                                                                                  | 6                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | 73-15-11.27 | 0.09 |   |     |   |   |   |            |      |   |            |      |   |            |      |   |             |      |   |             |      |   |             |      |   |            |      |   |            |      |
| 3                                                                                                                                                                                                                                                                                                       | 38-58-35.75                                                                                                                                                                                | 0.06                                                                                                                                                                                                                                                                  | 7                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | 32-57-39.54 | 0.07 |   |     |   |   |   |            |      |   |            |      |   |            |      |   |             |      |   |             |      |   |             |      |   |            |      |   |            |      |
| 4                                                                                                                                                                                                                                                                                                       | 30-27-8.60                                                                                                                                                                                 | 0.06                                                                                                                                                                                                                                                                  | 8                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | 36-28-3.67  | 0.06 |   |     |   |   |   |            |      |   |            |      |   |            |      |   |             |      |   |             |      |   |             |      |   |            |      |   |            |      |

**Determination of Right hand Members, or Errors, of Equations.**

Side Error = 10,000,000 x (Sum of Log Sines on right - Sum of Log Sines on left).

| Seconds of X<br>A, I, II, IV | Final Error<br>tested | Seconds of X<br>A, I, II, III | Final Error<br>tested | Seconds of X<br>A, I, III, IV | Final Error<br>tested | X - $\frac{c}{s}$                       | Log Sine   | $c$ | Test<br>Final Error<br>Change in<br>Log Sine | X - $\frac{c}{s}$ | Log Sine                 | $c$        | Test<br>Final Error<br>Change in<br>Log Sine |          |       |
|------------------------------|-----------------------|-------------------------------|-----------------------|-------------------------------|-----------------------|-----------------------------------------|------------|-----|----------------------------------------------|-------------------|--------------------------|------------|----------------------------------------------|----------|-------|
| $X_1$ 0.17                   | +0.087                | $X_2$ 35.75                   | +0.193                | $X_3$ 2.51                    | +0.240                | $X_1$ 68-32-58.803                      | 7.96882602 | 9   | +0.087                                       | +0.78             | $X_1 + X_2$ 80-59-54.162 | 7.99461796 | 4                                            | +0.161   | +0.66 |
| $X_1 + X_2$ 55.53            | +0.161                | $X_1 + X_3$ 18.11             | +0.190                | $X_1 + X_3$ 50.81             | +0.336                | $X_2$ 38-58-36.208                      | 7.98668863 | 26  | +0.193                                       | +5.02             | $X_2 + X_3$ 73-15-9.726  | 7.98117733 | 6                                            | +0.110   | +0.66 |
| $X_2$ 3.60                   | +0.050                | $X_2$ 11.27                   | +0.110                | $X_2$ 3.62                    | +0.059                | $X_1 + X_2$ 106-12-49.735               | 7.95257366 | 6   | +0.336                                       | -2.02             | $X_2 + X_3$ 36-48-23.660 | 7.76052600 | 29                                           | +0.059   | -1.71 |
| Sum 4.30                     |                       | Sum 5.12                      |                       | Sum 3.74                      |                       | Sum = $S_1 =$                           | 7.76936331 |     | $S'_1 =$                                     | +3.78             | Sum = $S_2 =$            | 7.76034739 |                                              | $S'_2 =$ | -0.41 |
| $e$ 4.132                    |                       | $e$ 4.617                     |                       | $e$ 3.222                     |                       | $e_1 = 10,000,000 \times (S_1 - S_2) =$ | -4.2       |     | $S'_1 - S'_2 =$                              | -4.19             |                          |            |                                              |          |       |
| $e_1 = +$                    | 0.198                 | $e_1 = +$                     | 0.493                 | $e_1 = +$                     | 0.517                 |                                         |            |     |                                              |                   |                          |            |                                              |          |       |

**Table I.—Numerical Statement of Co-efficients in Equations of Condition.**

|   | $w_1$  | $w_2$  | $w_3$  | $w_4$  | $w_5$  | $w_6$  | $w_7$  | $w_8$  |
|---|--------|--------|--------|--------|--------|--------|--------|--------|
| A | +1     | +1     | +1     | +1     |        |        |        |        |
| B |        |        | +1     | +1     | +1     |        |        |        |
| C |        |        |        |        | +1     | +1     | +1     | +1     |
| D |        |        |        |        |        | +12    | +6     | +29    |
|   | $-s_1$ | $+s_2$ | $+s_3$ | $-s_4$ | $-s_5$ | $+s_6$ | $-s_7$ | $+s_8$ |

**Table II.—Products of Co-efficients in Table I and the corresponding  $w$ .**

|     | $w_1$ | $w_2$ | $w_3$ | $w_4$ | $w_5$ | $w_6$ | $w_7$ | $w_8$ |
|-----|-------|-------|-------|-------|-------|-------|-------|-------|
| A.w | +0.05 | +0.05 | +0.06 | +0.06 |       |       |       |       |
| B.w |       |       | +0.06 | +0.06 | +0.07 | +0.09 |       |       |
| C.w |       |       |       |       | +0.07 | +0.09 | +0.07 | +0.06 |
| D.w |       |       |       |       |       | +1.08 | +0.42 | +1.74 |

**Table III.—Products of quantities in Table II and those in corresponding cols. of Table I.**

|       | $w_1$ | $w_2$ | $w_3$ | $w_4$ | $w_5$ | $w_6$ | $w_7$  | $w_8$  |
|-------|-------|-------|-------|-------|-------|-------|--------|--------|
| A.w.A | +0.05 | +0.05 | +0.06 | +0.06 |       |       |        |        |
| A.w.B |       |       | +0.06 | +0.06 |       |       |        |        |
| A.w.D |       |       |       |       |       |       |        |        |
| B.w.B |       |       | +0.06 | +0.06 | +0.07 | +0.09 |        |        |
| B.w.C |       |       |       |       | +0.07 | +0.09 |        |        |
| B.w.D |       |       |       |       |       | +1.08 |        |        |
| C.w.C |       |       |       |       | +0.07 | +0.09 | +0.07  | +0.06  |
| C.w.D |       |       |       |       |       | +1.08 | +0.42  | +1.74  |
| D.w.D |       |       |       |       |       |       | +12.96 | +2.52  |
|       |       |       |       |       |       |       |        | +50.46 |
|       |       |       |       |       |       |       |        | +99.83 |

**Table IV.—Equations between Factors.**

| Eqn. No. | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $c$    |
|----------|-------|-------|-------|-------|--------|
| 1        | +0.22 | +0.12 |       | -1.57 | +0.198 |
| 2        |       | +0.28 | +0.16 | -0.26 | +0.493 |
| 3        |       |       | +0.29 | +3.26 | +0.517 |
| 4        |       |       |       | +9.83 | +4.2   |

**Table V.—Final Determination of Errors.**

|                  | $w_1$        | $w_2$        | $w_3$        | $w_4$   |
|------------------|--------------|--------------|--------------|---------|
| $w_1 = 0.05$     | $w_2 = 0.05$ | $w_3 = 0.06$ | $w_4 = 0.06$ |         |
| $f_1.A$          | +0.0872      | +0.0872      | +0.0872      | +0.0872 |
| $f_1.B$          |              |              | -0.9132      | -0.9132 |
| $f_1.C$          |              |              |              |         |
| $f_1.D$          | +1.6529      | -0.7347      | +4.0611      |         |
| Sum              | +1.7406      | -0.6465      | +3.2161      | -0.8260 |
| Sum $w \times e$ | +0.0870      | -0.0324      | +0.1929      | -0.0496 |
| $w_1 = 0.07$     | $w_2 = 0.09$ | $w_3 = 0.07$ | $w_4 = 0.06$ |         |
| $f_1.A$          | -0.9132      | -0.9132      |              |         |
| $f_1.B$          | +4.3381      | +4.3381      | +4.3381      | +4.3381 |
| $f_1.C$          |              |              | -2.2062      | -2.2062 |
| $f_1.D$          |              |              |              |         |
| Sum              | +3.4249      | +1.9207      | +3.2360      | -0.8686 |
| Sum $w \times e$ | +0.2347      | +0.1049      | +0.2265      | -0.0593 |

\* Note.—The Errors obtained in this Table are to be applied with changed signs, as corrections, in the Triangle sheets.





Survey of India Department.

NO. ... PARTY ( ... ) SEASON 18 ...

Reduction of *Thob* Polygon by the Method of Minimum Squares.

**Explanation of Symbols.**

$X_n$  denotes Concluded Angle.

$x_n$  " most Probable Error of  $X_n$ .

$w_n$  " Reciprocal Weight "

$e_n$  " change of Log Sin  $X_n$  for 1".

$c$  " Spherical Excess.

$e$  " right hand member, or Error, of equation.

$f$  " Factor (indeterminate).

[ ] " Summation.

**Statement of Data.**

| No. | X     | w     | No.  | X  | w     | No.   | X    | w  |       |       |      |
|-----|-------|-------|------|----|-------|-------|------|----|-------|-------|------|
| 1   | 73.29 | 40.52 | 0.06 | 7  | 40.23 | 3.39  | 0.04 | 13 | 65.7  | 39.43 | 0.08 |
| 2   | 43.42 | 31.60 | 0.06 | 8  | 53.28 | 4.46  | 0.03 | 14 | 57.6  | 10.78 | 0.04 |
| 3   | 57.47 | 50.94 | 0.03 | 9  | 31.8  | 9.46  | 0.05 | 15 | 57.66 | 12.47 | 0.06 |
| 4   | 54.5  | 20.92 | 0.05 | 10 | 54.17 | 19.28 | 0.07 | 16 | 72.36 | 56.03 | 0.07 |
| 5   | 78.57 | 36.91 | 0.07 | 11 | 53.27 | 4.58  | 0.06 | 17 | 59.31 | 56.48 | 0.04 |
| 6   | 47.2  | 5.03  | 0.03 | 12 | 72.14 | 54.72 | 0.05 | 18 | 47.51 | 12.26 | 0.05 |

**Determination of Right hand Members, or Errors, of Equations.**  
Triangular Errors = Sum of Angles - (180° + e).

| A 1       | Seconds of X | Final Errors tested | A 2       | Seconds of X | Final Errors tested | A 3       | Seconds of X | Final Errors tested | A 4       | Seconds of X | Final Errors tested | A 5       | Seconds of X | Final Errors tested | A 6       | Seconds of X | Final Errors tested |
|-----------|--------------|---------------------|-----------|--------------|---------------------|-----------|--------------|---------------------|-----------|--------------|---------------------|-----------|--------------|---------------------|-----------|--------------|---------------------|
| $X_1$     | 40.52        | -0.049              | $X_7$     | 20.92        | -0.101              | $X_3$     | 3.39         | -0.200              | $X_{10}$  | 19.28        | -0.054              | $X_{16}$  | 54.43        | +0.089              | $X_{12}$  | 56.03        | -0.110              |
| $X_2$     | 31.60        | +0.072              | $X_8$     | 36.91        | -0.009              | $X_4$     | 4.46         | -0.091              | $X_{11}$  | 43.55        | +0.046              | $X_{14}$  | 10.78        | +0.124              | $X_{17}$  | 56.48        | +0.017              |
| $X_3$     | 50.94        | +0.072              | $X_9$     | 5.03         | -0.011              | $X_5$     | 9.46         | -0.158              | $X_{13}$  | 54.72        | +0.047              | $X_{15}$  | 12.47        | +0.113              | $X_{18}$  | 12.26        | +0.005              |
| Sum       | 3.06         |                     | Sum       | 2.86         |                     | Sum       | 2.31         |                     | Sum       | 2.55         |                     | Sum       | 2.68         |                     | Sum       | 2.77         |                     |
| $e_1 = +$ | 2.964        |                     | $e_2 = -$ | 2.981        |                     | $e_3 = -$ | 2.759        |                     | $e_4 = +$ | 2.516        |                     | $e_5 = +$ | 2.354        |                     | $e_6 = -$ | 2.858        |                     |
| $e_1 = +$ | 0.096        | +0.046              | $e_2 = -$ | 0.121        | -0.121              | $e_3 = -$ | 0.449        | -0.449              | $e_4 = +$ | 0.034        | +0.034              | $e_5 = +$ | 0.326        | +0.326              | $e_6 = -$ | 0.088        | -0.088              |

Central Error = Sum of Angles at III - 360°

Side Error = 10,000,000 x [Sum of Log Sines on right - Sum of Log Sines on left]

| Seconds of X | Final Errors tested | X - $\frac{1}{2}$ | Log Sine  | Test        |                    | X - $\frac{1}{3}$ | Log Sine | Test        |                    |       |        |           |       |        |           |       |        |
|--------------|---------------------|-------------------|-----------|-------------|--------------------|-------------------|----------|-------------|--------------------|-------|--------|-----------|-------|--------|-----------|-------|--------|
|              |                     |                   |           | Final Error | Change in Log Sine |                   |          | Final Error | Change in Log Sine |       |        |           |       |        |           |       |        |
| $X_1$        | 40.52               | -0.049            | $X_7$     | 20.92       | -0.101             | $X_3$             | 3.39     | -0.200      | $X_{10}$           | 19.28 | -0.054 | $X_{16}$  | 54.43 | +0.089 | $X_{12}$  | 56.03 | -0.110 |
| $X_2$        | 31.60               | +0.072            | $X_8$     | 36.91       | -0.009             | $X_4$             | 4.46     | -0.091      | $X_{11}$           | 43.55 | +0.046 | $X_{14}$  | 10.78 | +0.124 | $X_{17}$  | 56.48 | +0.017 |
| $X_3$        | 50.94               | +0.072            | $X_9$     | 5.03        | -0.011             | $X_5$             | 9.46     | -0.158      | $X_{13}$           | 54.72 | +0.047 | $X_{15}$  | 12.47 | +0.113 | $X_{18}$  | 12.26 | +0.005 |
| Sum          | 3.06                |                   | Sum       | 2.86        |                    | Sum               | 2.31     |             | Sum                | 2.55  |        | Sum       | 2.68  |        | Sum       | 2.77  |        |
| $e_1 = -$    | 0.43                | -0.430            | $e_2 = -$ | 0.430       | -0.430             | $e_3 = +$         | 0.279    | +0.279      | $e_4 = -$          | 0.60  | -0.60  | $e_5 = +$ | 0.60  | +0.60  | $e_6 = -$ | 0.60  | -0.60  |

Table V.—Final Determination of Errors†

|                | $x_1$    | $x_2$    | $x_3$    | $x_4$    | $x_5$    | $x_6$    | $x_7$    | $x_8$    | $x_9$    | $x_{10}$ | $x_{11}$ | $x_{12}$ | $x_{13}$ | $x_{14}$ | $x_{15}$ | $x_{16}$ | $x_{17}$ | $x_{18}$ |
|----------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| $f_1$ A        | + 1.0346 | + 1.0346 | + 1.0346 |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          |
| $f_2$ B        |          |          |          | - 0.1704 | - 0.1704 | - 0.1704 |          |          |          |          |          |          |          |          |          |          |          |          |
| $f_3$ C        |          |          |          |          |          |          | - 3.1415 | - 3.1415 | - 3.1415 |          |          |          |          |          |          |          |          |          |
| $f_4$ D        |          |          |          |          |          |          |          |          |          | + 1.0044 | + 1.0044 | + 1.0044 |          |          |          |          |          |          |
| $f_5$ E        |          |          |          |          |          |          |          |          |          |          |          |          | + 2.4635 | + 2.4635 | + 2.4635 |          |          |          |
| $f_6$ F        |          |          |          |          |          |          |          |          |          |          |          |          |          |          |          | + 2.2566 | + 2.2566 | + 2.2566 |
| $f_7$ G        | + 1.8520 |          |          | + 1.8520 |          |          | + 1.8520 |          |          | + 1.8520 |          |          | + 1.8520 |          |          | + 1.8520 |          |          |
| $f_8$ H        |          | + 0.1862 | - 0.1872 |          | + 0.0292 | - 0.1862 |          | + 0.1176 | - 0.0296 |          | + 0.1670 | - 0.0583 |          | + 0.1274 | - 0.1274 |          | + 0.1274 | - 0.1862 |
| Sum            | - 2.8176 | + 1.2208 | + 0.8974 | - 2.0214 | - 0.1312 | - 0.3546 | - 4.9926 | - 3.0229 | - 3.1709 | - 0.3674 | + 1.1514 | + 0.9458 | + 1.1115 | + 3.0908 | + 2.3861 | - 1.5607 | + 0.4164 | + 0.1006 |
| Sum $x_n = x'$ | + 0.0490 | + 0.0732 | + 0.0718 | - 0.1011 | - 0.0092 | - 0.0107 | - 0.1997 | - 0.2702 | - 0.1505 | - 0.0579 | + 0.0461 | + 0.0473 | + 0.0589 | + 0.1234 | + 0.1134 | - 0.1096 | + 0.0168 | + 0.0058 |

† Note.—The Errors computed in this Table are to be applied with changed signs, as corrections, to the Triangle sheets.



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NO. \_\_\_\_\_ PARTY ( \_\_\_\_\_ ) SEASON 18 \_\_\_\_\_  
 Reduction of \_\_\_\_\_ Polygon by the Method of Minimum Squares.

| Eq. No. | Co-eff. of Factor | In Eq. No. | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ | $x_{11}$ | $x_{12}$ | $x_{13}$ | $x_{14}$ | $x_{15}$ | $x_{16}$ | $x_{17}$ | $x_{18}$ | $x_{19}$ | $x_{20}$ |       |
|---------|-------------------|------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|-------|
| A       | +1                | +1         | +1    | +1    | +1    | +1    | +1    | +1    | +1    | +1    | +1    | +1       | +1       | +1       | +1       | +1       | +1       | +1       | +1       | +1       | +1       | +1       | +1    |
| G       | +1                | +1         | +1    | +1    | +1    | +1    | +1    | +1    | +1    | +1    | +1    | +1       | +1       | +1       | +1       | +1       | +1       | +1       | +1       | +1       | +1       | +1       | +1    |
| H       | +1                | +1         | +1    | +1    | +1    | +1    | +1    | +1    | +1    | +1    | +1    | +1       | +1       | +1       | +1       | +1       | +1       | +1       | +1       | +1       | +1       | +1       | +1    |
| A.      | +0.06             | +0.06      | +0.07 | +0.07 | +0.07 | +0.07 | +0.07 | +0.07 | +0.07 | +0.07 | +0.07 | +0.07    | +0.07    | +0.07    | +0.07    | +0.07    | +0.07    | +0.07    | +0.07    | +0.07    | +0.07    | +0.07    | +0.07 |
| G.      | +0.06             | +0.06      | +0.07 | +0.07 | +0.07 | +0.07 | +0.07 | +0.07 | +0.07 | +0.07 | +0.07 | +0.07    | +0.07    | +0.07    | +0.07    | +0.07    | +0.07    | +0.07    | +0.07    | +0.07    | +0.07    | +0.07    | +0.07 |
| H.      | +0.06             | +0.06      | +0.07 | +0.07 | +0.07 | +0.07 | +0.07 | +0.07 | +0.07 | +0.07 | +0.07 | +0.07    | +0.07    | +0.07    | +0.07    | +0.07    | +0.07    | +0.07    | +0.07    | +0.07    | +0.07    | +0.07    | +0.07 |

Table I.—Numerical Statement of Co-efficients in Equations of Condition.

Table II.—Products of Co-efficients in Table I and the corresponding  $x_i$ .

Table III.—Products of the quantities in Table II and those in corresponding columns of Table I.

Table IV.—Equations between Factors, and determination of the Values of  $f_i$ .

| Eq. No. | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | $f_6$ | $f_7$ | $f_8$ | $f_9$ | $f_{10}$ | $f_{11}$ | $f_{12}$ | $f_{13}$ | $f_{14}$ | $f_{15}$ | $f_{16}$ | $f_{17}$ | $f_{18}$ | $f_{19}$ | $f_{20}$ |       |       |       |
|---------|-------|-------|-------|-------|-------|-------|-------|-------|-------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|-------|-------|-------|
| 1       | +0.20 | +0.15 | +0.12 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16 | +0.16 |       |
| 2       | +0.20 | +0.15 | +0.12 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16 | +0.16 | +0.16 |
| 3       | +0.20 | +0.15 | +0.12 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16 | +0.16 | +0.16 |
| 4       | +0.20 | +0.15 | +0.12 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16 | +0.16 | +0.16 |
| 5       | +0.20 | +0.15 | +0.12 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16 | +0.16 | +0.16 |
| 6       | +0.20 | +0.15 | +0.12 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16 | +0.16 | +0.16 |
| 7       | +0.20 | +0.15 | +0.12 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16 | +0.16 | +0.16 |
| 8       | +0.20 | +0.15 | +0.12 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16 | +0.16 | +0.16 |
| 9       | +0.20 | +0.15 | +0.12 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16 | +0.16 | +0.16 |
| 10      | +0.20 | +0.15 | +0.12 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16 | +0.16 | +0.16 |
| 11      | +0.20 | +0.15 | +0.12 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16 | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16    | +0.16 | +0.16 | +0.16 |



## PART III.

**Azimuths.**

## CHAPTER I. CIRCUMPOLAR-STAR OBSERVATIONS FOR AZIMUTH.

1. In order that any network of triangulation may be correctly orientated, it is necessary that at least one side of one of the triangles should have its azimuth rigorously determined by astronomical observations. Theoretically, if the azimuth of one side is known, all the others follow from it, by a simple computation based on assumed elements of the earth's figure in combination with the measured angles of the triangles. Practically, however, the difficulty comes in that the elements of the earth's figure are not yet known with absolute certainty, and also that the unavoidable errors in the measurement of the angles become involved in the computed azimuths. To meet this, it was originally intended that the computed azimuths should at certain stated distances be *corrected* by means of celestial observations, and the accumulation of error thus kept within narrow bounds. The modern view is, however, opposed to this, and the practice now obtaining is to observe astronomical azimuths at convenient intervals of about  $2^{\circ}$  apart, and, by comparing these with the azimuths brought up by computation from the origin at Kaliánpur\*, to form equations whose solution will, when the whole scheme is complete, afford corrections to the adopted elements, and aid in pointing out the existence of local attractions or other anomalies.

2. In the Third Edition of the Auxiliary Tables, pages 78 to 81, six methods of measuring an azimuth are given: of these, however, only one—the second somewhat modified—is ever used in the Great Trigonometrical Survey where a rigorous determination is required. Circumpolar stars, *i.e.*, those stars whose N.P.D. is less than the latitude of the station of observation, are, in their diurnal course, twice stationary in azimuth, once on the eastern, and once on the western side of the pole; at which times they are said

\* The adopted initial azimuth is that of Súrntál at Kaliánpur taken at  $190^{\circ} 27' 5'' \cdot 10$ , *vide* page 141 of Volume II of the *Account of the Operations of the Great Trigonometrical Survey of India*.

to be *at elongation*. At these instants their true azimuths are easily computed, and they afford excellent objects for observation, being almost stationary in the same direction for a considerable time. The principle of the observation consists merely in measuring the angle in the horizontal plane between the star at elongation and a referring mark, whence the azimuth of the latter, and through it the azimuth of any side of the triangulation at that station, becomes known. The method of observation, the management of the levels, which is especially important, and other valuable details will be found at pages 38 *et seq.* of the Auxiliary Tables, to which the reader is referred. Further information on this subject will also be found in Chapter XII, Volume II of the *Account of the Operations of the Great Trigonometrical Survey of India*. Practically the time during which the observations may be made, is not limited to the exact moment of elongation; the limit of safety depending partly on the latitude, and partly on the star's polar distance. The measurements made before and after the elongation are corrected for the interval by a process explained in the next chapter, and thus become of equal value with those made at the moment of elongation in determining the final azimuth. The zeros and changes of face should be the same as those laid down for horizontal angles, *vide* Part II, Chapter I, para. 24 of this handbook, with this exception only, that four rounds instead of three should be taken on each face, divided up as exemplified on page 42 of the Auxiliary Tables.

3. The referring mark should be at a distance of about 2 miles, so as to be well defined in the telescope with solar focus, and yet sufficiently close to ensure easy visibility. It is convenient also, though not indispensable, that it should be about on the same level as the station from which the observations are made, in order to avoid corrections for collimation and level, which would otherwise be necessary. The aperture of the lamp should be reduced to one inch or thereabouts.

4. Some criterion of this limit of safety is desirable in order that the observer may lay out his time to the best advantage. In imposing such a limit it may be as well to state that the formulæ now employed in the computation of azimuths are sufficiently rigorous to justify an observer in selecting a circumpolar star at almost any point of its diurnal course for determination of azimuth. It must be remembered on the other hand that a star

Referring mark.

Time-limits of observations.

is only stationary in azimuth at elongation, and the farther it is from that point the quicker it moves (in azimuth), and the more difficult of accurate intersection it becomes in consequence. The following limitation may be accepted, *viz.*, that a circumpolar star should not be used for observations of azimuth when its change of azimuth exceeds 1" in 10 seconds of time. The following table, which is founded on this limitation, gives the approximate value of the hour angle reckoned from time of elongation (either before or after), which should not be exceeded in observations of this kind.

| N.P.D. | Latitude.  |            |            |            |            |            |            |
|--------|------------|------------|------------|------------|------------|------------|------------|
|        | 8°         | 12°        | 16°        | 20°        | 24°        | 28°        | 32°        |
|        | <i>h m</i> | <i>h m</i> | <i>h m</i> | <i>m h</i> | <i>h m</i> | <i>h m</i> | <i>h m</i> |
| 1° 0'  | 2 20       | 2 12       | 2 5        | 1 57       | 1 50       | 1 43       | 1 36       |
| 1 15   | 1 49       | 1 43       | 1 38       | 1 32       | 1 27       | 1 21       | 1 16       |
| 1 30   | 1 30       | 1 25       | 1 21       | 1 16       | 1 12       | 1 7        | 1 3        |
| 1 45   | 1 17       | 1 13       | 1 9        | 1 5        | 1 1        | 0 57       | 0 54       |
| 2 0    | 1 7        | 1 3        | 1 0        | 0 57       | 0 53       | 0 50       | 0 47       |
| 2 30   | 0 53       | 0 50       | 0 48       | 0 45       | 0 43       | 0 40       | 0 37       |
| 3 0    | 0 44       | 0 42       | 0 40       | 0 38       | 0 35       | 0 33       | 0 31       |
| 4 0    | 0 33       | 0 31       | 0 30       | 0 28       | 0 27       | 0 25       | 0 23       |
| 5 0    | 0 26       | 0 25       | 0 24       | 0 23       | 0 21       | 0 20       | 0 19       |
| 6 0    | 0 22       | 0 21       | 0 20       | 0 19       | 0 18       | 0 17       | 0 16       |

The table is based on the following approximate formula,  $\sin P = \frac{\delta A}{\delta P} \cdot \frac{\text{colat}}{\text{N.P.D.}}$ , where  $P$  is the hour angle from culmination, and  $\delta A$  is the change of azimuth corresponding to a change  $\delta P$  in the hour angle.

The value of the fraction  $\frac{\delta A}{\delta P}$  is taken at  $\cdot 007$ , corresponding to the change of 1" of azimuth in 10 seconds of time, as explained above. It is by no means intended that the observer should use this table to extend his work to an undue amount on any one night with a view of thereby decreasing the number of nights' work required. It is a well known fact, though as yet quite unexplained, that even the best instruments will give different results at different times, or even on consecutive nights, when all the circumstances are *apparently* the same; and it is therefore always better, when a certain quantity of measures of one



thing have to be accumulated, to spread them over several days, rather than to take them all in one. The chief use of the table is to enable the observer to see how long before or after elongation, it will be safe for him to work, supposing that some accidental cause, such as clouds, prevents him observing at or very near the most favourable position.

## CHAPTER II. COMPUTATION OF CIRCUMPOLAR-STAR OBSERVATIONS.

1. The form provided for these calculations is *P. 32*. Specimens of the calculation are given at the end of this chapter.

2. Page 1 commences with a statement of data. First are entered the circumpolar stars, the one observed at Eastern Elongation after E. and that at Western Elongation after W. Below are entered the stars employed for finding time, together with their approximate declinations and zenith distances, provision being made for the case when the stars employed for time of western elongation are not the same as those for eastern elongation. On the right-hand side of the same statement are given the Azimuth of the Referring Mark as obtained through the triangulation, the Latitude and Longitude of the observing station obtained in the same way—the longitude being reduced to time and then to decimals of a day—the names and numbers of the Theodolite and Chronometer employed, the values of the level scales of the theodolite, and lastly the name of the observer. The rest of page 1 is taken up with the determination of correction and rate of the chronometer from the time-star observations. The correction is found for each night by ascertaining the chronometer times at which certain selected stars pass the meridian and comparing them with the Stars' Right Ascensions, and the rate is given by the successive differences of these comparisons from night to night.

3. The first thing to be done in reducing the time-star observations, is to compute the corrections necessary for error of *Collimation*, error of Level or *Inclination* in the transit axis, and error of setting of the instrument in the meridian, otherwise called *Deviation*. The method of obtaining these corrections is already fully explained in the Auxiliary Tables,

Corrections for Instrumental Errors.

Third Edition, pages 42 and 43. Provision is made in the form for the necessary calculations under the heading of "Transit Corrections for Instrumental Errors." The portion of the form at the bottom of page 1, headed "Deduced Chronometer Corrections and Rates," does not need explanation as the headings of the several columns are sufficiently explicit.

4. The next step is to find the apparent places of the circumpolar stars at elongation. If they are Nautical Almanac stars, page 2 of the form is not required, as the apparent places are given for Upper Transit at Greenwich, and the R.A. and N.P.D. can be entered at once on page 3 for each day of observation in the columns set apart for the purpose. The corrections to these values for difference of longitude and for time of elongation must next be computed and entered in the two following columns. Taking the change for one day and multiplying this by the longitude reduced to the decimal of a day (see page 1) gives the first correction. As stars transit in India before they transit at Greenwich, the correction is always a backward one. For the second correction it is first necessary to know the horary angle at elongation: this is computed on the lower portion of page 3 for the first date of observation, and for the present purpose will be sufficient. The angle which is obtained in arc must be reduced to time and to the decimal of a day and then multiplied by the daily changes in R.A. and N.P.D. to give the correction to elongation. If the elongation is eastern the correction will be a backward one, if western, a forward one. Columns 2, 3 and 4 of each compartment added together algebraically, give the star's apparent place at elongation.

5. If now the circumpolar stars are not Nautical Almanac stars, but taken from a Star Catalogue, page 2 of the form must be employed for calculating their *apparent* places on the first and last days of observation. The catalogue gives the *mean* places of stars on the 1st January of its epoch. The corrections required to reduce to *mean* places on the 1st January of the year of observation are similar for R.A. and N.P.D., and are of the following form.

$$\left\{ p + \mu + \frac{s}{100} \cdot \frac{y}{2} \right\} y$$

in which  $p$  is the Annual Precession,  $\mu$  the Annual Proper Motion,  $s$  the

Secular Variation, in R.A. and N.P.D. respectively, and  $y$  the number of years from the epoch of the catalogue to the 1st January of year of observation. If the star is near the pole, and the period elapsed from the epoch of the catalogue be considerable, a further correction is necessary, as stated in the foot-note to page 2 of the form (see page 62 of the Auxiliary Tables). The *apparent* places at Greenwich *mean midnight* are then obtained by applying the corrections

$$Ee + Ff + Gg + Hh + l + L - 300 \text{ to the R.A.,}$$

$$\text{and } Ee' + Ff' + Gg' + Hh' + l' + L - 300 \text{ to the N.P.D ;}$$

$e, f, \&c.,$  and  $e', f', \&c.,$  being furnished by the catalogue and  $E, F, \&c.,$  called Airy's Day Numbers, being taken from the Nautical Almanac for the first and last days of observation. Provision for these calculations is made on page 2 of the form. Having obtained the *apparent* places of the circumpolar stars for Greenwich *mean midnight* (Gr. M. M.) on the first and last days of observation, the next step is to reduce these to the times of Upper Transit at Greenwich. This calculation is made independently of the form. From page II of the Nautical Almanac for the required month take out the sidereal time of Greenwich mean midnight for the astronomical date of observation, *i.e.*, take the mean of the successive sidereal times of mean noon and subtract 12 hours, and compare it with the R.A. of the star.\* Four cases will present themselves, *viz.*,

(1) R.A. of star *greater* than Sid. T. at Gr. M. M. by *less* than 12 hours.

(2) " " " " *more* "

(3) " *less* " " *less* "

(4) " " " " *more* "

In case (1) the star transits at Greenwich on the date of observation after Gr. M. M. by a period of time represented by

$$\text{R.A. of star} - \text{Sid. Time at Gr. M. M.}$$

This reduced to the fraction of a day and multiplied by the daily change

\* If the R.A. of the star is greater than the R.A. of the Sun at Gr. mean noon by less than  $3^m 56^s.5$  the star will transit twice during the date of observation, and care must be taken to compare with that transit which occurs nearest the elongation observed.

in R.A. or N.P.D. gives the correction to the time of transit at Greenwich and this correction is a *forward* one.

(2) The star has transited between Gr. M. N. and Gr. M. M. on the date of observation, but has not passed its lower culmination. The period from transit is represented by

$$\text{Sid. Time at Gr. M. M.} + 24 \text{ hours} - \text{R.A. of star,}$$

and the correction is a *backward* one.

(3) The star has transited on the date of observation before Gr. M. M. by a period of time represented by

$$\text{Sid. Time at Gr. M. M.} - \text{R.A. of star,}$$

and the correction is a *backward* one.

(4) The star will transit after a period represented by

$$\text{R.A. of star} + 24 \text{ hours} - \text{Sid. Time at Gr. M. M.,}$$

and the correction is a *forward* one. The difference in R.A. or N.P.D. between the first day and last day, divided by the number of days gives the daily change in each for entry in the heading of the columns on page 3, "R.A. at Upper Transit Greenwich," and "N.P.D. at Upper Transit Greenwich." Page 3 may now be entered and employed as described for Nautical Almanac stars.

6. After the *apparent* places of the circumpolar stars have been found at times of elongation, the remainder of the computation at the bottom of page 3 may be proceeded with. The horary angles and azimuths at elongation on the first days of observation may be computed, and then what are called the "Factors" for horary angle and azimuth. These factors are obtained to save the labour of computing the horary angle and azimuth independently each day with the corresponding N.P.D.

7. Having obtained the horary and azimuth angles for the first day, it is only necessary to correct them for subsequent days by multiplying the changes in N.P.D. by the 'factors'. For this calculation employ the upper part of

Horary Angle and Azimuth, and their Factors.

Deduction of Star's Azimuth &c.

page 4 of the form. Enter the table on page 4 headed "Deduction of Star's Azimuth &c.," and fill in the first three columns according to the dates of observation, and in the fourth column, omitting the first line corresponding to the date on which the N.P.D. has been actually calculated, fill in successively the accumulated change in N.P.D. from that date from page 3.

8. Now multiply each of these by the factor for the horary angle and enter in the fifth column. In the sixth column enter the horary angle for the first date—as computed on page 3—in degrees, minutes and seconds to one place of decimals, and by applying the corrections from the fifth column obtain the horary angles for the subsequent dates. In the seventh column change these angles into time. In the eighth column enter the star's R.A. at elongation from page 3 for the several dates. If the elongation is eastern subtract the horary angle from the R.A. at elongation adding 24 hours to the R.A. if necessary and if western add. The results are the sidereal times of elongation, and are to be entered in the ninth column to one place of decimals.

Sidereal Times of Elongation.

9. In the tenth column, omitting the first line, are entered the corrections to the azimuth of the first date obtained by multiplying the quantities in the fourth column by the factor for azimuth from page 3. In the last column is first entered the Azimuth at Elongation as computed for the first date and below the successive azimuths obtained by applying the corrections in the preceding column, keeping two places of decimals.

Azimuths at Elongation.

10. Up to this point the calculations have only been concerned with the star or stars at elongation. But they are observed many times both before and after elongation; and it is necessary to calculate, for each observation, a quantity  $\delta A$ , by which the azimuth of the star at the moment of observation, differs from its azimuth at elongation. The formula employed is

Formula for  $\delta A$ .

$$\delta A = \frac{2 \sin^2 \frac{1}{2} \delta P \operatorname{cosec} 1'' \tan A \cos^3 \text{N.P.D.}}{1 - 2 \sin^2 \text{N.P.D.} \sin^2 \frac{1}{2} \delta P \pm \cot P \sin \delta P'}$$

in which  $A$  is the azimuth of the circumpolar star at elongation,  $P$  the corresponding hour angle and  $\delta P$  the interval from elongation of the star for which  $\delta A$  is required in seconds of arc. The last term of the denominator is + or - according as the star is *below* or

above the position of elongation. Of the above formula the factor  $\tan A \cdot \cos^2 N.P.D.$  may be considered as constant for each night, and the successive values are computed in the table in the centre of page 4 of the form. The headings of the several columns of this table sufficiently explain themselves.

11. After this table follow a series of compartments for computing  $\delta P$  for each observation on each night for a pair of zeros, Face Right and Face Left. The chief part of the calculation is necessitated by the Chronometer times of observation needing correction for error and rate of the chronometer.

12. The first four columns need no remark except that the name of the star is to be entered in the first. In the heading of the fifth insert the 'Chronometer Time of Corrected Transit' of the time-star from page 1, or if more than one star has been observed for time, the mean time of transit of all the stars may be employed; but this should only be done when all the stars have been regularly observed. Below enter the individual chronometer times of the observations of the circumpolar star. The difference between these and the time in the heading of the column are the chronometer intervals from transit of the time-star, at which moment the chronometer correction is known. These are given in the form in seconds, and when multiplied by the rate per second, taken from page 1 and entered in the heading of the seventh column, give the corrections to be recorded in the seventh column for the rate of the chronometer during the intervals. In the heading of the next, or eighth column enter the correction to the chronometer at time of transit of the time-star from page 1, and below it the algebraical sum of this quantity and those in the preceding column. In the heading of the ninth column enter the sidereal time of elongation of the circumpolar star from page 4, for the particular date and below it the algebraical sum of the fifth and eighth columns. The difference of the quantity in the heading of the column, and each of those below is the interval from elongation, or  $\delta P$ , and is to be given in the form in minutes and seconds and also in minutes and decimals of a minute to two places.

13. When the above process has been gone through for each pair of zeros in turn, the next part of the calculation is the finding of the corresponding values

Calculation of  $\delta A$ .

of  $\delta A$ . This is commenced on page 7 under the heading "Computation of  $\delta A$  and Corrected Angle between Referring Mark and Star." First enter  $\delta P$  in minutes and decimals of a minute as previously computed on page 4, and with this as one argument and the N.P.D. of the circumpolar star as the other, enter Table XXV of the Auxiliary Tables, Part I, and take out the value of  $(2 \sin^2 \text{N.P.D.} \sin^2 \frac{1}{2} \delta P)$ , subtract it from unity and enter the result in the form. Next with  $\delta P$  and the Hour Angle,  $P$ , as arguments, enter Part II of Table XXV, and take out  $(\cot P \sin \delta P)$ : this quantity will be + when the altitude of the star is less than at elongation and - when greater. Take the algebraical sum of the two quantities and then the log and the co-log. Again refer to Table XXV and to Part III, and with  $\delta P$  as argument in minutes and seconds, from page 4, take out the quantity  $(\log 2 \sin^2 \frac{1}{2} \delta P \operatorname{cosec} 1'')$ . Insert  $(\log \tan A \cos^2 \text{N.P.D.})$  from page 4 and add. The sum is equal to  $\log \delta A$ . Taking out the natural number, which is in seconds of arc, and adding to or subtracting from the observed angle, Referring Mark and Star, according to the position of the Referring Mark relatively to the star, gives the angle Referring Mark and Star at elongation, or as it is called in the form "Corrected Angle, Referring Mark and Star."

14. When this has been done for all the observations, the next step is to calculate any correction which may be due to the body of the instrument employed for the observations not being perfectly level as shewn by the level readings taken for the purpose.

*Instrumental Correction.*

15. For the calculation of this correction the tangent of the circumpolar star's altitude at elongation is required.

*Continued.* This is computed at the bottom of page 11 of the form, from the formula

$$\sin \text{Alt.} = \sec \text{N.P.D.} \sin \text{Lat.}$$

This gives the correct altitude, which has to be converted to apparent altitude by the addition of refraction. Taking the tangent of this angle and multiplying it by the value of 1 division of the scale of the Level on the body of the instrument, or adding their logs, as in the form, gives a factor for the level error to be employed on page 12.

16. Now enter page 12 and to the mean of all the east readings of the level on Face Right add the mean of all the east readings on Face Left, for each pair of zeros independently: do the same for the west readings, then  $\frac{1}{4}$ th of the difference of the two resulting quantities is the level correction, denoted in the form by N in divisions of the level. Multiplying by the factor on the preceding page gives the correction in seconds of arc to be applied to the Angles Referring Mark and Star at elongation. The sign of the correction is found by the rule at the bottom of the page in the form.\* The rule given on page 44 of the Auxiliary Tables applies to the angle Referring Mark and North Point. The correction will have the same sign for the observations on the same pairs of zeros, Face Right and Face Left.

17. The lower half of page 12 may now be filled up. First abstract all the results "Corrected Angle Referring Mark and Star" from pages 7 to 11. Take out the means of the results on each face of each pair of zeros separately and then the mean of Face Right and Face Left. The sign of this quantity will be + if the R.M. is east of the star and - if west. Apply the level corrections from the upper half of the page and there result as many values of the angle Referring Mark and Star at Elongation as there are pairs of zeros. Now apply the Corrected Angle to the Star's Azimuth at Elongation from the south, which is obtained thus:—if the elongation is east add  $180^\circ$  to the azimuths computed on page 4 for the respective dates, but if west subtract the azimuths from  $180^\circ$  and there result as many values of the Azimuth of the Referring Mark. Take the mean of the values of Azimuth by Western Elongation and also that by Eastern Elongation and then the mean of both. This angle is then applied to the angle between one of the surrounding stations and the Referring Mark for comparison with the value brought up by triangulation.

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\* The following consideration may help in the application of the sign:—If the E. end readings are the greater the apparent, or instrumental, zenith is west of the true zenith, and a plane through the instrumental zenith, the instrument and star will cut the horizon to the east of the point to which the star would be referred from the true zenith. If the Referring Mark is to the east of the star the angle Referring Mark and Star is consequently too small and the correction is therefore additive. Similar reasoning applies to other positions of the Referring Mark and the apparent zenith.

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NO. \_\_\_\_\_ PARTY ( \_\_\_\_\_ ) SEASON 18 \_\_\_\_\_

Computation of Circumpolar Star Observations for Azimuth at Toungzin H.S.

|                                                             |                                                                                        |  |
|-------------------------------------------------------------|----------------------------------------------------------------------------------------|--|
| Circumpolar Stars. E. <u>N: 1302 Gr. 9-Yr. W. Polaris</u>   | Azimuth of Referring Mark (by Triangulation) <u>31-16-25.685</u>                       |  |
| Transit Stars. Approx. Dec. N. or S. Z. D.                  | Latitude of Station of Obsn. <u>16-25-50.13</u>                                        |  |
| <u>N: 893 Gr. 9-Yr.</u> } E.E. <u>29-54 S.</u> <u>46-20</u> | Longitude <u>97-42-46.67 = 6-30-51.11 = 0.271</u>                                      |  |
| <u>Sirius</u> } W.E. <u>16-34 S.</u> <u>33-0</u>            | Instrument <u>Barrow's 24-in. Theod. No. 2</u> Chron. <u>Sideral No. 3402 for 1884</u> |  |
|                                                             | 1 Divn. of Transit Axis Level = <u>l = 0.875</u>                                       |  |
|                                                             | 1 " Body Level, <u>l<sub>1</sub> = 1.031</u> ; <u>l<sub>2</sub> =</u>                  |  |
|                                                             | Observer <u>Major G. Strachan R.E.</u>                                                 |  |

Transit Corrections for Instrumental Errors.

| Date 1884       | 18 <sup>th</sup> Mar | 18 <sup>th</sup> Mar | 19 <sup>th</sup> Mar | 19 <sup>th</sup> Mar |  |  |  |  |  |  |  |  |
|-----------------|----------------------|----------------------|----------------------|----------------------|--|--|--|--|--|--|--|--|
| Star            | <u>Sirius N: 893</u> | <u>Sirius</u>        | <u>N: 893</u>        | <u>N: 893</u>        |  |  |  |  |  |  |  |  |
| Collimation     | + 6.12               | - 4.14               | + 2.96               | - 2.28               |  |  |  |  |  |  |  |  |
|                 | + 4.92               | + 1.40               | + 1.76               | + 1.20               |  |  |  |  |  |  |  |  |
|                 | - 0.72               | + 1.44               | + 0.10               | + 0.46               |  |  |  |  |  |  |  |  |
|                 | - 4.14               | + 4.90               | - 2.28               | - 4.68               |  |  |  |  |  |  |  |  |
|                 | + 1.40               | - 5.10               | + 1.20               | - 1.08               |  |  |  |  |  |  |  |  |
| Sum             | + 7.58               | - 1.50               | + 3.76               | - 6.38               |  |  |  |  |  |  |  |  |
| 1/2 Mean = a    | + 0.8                | - 0.2                | + 0.4                | - 0.6                |  |  |  |  |  |  |  |  |
| Inclination     | div. <u>-27.5</u>    | div. <u>-46.0</u>    | div. <u>-42.0</u>    | div. <u>-53.0</u>    |  |  |  |  |  |  |  |  |
|                 | +141.5               | +139.1               | +109.6               | +126.9               |  |  |  |  |  |  |  |  |
| Sum             | +114.0               | +93.1                | +67.6                | +71.9                |  |  |  |  |  |  |  |  |
| 1/2 Mean = i    | +28.50               | +23.28               | +16.90               | +17.98               |  |  |  |  |  |  |  |  |
| i x l           | +24.9                | +20.4                | +16.8                | +15.7                |  |  |  |  |  |  |  |  |
| Coef Z.D.*      | <u>0.839</u>         | <u>0.691</u>         | <u>0.839</u>         | <u>0.691</u>         |  |  |  |  |  |  |  |  |
| Product = β     | +20.9                | +14.1                | +14.1                | +10.8                |  |  |  |  |  |  |  |  |
| Deviation       | - 7.52               | - 7.95               | - 15.78              | - 21.70              |  |  |  |  |  |  |  |  |
| Sin Z.D.*       | <u>0.545</u>         | <u>0.728</u>         | <u>0.545</u>         | <u>0.728</u>         |  |  |  |  |  |  |  |  |
| Product = γ     | - 4.1                | - 5.7                | - 8.6                | - 15.7               |  |  |  |  |  |  |  |  |
| a + β + γ       | +17.6                | + 8.2                | + 4.2                | - 5.5                |  |  |  |  |  |  |  |  |
| 15 Sin N.P.D.*  | <u>0.070</u>         | <u>0.077</u>         | <u>0.070</u>         | <u>0.077</u>         |  |  |  |  |  |  |  |  |
| Product = Corr. | + 1.23               | + 0.63               | + 0.29               | - 0.42               |  |  |  |  |  |  |  |  |

Form.—Keep a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z, and the correction to the Observed Time of Transit to 2 places of Decimals.  
\* Take from Table XXIV of Auxiliary Tables, 3rd Edition, 1897.

Rules for the signs of Corrections, the Star being above the Pole.  
Collimation + when P. L. reads highest with scale reading from left to right and the Transit is observed with the Face to the East. Inclination + when West pilot of chronometer wire is highest. Deviation + for Stars of North aspect and - for Stars of South aspect, when reading of Azimuth circle is too high with instruments reading from left to right.

Deduced Chronometer Corrections and Rates.

| Transit Star  | Date                       | Chronometer Time of Observed Transit | Correction from above | Chronometer Time of Corrected Transit - T | Star's R.A.       | Chronometer Correction - R.A. - T | Rate Correction per day | Mean Rate Corr from per second |
|---------------|----------------------------|--------------------------------------|-----------------------|-------------------------------------------|-------------------|-----------------------------------|-------------------------|--------------------------------|
| <u>Sirius</u> | <u>1884</u>                | <u>h m s</u>                         | <u>s</u>              | <u>h m s</u>                              | <u>h m s</u>      | <u>h m s</u>                      | <u>" "</u>              |                                |
| <u>Sirius</u> | <u>18<sup>th</sup> Mar</u> | <u>6-39-30</u>                       | <u>+ 1.23</u>         | <u>6-39-31.23</u>                         | <u>6-40-3.32</u>  | <u>+ 0. 0. 32.09</u>              |                         |                                |
| <u>N: 893</u> | <u>" "</u>                 | <u>9- 4-32</u>                       | <u>+ 0.63</u>         | <u>9- 4-32.63</u>                         | <u>9- 5- 3.64</u> | <u>+ - 3.02</u>                   |                         |                                |
| <u>Sirius</u> | <u>19<sup>th</sup> "</u>   | <u>6-39-50</u>                       | <u>+ 0.29</u>         | <u>6-39-50.29</u>                         | <u>6-40- 3.30</u> | <u>+ - 13.01</u>                  | <u>0. 19.08</u>         |                                |
| <u>N: 893</u> | <u>" "</u>                 | <u>9- 4-54</u>                       | <u>- 0.42</u>         | <u>9- 4-53.58</u>                         | <u>9- 5- 3.64</u> | <u>+ - 10.06</u>                  | <u>0. 20.97</u>         | <u>0.00023</u>                 |

Survey of India Department.

NO. \_\_\_\_\_ PARTY ( \_\_\_\_\_ ) SEASON 18 \_\_\_\_\_

Computation of Circumpolar Star Observations for Azimuth at Toungtzein H.S.

Deduction of Apparent places of Circumpolar Stars at Greenwich mean midnight on 18. March 1884 and 22. March 1884

| Star <u>N. 1302 Gr. 9-Vr. Catalogue</u> |           |                               |           |            |         |        |          |        |                               |             |                 |    |         |    |          |    |       |    |       |  |
|-----------------------------------------|-----------|-------------------------------|-----------|------------|---------|--------|----------|--------|-------------------------------|-------------|-----------------|----|---------|----|----------|----|-------|----|-------|--|
| For R.A.                                |           |                               |           | For N.P.D. |         |        |          |        |                               |             |                 |    |         |    |          |    |       |    |       |  |
| e                                       | T. 96649  | f                             | 0.01127   | g          | 1.39205 | h      | T. 96922 | e'     | T. 76886                      | f'          | 0.30682         | g' | 1.62265 | h' | T. 82281 |    |       |    |       |  |
| E                                       | 0.79576   | F                             | 1.40522   | G          | 0.19307 | H      | 1.51724  | E      | 0.79576                       | F           | 1.40522         | G  | 0.19307 | H  | 1.51724  |    |       |    |       |  |
| Sum                                     | 0.76225   |                               | 1.41649   |            | 1.58512 |        | 1.48666  | Sum    | 0.56462                       |             | 1.71204         |    | 1.81572 |    | 1.34005  |    |       |    |       |  |
| e. E                                    | 5.784     | 1st January 1872              | Mean R.A. | 14         | 9       | 22.991 | e'. E    | 3.670  | 1st January 1872              | Mean N.P.D. | 11              | 51 | 4.09    |    |          |    |       |    |       |  |
| f. F                                    | 26.091    | Corrn. to 1st January 1884    |           |            |         | 4.105  | f'. F    | 51.528 | Corrn. to 1st January 1884    |             |                 |    | 4       | 3  | 22.97    |    |       |    |       |  |
| g. G                                    | 38.470    | 1st January 1884              | Mean R.A. | 14         | 9       | 18.886 | g'. G    | 65.421 | 1st January 1884              | Mean N.P.D. | 11              | 54 | 27.06   |    |          |    |       |    |       |  |
| h. H                                    | 30.652    | Corrn. to first date of Obsn. |           |            |         | 2.827  | h'. H    | 21.880 | Corrn. to first date of Obsn. |             |                 |    |         |    | 13.71    |    |       |    |       |  |
| l                                       | 108.314   | Apparent R.A.                 |           |            |         | 14     | 9        | 21.713 | l'                            | 77.691      | Apparent N.P.D. |    |         |    |          |    | 11    | 54 | 40.77 |  |
| L                                       | 93.516    |                               |           |            |         |        |          |        | L                             | 93.516      |                 |    |         |    |          |    |       |    |       |  |
| Sum                                     | 302.827   |                               |           |            |         |        |          |        | Sum                           | 313.706     |                 |    |         |    |          |    |       |    |       |  |
|                                         | - 300.000 |                               |           |            |         |        |          |        |                               | - 300.000   |                 |    |         |    |          |    |       |    |       |  |
| Corrn.                                  | + 2.827   |                               |           |            |         |        |          |        | Corrn.                        | + 13.706    |                 |    |         |    |          |    |       |    |       |  |
| e                                       | T. 96649  | f                             | 0.01127   | g          | 1.39205 | h      | T. 96922 | e'     | T. 76886                      | f'          | 0.30682         | g' | 1.62265 | h' | T. 82281 |    |       |    |       |  |
| E                                       | 0.79703   | F                             | 1.38033   | G          | 0.19482 | H      | 1.51745  | E      | 0.79703                       | F           | 1.38033         | G  | 0.19482 | H  | 1.51745  |    |       |    |       |  |
| Sum                                     | 0.76252   |                               | 1.39160   |            | 1.58687 |        | 1.48667  | Sum    | 0.56539                       |             | 1.68715         |    | 1.81747 |    | 1.34026  |    |       |    |       |  |
| e. E                                    | 5.801     | 1st January 1884              | Mean R.A. | 14         | 9       | 18.886 | e'. E    | 3.680  | 1st January 1884              | Mean N.P.D. | 11              | 54 | 27.06   |    |          |    |       |    |       |  |
| f. F                                    | 24.638    | Corrn. to last date of Obsn.  |           |            |         | 3.061  | f'. F    | 48.658 | Corrn. to last date of Obsn.  |             |                 |    | 4       | 3  | 12.62    |    |       |    |       |  |
| g. G                                    | 38.625    | Apparent R.A.                 |           |            |         | 14     | 9        | 21.967 | g'. G                         | 65.686      | Apparent N.P.D. |    |         |    | 11       | 54 | 39.68 |    |       |  |
| h. H                                    | 30.667    |                               |           |            |         |        |          |        | h'. H                         | 21.841      |                 |    |         |    |          |    |       |    |       |  |
| l                                       | 108.314   |                               |           |            |         |        |          |        | l'                            | 77.691      |                 |    |         |    |          |    |       |    |       |  |
| Li                                      | 95.016    |                               |           |            |         |        |          |        | L                             | 95.016      |                 |    |         |    |          |    |       |    |       |  |
| Sum                                     | 303.061   |                               |           |            |         |        |          |        | Sum                           | 312.622     |                 |    |         |    |          |    |       |    |       |  |
|                                         | - 800.000 |                               |           |            |         |        |          |        |                               | - 300.000   |                 |    |         |    |          |    |       |    |       |  |
| Corrn.                                  | + 3.061   |                               |           |            |         |        |          |        | Corrn.                        | + 12.622    |                 |    |         |    |          |    |       |    |       |  |

| Star _____ |                               |           |   |        |                               |             |    |
|------------|-------------------------------|-----------|---|--------|-------------------------------|-------------|----|
| e          | f                             | g         | h | e'     | f'                            | g'          | h' |
| E          | F                             | G         | H | E      | F                             | G           | H  |
| Sum        |                               |           |   | Sum    |                               |             |    |
| e. E       | 1st January 18                | Mean R.A. |   | e'. E  | 1st January 18                | Mean N.P.D. |    |
| f. F       | Corrn. to 1st January 18      |           |   | f'. F  | Corrn. to 1st January 18      |             |    |
| g. G       | 1st January 18                | Mean R.A. |   | g'. G  | 1st January 18                | Mean N.P.D. |    |
| h. H       | Corrn. to first date of Obsn. |           |   | h'. H  | Corrn. to first date of Obsn. |             |    |
| l          | Apparent R.A.                 |           |   | l'     | Apparent N.P.D.               |             |    |
| L          |                               |           |   | L      |                               |             |    |
| Sum        |                               |           |   | Sum    |                               |             |    |
|            | - 300.000                     |           |   |        | - 300.000                     |             |    |
| Corrn.     |                               |           |   | Corrn. |                               |             |    |
| e          | f                             | g         | h | e'     | f'                            | g'          | h' |
| E          | F                             | G         | H | E      | F                             | G           | H  |
| Sum        |                               |           |   | Sum    |                               |             |    |
| e. E       | 1st January 18                | Mean R.A. |   | e'. E  | 1st January 18                | Mean N.P.D. |    |
| f. F       | Corrn. to last date of Obsn.  |           |   | f'. F  | Corrn. to last date of Obsn.  |             |    |
| g. G       | Apparent R.A.                 |           |   | g'. G  | Apparent N.P.D.               |             |    |
| h. H       |                               |           |   | h'. H  |                               |             |    |
| l          |                               |           |   | l'     |                               |             |    |
| L          |                               |           |   | L      |                               |             |    |
| Sum        |                               |           |   | Sum    |                               |             |    |
|            | - 300.000                     |           |   |        | - 300.000                     |             |    |
| Corrn.     |                               |           |   | Corrn. |                               |             |    |

\* Note.—If p = Annual Precession, p = Annual Proper Motion, s = Secular Variation, y = Number of years between epoch of Catalogue & 1st January of year of Observation, the Correction to the R.A. or N.P.D. given in the Catalogue (as the case may be) to obtain the place of the Star on the 1st January of the year of Observation is { p + p + (s/2) } y, to which if the interval between the epoch of the Catalogue and the year of Observation is large and the Star is near the pole, a further correction is necessary, viz. - (p R.A. - s Dm.) / (2y), see pp. 63 and (117) of Table LIV of the Auxiliary Tables, 3rd Edition, 1867.

NO. \_\_\_\_\_ PARTY ( \_\_\_\_\_ ) SEASON 18\_\_

Computation of Circumpolar Star Observations for Azimuth at Towangsim H.S.

Deduction of Star's place at times of Maximum Elongation on days of Observation.

| Star <u>N<sup>o</sup> 1302 Gr. 7-Yr. Cat.</u> |                                 |                |               |               |                    | Star _____         |                 |                                 |  |               |               |                    |                    |
|-----------------------------------------------|---------------------------------|----------------|---------------|---------------|--------------------|--------------------|-----------------|---------------------------------|--|---------------|---------------|--------------------|--------------------|
| Date<br>18 <u>84</u>                          | R.A. at Upper Transit Greenwich |                | Corrections   |               | R.A. at            |                    | Date<br>18_____ | R.A. at Upper Transit Greenwich |  | Corrections   |               | R.A. at            |                    |
|                                               | Diff. for 1 Day                 |                | for Longitude | to Elongation | Eastern Elongation | Western Elongation |                 | Diff. for 1 Day                 |  | for Longitude | to Elongation | Eastern Elongation | Western Elongation |
|                                               | - +                             | <u>0.058</u>   |               |               |                    |                    |                 |                                 |  |               |               |                    |                    |
|                                               | h m s                           |                | s             | s             | h m s              | h m s              |                 | h m s                           |  | s             | s             | h m s              | h m s              |
| <u>Mar. 18</u>                                | <u>14</u>                       | <u>9.21.72</u> | <u>- 0.02</u> | <u>- 0.01</u> | <u>14</u>          | <u>9.21.69</u>     |                 |                                 |  |               |               |                    |                    |
| <u>19</u>                                     |                                 | <u>21.75</u>   | <u>- .02</u>  | <u>- .01</u>  |                    | <u>21.75</u>       |                 |                                 |  |               |               |                    |                    |
| <u>20</u>                                     |                                 | <u>21.84</u>   | <u>- .02</u>  | <u>- .01</u>  |                    | <u>21.81</u>       |                 |                                 |  |               |               |                    |                    |
| <u>21</u>                                     |                                 | <u>21.90</u>   | <u>- .02</u>  | <u>- .01</u>  |                    | <u>21.87</u>       |                 |                                 |  |               |               |                    |                    |
| <u>22</u>                                     |                                 | <u>21.95</u>   | <u>- .02</u>  | <u>- .01</u>  |                    | <u>21.92</u>       |                 |                                 |  |               |               |                    |                    |

| Date<br>18 <u>84</u> | N.P.D. at Upper Transit Greenwich |               | Corrections   |               | N.P.D. at          |                    | Date<br>18_____ | N.P.D. at Upper Transit Greenwich |  | Corrections   |               | N.P.D. at          |                    |
|----------------------|-----------------------------------|---------------|---------------|---------------|--------------------|--------------------|-----------------|-----------------------------------|--|---------------|---------------|--------------------|--------------------|
|                      | Diff. for 1 Day                   |               | for Longitude | to Elongation | Eastern Elongation | Western Elongation |                 | Diff. for 1 Day                   |  | for Longitude | to Elongation | Eastern Elongation | Western Elongation |
|                      |                                   | <u>0.270</u>  |               |               |                    |                    |                 |                                   |  |               |               |                    |                    |
|                      | o ' "                             |               | "             | "             | o ' "              | o ' "              |                 | o ' "                             |  | "             | "             | o ' "              | o ' "              |
| <u>Mar. 18</u>       | <u>11.54.40.74</u>                | <u>+ 0.07</u> | <u>+ 0.07</u> |               | <u>11.54.40.88</u> |                    |                 |                                   |  |               |               |                    |                    |
| <u>19</u>            | <u>40.47</u>                      | <u>+ .07</u>  | <u>+ .07</u>  |               | <u>40.61</u>       |                    |                 |                                   |  |               |               |                    |                    |
| <u>20</u>            | <u>40.20</u>                      | <u>+ .07</u>  | <u>+ .07</u>  |               | <u>40.34</u>       |                    |                 |                                   |  |               |               |                    |                    |
| <u>21</u>            | <u>39.93</u>                      | <u>+ .07</u>  | <u>+ .07</u>  |               | <u>40.07</u>       |                    |                 |                                   |  |               |               |                    |                    |
| <u>22</u>            | <u>39.66</u>                      | <u>+ .07</u>  | <u>+ .07</u>  |               | <u>39.80</u>       |                    |                 |                                   |  |               |               |                    |                    |

Computation of Factors for Hourly Angle and Azimuth.

|                                                                                   |                                                            |
|-----------------------------------------------------------------------------------|------------------------------------------------------------|
| For Star <u>N<sup>o</sup> 1302</u>                                                | For Star _____                                             |
| Hourly Angle and Azimuth on <u>18. March</u> 18 <u>84</u> at <u>E.</u> Elongation | Hourly Angle and Azimuth on _____ 18__ at _____ Elongation |
| o ' " Hourly Angle. Azimuth.                                                      | o ' " Hourly Angle. Azimuth.                               |
| Lat. of Station <u>16.25.50.12</u> Tan <u>7.4696693</u> Sec <u>0.0181076</u>      | Lat. of Station _____ Tan _____ Sec _____                  |
| Star's N.P.D. <u>11.54.40.88</u> Tan <u>7.3241591</u> Sin <u>7.3147057</u>        | Star's N.P.D. _____ Tan _____ Sin _____                    |
| Cos <u>2.7938284</u> Sin <u>7.3328123</u>                                         | Cos _____ Sin _____                                        |
| <u>86.26 - 0.92 12.25 - 35.11</u>                                                 | _____                                                      |
| * Factor for Hourly Angle. * Factor for Azimuth.                                  | * Factor for Hourly Angle. * Factor for Azimuth.           |
| Sec <sup>2</sup> N.P.D. <u>0.0189063</u> Cos N.P.D. <u>7.9905466</u>              | Sec <sup>2</sup> N.P.D. _____ Cos N.P.D. _____             |
| Tan Latitude <u>7.4696693</u> Sec Latitude <u>0.0181076</u>                       | Tan Latitude _____ Sec Latitude _____                      |
| Cosec Hor. Angle <u>0.0008419</u> Sec Azimuth <u>0.0102952</u>                    | Cosec Hor. Angle _____ Sec Azimuth _____                   |
| Sum <u>7.4894180</u> Sum <u>0.0189494</u>                                         | Sum _____ Sum _____                                        |
| Nat. No. = - <u>0.309</u> Nat. No. = + <u>1.045</u>                               | Nat. No. = - _____ Nat. No. = + _____                      |

\* Note.—The sign of the Factor for Hourly Angle is always minus, and that of Factor for Azimuth always plus.

NO. \_\_\_\_\_ PARTY ( \_\_\_\_\_ ) SEASON 18 \_\_\_\_\_

Computation of Circumpolar Star Observations for Azimuth at Tourngyin H.S.

Deduction of Star's Azimuth and the Sidereal time at Maximum Elongation on days of Observation.

| Period |            | Star | Change in N.P.D. = $\delta$ | Correction to Hourly Angle = $\delta \times$ Factor computed on p. 8 | Hourly Angle at Maximum |            | Star's R.A. at Maximum | Sidereal Time of Maximum | Correction to Azimuth = $\delta \times$ Factor computed on p. 8 | Azimuth at Maximum |
|--------|------------|------|-----------------------------|----------------------------------------------------------------------|-------------------------|------------|------------------------|--------------------------|-----------------------------------------------------------------|--------------------|
| Date   | Elongation |      |                             |                                                                      | In Space                | In Time    |                        |                          |                                                                 |                    |
| 1884   |            |      |                             |                                                                      |                         |            |                        |                          |                                                                 |                    |
| Mar 18 | E. 1302    |      |                             |                                                                      | 86.26 0.92              | 5 45 44.06 | 14 9 21.69             | 8 23 37.63               |                                                                 | 12 25 35.11        |
| 19     |            |      | -0.27                       | +0.08                                                                | 1.00                    | 44.07      | 21.75                  | 37.68                    | -0.28                                                           | 34.83              |

If  $\delta A$  = the difference of Azimuth of a Star at Maximum Elongation and at a time  $\delta P$  before or after Elongation,  $\delta A = \frac{(2 \sin^2 \frac{1}{2} \delta P \cdot \text{Cosec } 1'') (\tan A \text{ Cos }^2 \text{ N.P.D.})}{1 - 2 \sin^2 \text{ N.P.D.} \cdot \sin^2 \frac{1}{2} \delta P \pm \text{Cot } P \sin \delta P}$  in which A is the Azimuth of the Star at Elongation, and P the Hour Angle in space.  
 Note.—The last term of the denominator is - or + according as the Star is above or below the position of Maximum Elongation.

Computation of Constant  $\tan \text{ Azimuth} \times \text{Cos}^2 \text{ N.P.D.}$  for each day of Observation.

| Date   | Elongation | Star | Log. Tan Azimuth | Log. Cos <sup>2</sup> N.P.D. | Sum = Log Constant | Date | Elongation | Star | Log. Tan Azimuth | Log. Cos <sup>2</sup> N.P.D. | Sum = Log Constant |
|--------|------------|------|------------------|------------------------------|--------------------|------|------------|------|------------------|------------------------------|--------------------|
| 1884   |            |      |                  |                              |                    |      |            |      |                  |                              |                    |
| Mar 18 | E          | 1302 | T. 34.311        | T. 9.9109                    | T. 32.420          |      |            |      |                  |                              |                    |
| 19     |            |      | 11               | 09                           | 20                 |      |            |      |                  |                              |                    |
| 20     |            |      | 10               | 09                           | 19                 |      |            |      |                  |                              |                    |

Correction of Observed Times, and Deduction of  $\delta P$ .

| Date and Elongation          | No. of Observation | Face | Zero | Observed Times  |        | Interval from Transit | Correction for Rate |                           | Total Correction | Corrected Times          |                        | $\delta P$ † In Time    |  | REMARKS. |  |
|------------------------------|--------------------|------|------|-----------------|--------|-----------------------|---------------------|---------------------------|------------------|--------------------------|------------------------|-------------------------|--|----------|--|
|                              |                    |      |      | Time of Transit |        |                       | Rate per Second =   | Chronometric Correction = |                  | Interval Time of Maximum | In Minutes and Seconds | In Minutes and Decimals |  |          |  |
| 1884                         |                    |      |      | 7 52 1.93       |        |                       | -0.00238            | + 0 0 31.6                |                  | 8 23 37.6                |                        |                         |  |          |  |
| 18-Mar. Eastern Star N. 1302 | 1                  | R    | 0    | 7 59 10         | + 4.28 | - 0.1                 | + 0 0 31.5          |                           | 7 59 41.5        | 23 56.1                  | 23.94                  |                         |  |          |  |
|                              | 2                  | "    |      | 8 2 47          | 6.45   | - 0.2                 | - 31.4              |                           | 8 3 18.4         | 20 19.2                  | 20.32                  |                         |  |          |  |
|                              | 3                  | L    | 180  | 14 18           | 13.36  | - 0.3                 | - 31.3              |                           | 14 49.3          | 8 48.3                   | 8.81                   |                         |  |          |  |
|                              | 4                  | "    |      | 17 30           | 15.28  | - 0.4                 | - 31.2              |                           | 18 1.2           | 5 36.4                   | 5.61                   |                         |  |          |  |
|                              | 5                  | "    |      | 26 24           | 20.62  | - 0.5                 | - 31.1              |                           | 26 55.1          | 3 17.5                   | 3.29                   |                         |  |          |  |
|                              | 6                  | "    |      | 29 4            | 22.22  | - 0.5                 | - 31.1              |                           | 29 35.1          | 5 57.5                   | 5.96                   |                         |  |          |  |
|                              | 7                  | R    | 0    | 39 1            | 28.19  | - 0.7                 | - 30.9              |                           | 39 31.9          | 15 54.3                  | 15.90                  |                         |  |          |  |
|                              | 8                  | "    |      | 41 56           | 29.94  | - 0.7                 | - 30.9              |                           | 42 26.9          | 18 49.3                  | 18.82                  |                         |  |          |  |
|                              | 9                  |      |      |                 |        |                       |                     |                           |                  |                          |                        |                         |  |          |  |
|                              | 10                 |      |      |                 |        |                       |                     |                           |                  |                          |                        |                         |  |          |  |

\* Note.—The quantities in these columns should be kept to only 1 place of decimals.

† Note.—The reduction of  $\delta P$  into space is not required, as the terms in which functions of P and  $\delta P$  occur have been tabulated in the 3rd Edition of the Auxiliary Tables (No. XXV, Parts I, II and III) with  $\delta P$  in time as the argument.

NO. \_\_\_\_\_ PARTY ( \_\_\_\_\_ ) SEASON 18 \_\_\_\_\_

Computation of Circumpolar Star Observations for Azimuth at Taunggyin H.S.

Computation of  $\delta A$  and Corrected Angle between Referring Mark and Star.

| Date _____                                                        |                 | Star _____ |          |          |          |          |          | Elongation _____ |  |
|-------------------------------------------------------------------|-----------------|------------|----------|----------|----------|----------|----------|------------------|--|
| Elements of Computation                                           | Observation 1st | 2nd        | 3rd      | 4th      | 5th      | 6th      | 7th      | 8th              |  |
|                                                                   | F. zero         | F. _____   | F. _____ | F. _____ | F. _____ | F. _____ | F. _____ | F. _____         |  |
| $\delta P$ in minutes and decimals                                |                 |            |          |          |          |          |          |                  |  |
| $1 - 2 \sin^2 N.P.D. \sin^2 \frac{1}{2} \delta P$ (1)             |                 |            |          |          |          |          |          |                  |  |
| * Cot P. sin $\delta P$ (2)                                       |                 |            |          |          |          |          |          |                  |  |
| Sum                                                               |                 |            |          |          |          |          |          |                  |  |
| Log.                                                              |                 |            |          |          |          |          |          |                  |  |
| Co-log.                                                           |                 |            |          |          |          |          |          |                  |  |
| Log. $2 \sin^2 \frac{1}{2} \delta P \operatorname{cosec} 1''$ (3) |                 |            |          |          |          |          |          |                  |  |
| Log. $\tan A \cos^2 N.P.D.$                                       |                 |            |          |          |          |          |          |                  |  |
| Sum = log. $\delta A$ .                                           |                 |            |          |          |          |          |          |                  |  |
| $\delta A$                                                        |                 |            |          |          |          |          |          |                  |  |
| Observed $\angle$ , R.M. and Star                                 |                 |            |          |          |          |          |          |                  |  |
| Corrected $\angle$ Do.                                            |                 |            |          |          |          |          |          |                  |  |

Date 18. March Star N. 1302 Elongation E.

| Elements of Computation                                           | Observation  |          | F.L. 180° |           | F.L. 180° |           | F.L. 180° |          | F.L. 180° |         |
|-------------------------------------------------------------------|--------------|----------|-----------|-----------|-----------|-----------|-----------|----------|-----------|---------|
|                                                                   | F.R. zero 0° | F.R. 0°  | F.L. 180° | F.L. 180° | F.L. 180° | F.L. 180° | F.R. 0°   | F.R. 0°  | F.R. 0°   | F.R. 0° |
| $\delta P$ in minutes and decimals                                | 23.94        | 20.32    | 8.81      | 5.61      | 3.29      | 5.96      | 15.90     | 18.82    |           |         |
| $1 - 2 \sin^2 N.P.D. \sin^2 \frac{1}{2} \delta P$ (1)             | 0.99976      | 0.99983  | 0.99997   | 0.99999   | 1.00000   | 0.99999   | 0.99990   | 0.99986  |           |         |
| * Cot P. sin $\delta P$ (2)                                       | +0.00649     | +0.00552 | +0.00240  | +0.00152  | -0.00089  | -0.00162  | -0.00432  | -0.00513 |           |         |
| Sum                                                               | 1.00625      | 1.00535  | 1.00237   | 1.00151   | 0.99911   | 0.99837   | 0.99558   | 0.99475  |           |         |
| Log.                                                              | 0.00271      | 0.00232  | 0.00103   | 0.00066   | T.99961   | T.99929   | T.99808   | T.99771  |           |         |
| Co-log.                                                           | T.99729      | T.99768  | T.99897   | T.99934   | 0.00039   | 0.00071   | 0.00192   | 0.00229  |           |         |
| Log. $2 \sin^2 \frac{1}{2} \delta P \operatorname{cosec} 1''$ (3) | 3.05074      | 2.99861  | 2.18257   | 1.79049   | 1.32759   | 1.84304   | 2.69590   | 2.84204  |           |         |
| Log. $\tan A \cos^2 N.P.D.$                                       | T.32420      | T.32420  | T.32420   | T.32420   | T.32420   | T.32420   | T.32420   | T.32420  |           |         |
| Sum = log. $\delta A$                                             | 2.37223      | 2.23049  | 1.50574   | 1.11403   | 0.65218   | 1.16795   | 2.02202   | 2.16853  |           |         |
| $\delta A$                                                        | + 3.55.63    | 2.50.02  | 0.32.04   | 0.13.00   | 0.4.49    | 0.16.72   | 1.45.20   | 2.27.44  |           |         |
| Observed $\angle$ , R.M. and Star                                 | 161-5-26.03  | 6-26.60  | 8-42.08   | 8-59.56   | 9-6.62    | 8-54.90   | 7-32.74   | 6-58.90  |           |         |
| Corrected $\angle$ Do.                                            | 161-9-21.71  | 16.62    | 14.12     | 12.56     | 11.11     | 9.62      | 17.94     | 26.31    |           |         |

Deduction of Correction for Dislevelment of Body of Instrument.

Correction =  $d'' \times \tan$  Altitude of Star at Elongation, where  $d$  = Dislevelment in Divisions multiplied by Value of 1 Division of Level Scale.

| For Star <u>N. 1302</u> R.E.                           |                                     | For Star _____ W.E. |                          |
|--------------------------------------------------------|-------------------------------------|---------------------|--------------------------|
| N.P.D. of Circumpolar Star ( <u>11 54 41</u> )         | Log Sec <u>0.0294534</u>            | ( _____ )           | Log Sec _____            |
| Latitude of Stn. of Observation ( <u>16 25 50.13</u> ) | Log Sin <u>T.4515617</u>            | ( _____ )           | Log Sin _____            |
| Altitude <u>16-48-10</u>                               | Log Sin Alt. = Sum <u>T.4610151</u> |                     | Log Sin Alt. = Sum _____ |
| Correction for Refraction <u>+ 3.11</u>                |                                     |                     |                          |
| Apparent Altitude <u>16-51-21</u>                      | Log Tan <u>T.48142</u>              |                     | Log Tan _____            |
| 1 Division Level Scale = <u>1.031</u>                  | Log <u>0.01326</u>                  |                     | Log _____                |
|                                                        | Log Factor = Sum <u>T.49468</u>     |                     | Log Factor = Sum _____   |

\* Cot P sin  $\delta P$  is + when the star is below and - when above the position of maximum elongation. For (1) (2) (3) see Parts I, II, III of Table XXV of the Auxiliary Tables, 3rd Edition, 1887.

NO. \_\_\_\_\_ PARTY ( \_\_\_\_\_ ) SEASON 18 \_\_\_\_\_

Computation of Circumpolar Star Observations for Azimuth at Towngziu H.S.

Deduction of Correction for Displacement of Body of Instrument, and Resulting Azimuth of R.M.

*Eastern Elongation.*

| DATE.                                         | V. End           |         | W. End |  | E. End |  | W. End |  | E. End |  | W. End |  | E. End |  |
|-----------------------------------------------|------------------|---------|--------|--|--------|--|--------|--|--------|--|--------|--|--------|--|
|                                               |                  |         |        |  |        |  |        |  |        |  |        |  |        |  |
| Sum of Level readings on                      | F.L.             | 272.8   | 244.1  |  |        |  |        |  |        |  |        |  |        |  |
| Mean of above                                 | = M <sub>1</sub> | 68.2    | 61.03  |  |        |  |        |  |        |  |        |  |        |  |
| Sum of Level readings on                      | F.B.             | 210.4   | 306.7  |  |        |  |        |  |        |  |        |  |        |  |
| Mean of above                                 | = M <sub>2</sub> | 52.6    | 76.67  |  |        |  |        |  |        |  |        |  |        |  |
| Sum of M <sub>1</sub> and M <sub>2</sub> = Σ  |                  | 120.8   | 137.7  |  |        |  |        |  |        |  |        |  |        |  |
| Diff. between Σ for E. end & Σ for W. end = N |                  |         | 4.16   |  |        |  |        |  |        |  |        |  |        |  |
| Log N                                         |                  | 0.61909 |        |  |        |  |        |  |        |  |        |  |        |  |
| Log Correction = Log Factor + Log N           |                  | 0.11377 |        |  |        |  |        |  |        |  |        |  |        |  |
| *Correction                                   |                  | 1.30    |        |  |        |  |        |  |        |  |        |  |        |  |

*Western Elongation.*

| DATE.                                         | E. End           |  | W. End |  | E. End |  | W. End |  | E. End |  | W. End |  | E. End |  |
|-----------------------------------------------|------------------|--|--------|--|--------|--|--------|--|--------|--|--------|--|--------|--|
|                                               |                  |  |        |  |        |  |        |  |        |  |        |  |        |  |
| Sum of Level readings on                      | F.L.             |  |        |  |        |  |        |  |        |  |        |  |        |  |
| Mean of above                                 | = M <sub>1</sub> |  |        |  |        |  |        |  |        |  |        |  |        |  |
| Sum of Level readings on                      | F.B.             |  |        |  |        |  |        |  |        |  |        |  |        |  |
| Mean of above                                 | = M <sub>2</sub> |  |        |  |        |  |        |  |        |  |        |  |        |  |
| Sum of M <sub>1</sub> and M <sub>2</sub> = Σ  |                  |  |        |  |        |  |        |  |        |  |        |  |        |  |
| Diff. between Σ for E. end & Σ for W. end = N |                  |  |        |  |        |  |        |  |        |  |        |  |        |  |
| Log N                                         |                  |  |        |  |        |  |        |  |        |  |        |  |        |  |
| Log Correction = Log Factor + Log N           |                  |  |        |  |        |  |        |  |        |  |        |  |        |  |
| *Correction                                   |                  |  |        |  |        |  |        |  |        |  |        |  |        |  |

*Eastern Elongation.*

*Western Elongation.*

| Face L.                                              | Zero and Date |          |  |  |  |  | Face L.                                              | Zero and Date |  |  |  |  |  |  |
|------------------------------------------------------|---------------|----------|--|--|--|--|------------------------------------------------------|---------------|--|--|--|--|--|--|
|                                                      |               |          |  |  |  |  |                                                      |               |  |  |  |  |  |  |
|                                                      | 180           |          |  |  |  |  |                                                      |               |  |  |  |  |  |  |
|                                                      | 18:40         |          |  |  |  |  |                                                      |               |  |  |  |  |  |  |
| Observed Angle R.M. and Star corrected to Elongation | 16.12         |          |  |  |  |  | Observed Angle R.M. and Star corrected to Elongation |               |  |  |  |  |  |  |
|                                                      | 12.54         |          |  |  |  |  |                                                      |               |  |  |  |  |  |  |
|                                                      | 11.11         |          |  |  |  |  |                                                      |               |  |  |  |  |  |  |
|                                                      | 9.62          |          |  |  |  |  |                                                      |               |  |  |  |  |  |  |
| Mean                                                 | 11.85         |          |  |  |  |  | Mean                                                 |               |  |  |  |  |  |  |
| Face R.                                              | Zero and Date |          |  |  |  |  | Face B.                                              | Zero and Date |  |  |  |  |  |  |
|                                                      | 0             |          |  |  |  |  |                                                      |               |  |  |  |  |  |  |
|                                                      | 18:40         |          |  |  |  |  |                                                      |               |  |  |  |  |  |  |
| Observed Angle R.M. and Star corrected to Elongation | 21.71         |          |  |  |  |  | Observed Angle R.M. and Star corrected to Elongation |               |  |  |  |  |  |  |
|                                                      | 16.62         |          |  |  |  |  |                                                      |               |  |  |  |  |  |  |
|                                                      | 17.94         |          |  |  |  |  |                                                      |               |  |  |  |  |  |  |
|                                                      | 26.31         |          |  |  |  |  |                                                      |               |  |  |  |  |  |  |
| Mean                                                 | 20.65         |          |  |  |  |  | Mean                                                 |               |  |  |  |  |  |  |
| Mean of both Faces                                   | -161          | 9.16.25  |  |  |  |  | Mean of both Faces                                   |               |  |  |  |  |  |  |
| Level Corr.                                          | -             | 1.30     |  |  |  |  | Level Corr.                                          |               |  |  |  |  |  |  |
| Corrected Mean                                       | -161          | 9.17.55  |  |  |  |  | Corrected Mean                                       |               |  |  |  |  |  |  |
| Asth. of Star                                        | 192           | 25.55.11 |  |  |  |  | Asth. of Star                                        |               |  |  |  |  |  |  |
| Asth. of R.M.                                        | 31            | 16.47.56 |  |  |  |  | Asth. of R.M.                                        |               |  |  |  |  |  |  |

Resulting Azimuth of Referring Mark (Konlah H.S.) ... } by Eastern Elongation 31-16-18.542  
 } by Western " 18.622  
 Mean 31-16-18.582

Angle between \_\_\_\_\_ and Referring Mark \_\_\_\_\_  
 Azimuth of \_\_\_\_\_ Do. \_\_\_\_\_ by Observation \_\_\_\_\_

\* Note.—When the sum of the E. End readings is greater than that of the W. End and the R.M. is <sup>east</sup> of the star, this is subtracted from the angle, R.M. and star, and vice versa when the sum of the W. End readings is the greater.



## PART IV.

## Electro-Telegraphic Longitudes.

## CHAPTER I. OBSERVATIONS FOR DIFFERENCE OF LONGITUDE.

1. All the known methods of determining absolute longitude by direct observation of celestial bodies are deficient in the accuracy that is indispensable for the requirements of Geodesy, and are therefore never resorted to in the Trigonometrical Branch for such a purpose. By the term "determining absolute longitude" is here meant the ascertaining of both local and Greenwich time by direct celestial observations at any one station. A knowledge of the *difference* of longitude of two selected stations within reasonable distance of each other is however far easier of attainment, and supplies equally well the data required for geodetical purposes.

Introductory.

2. Even this process was troublesome prior to the invention of the Electric Telegraph; but in the present day by its aid results are obtained of an order of accuracy far beyond what had been previously thought possible. It must be borne in mind that difference of longitude is merely the difference of local times (either solar or sidereal) at any given instant; if therefore an observer at one station can by any means ascertain the time at another place, which corresponds to a given time at his own station, he is at once in possession of the difference of longitude.

3. Before the invention of the Electric Telegraph it was customary to make instantaneous signals by the explosion of rockets, or of small quantities of gunpowder or by other similar devices, so arranged that they could be seen from both the stations whose difference of longitude was to be determined. The observers at both stations noted their own local times of the occurrence of the signal, and hence a comparison of the local times, and consequently a value of the difference of longitude was obtained. It was assumed that the transmission of light was instantaneous, and that the signal was seen precisely at the same moment at both stations, a

Early methods of determining differences of longitude.



perfectly legitimate assumption when the enormous velocity of light is considered.

4. This method and that of transporting chronometers were for a long time the most reliable; but they have been entirely superseded by the telegraphic method, wherever a telegraph wire is available for the purpose, as the latter furnishes a far more accurate and convenient mode of comparing the time at distant stations: suppose, for the sake of example, an observer at the eastern station, when his local time is exactly noon, sends an arbitrary signal through the wire to an observer at the western station, who, on receiving the signal, notes his local time to be  $11^{\text{h}} 45^{\text{m}}$ : it is obvious that, neglecting the time of transmission of the signal along the wire, the difference of longitude is 15 minutes. This is the principle that underlies the various methods of finding differences of longitude by the Electric Telegraph, but in practice considerable modifications of such a simple and obvious plan as this are necessary: the rationale however of all is the same, *viz.*, the power of communicating (almost) instantaneously with a distant station.

5. In the first place each observer must be provided with a transit instrument and an astronomical clock for finding and keeping his local time with all possible accuracy. These, including the chronographs on which the transits are recorded by electricity, and the collimators, are described in great detail in the Introductory Chapters to Volumes IX and X of the "Account of the Operations of the Great Trigonometrical Survey of India" which should be in the possession of every one intending to take part in these operations.

6. The mounting and adjustment of the transit instruments will be first described. Sufficient stability for these instruments can only be secured by mounting them on carefully isolated brick pillars. A brief description of these, as well as of the pillars required for the clock and the collimators, and of the portable observatories in which the observations are made, will be found in Sections 1 to 5 of Chapter II of Volume X. The plan and sections of the pillars with full details and dimensions are given in a plate facing this page for convenience of reference. The pillars must be carefully built according to the dimensions in the drawings, and attention must be given to the bond and quality of the mortar.





7. The depth of foundations shown in the drawings will generally be sufficient, but more should be given if necessary. Great care should be taken that the inner faces of the transit piers are exactly parallel to the meridian line, laid down *vide* para. 10, and equidistant from it. The central lines of the collimator pillars should be 3 inches to the east side of the meridian line, for the better accommodation of the tripod stands on which the collimators are mounted.

8. The insulating space round the piers should be about 2 inches in breadth; and in excavating for the foundation, the greater space required for the lowest courses may be obtained by undermining, if the soil will admit of it, care being taken to preserve the edges as sharp as possible. The iron clamps for the lamp stand are formed of flat iron about  $1\frac{1}{2}$ " broad and  $\frac{1}{2}$ " thick, built firmly into the transit piers. Each pair is meant to support an upright iron bar  $1''\cdot05$  broad, and  $0''\cdot4$  thick by allowing it to pass through holes of about that size cut in each clamp  $\frac{1}{2}$ " clear of the brickwork. There are six pairs in all, *viz.*, one pair on the northern and one on the southern face of each pier, one on the eastern side of the east pier and one on the western side of the west pier. The holes in each pair must be carefully plumbed to ensure the iron bar for the lamp stand being supported in a vertical position. The two northern as well as the two southern pairs should be  $3' 3''$  apart from centre to centre. During the building the rough level must be occasionally applied, and the piers brought up evenly, so that when finished the stone caps may be level.

9. When the piers are finished the ground between them may be excavated, and then the whole filled up with perfectly dry sand. The excavation is finally covered over with a board made to fit it, a round hole the size of the mercury trough being cut therein and closed with a moveable cap. The construction of these pillars is generally entrusted to a sub-surveyor who precedes the observers by a week or two, so that the pillars may have time to settle and harden before being brought into use.

10. Before commencing their construction the sub-surveyor will have to lay out a meridian line which must be true within a few minutes of arc. As the pillars are almost always built in the vicinity of a Telegraph Office, this process

is very simply performed, since nearly all the Telegraphic Offices of India are now supplied every day at 4 P.M. with the exact Madras time by electric signal. About the time the daily signal is expected from Madras, the observer must repair to the office and note the exact time by his chronometer when the signal is received. Hence he knows the error on Madras time, and consequently (as his longitude is very approximately known) the error on local time. Now by a well known computation he can ascertain the mean local time of the transit of Polaris (which is the best star to employ for the purpose), and hence, by applying his chronometer error, the chronometer time of its transit. Armed with this knowledge the observer is in a position, with the help of a theodolite, to ascertain the direction of the meridian very closely. Take the following example.

11. An observer in longitude  $5^{\text{h}} 13^{\text{m}} 46^{\text{s}}$  east of Greenwich and  $7^{\text{m}} 13^{\text{s}}$  west of Madras notes that on November 1, 1889, his chronometer, when the Madras signal is received, stands at  $3^{\text{h}} 28^{\text{m}} 4^{\text{s}}$ : find the chronometer time when Polaris is on the observer's meridian.

|                                                              |     |                          |     |     |    |    |         |  |
|--------------------------------------------------------------|-----|--------------------------|-----|-----|----|----|---------|--|
| At $4^{\text{h}}$ Madras time the observer's true time in W. |     |                          |     |     |    |    |         |  |
| Longitude $7^{\text{m}} 13^{\text{s}}$ is                    | ... | ...                      | ... | ... | 3  | 52 | 47      |  |
| Chronometer time                                             | ... | ...                      | ... | ... | 3  | 28 | 4       |  |
| Therefore his chronometer is slow                            | ... | ...                      | ... | ... |    | 24 | 43      |  |
| Sidereal time Gr. Mean noon Nov. 1, 1889                     | ... | ...                      | ... | ... | 14 | 43 | 51      |  |
| Correction for longitude of observer                         | ... | ...                      | ... | ... |    | —  | 52      |  |
| Sidereal time at local mean noon                             | ... | ...                      | ... | ... | 14 | 42 | 59      |  |
| Right ascension of Polaris                                   | ... | ...                      | ... | ... | 25 | 19 | 13      |  |
| At Culmination, sid. interval after mean noon                | ... | ...                      | ... | ... | 10 | 36 | 14      |  |
| Do.                                                          | do. | reduced to mean interval | ... | ... | 10 | 34 | 30      |  |
| Chronometer slow                                             | ... | ...                      | ... | ... |    | 24 | 43      |  |
| Chronometer time of upper culmination                        | ... | ...                      | ... | ... | 10 | 9  | 47 P.M. |  |

If then the observer, at chronometer time  $10^{\text{h}} 9^{\text{m}} 47^{\text{s}}$  (having previously levelled and adjusted his theodolite), sets the middle wire on Polaris he will have obtained the direction of the meridian with sufficient accuracy. The chronometer rate is here neglected, it is generally in

practice sufficiently well known to allow of its application in determining the time of transit; the above example assumes that the observer is using a mean time chronometer. Suppose a sidereal chronometer to be employed, and the time shown by it on receiving the 4 o'clock signal from Madras to be  $18^{\text{h}} 3^{\text{m}} 8^{\text{s}}$  then, the other data remaining the same, the computation of chronometer time would be as follows:—

|                                                                       |        | h  | m  | s  |
|-----------------------------------------------------------------------|--------|----|----|----|
| Sidereal time local mean noon (as above)                              | ...    | 14 | 42 | 59 |
| ,, equivalent of $3^{\text{h}} 52^{\text{m}} 47^{\text{s}}$ mean time | ...    | 3  | 53 | 25 |
|                                                                       |        |    |    |    |
| Sidereal time of receiving signal                                     | ...    | 18 | 36 | 24 |
| Chronometer time ,, ,,                                                | ...    | 18 | 3  | 8  |
|                                                                       |        |    |    |    |
| Chronometer slow                                                      | ... .. |    | 33 | 16 |
| Right ascension of Polaris                                            | ... .. | 1  | 19 | 13 |
|                                                                       |        |    |    |    |
| Chronometer time of upper culmination                                 | ...    | 0  | 45 | 57 |

12. A still more accurate result could be obtained by observing Polaris or some other circumpolar star at elongation; but in most cases a sub-surveyor's knowledge of Astronomy would not be sufficient to enable him to make the necessary computations. If a sidereal chronometer be at hand and the sub-surveyor is able to obtain and regulate his time, Table LIII of the Auxiliary Tables might prove serviceable. But at all events the method described above has the advantage of extreme simplicity and is quite precise enough, as even at culmination the azimuth of Polaris alters only about 1 minute of arc in 3 minutes of time in Indian latitudes. The meridian line being once obtained, it should be marked by pegs at some distance north and south to enable the builders to refer to it when necessary during the construction of the pillars.

13. The building of the pillars should be taken up in the following order:—

Details of pillars &c.

1. Clock Pillar.
2. Transit Pillars.
3. Collimator ,,

The clock-room must not be less than 10 feet square, and conveniently close to the Telegraph Office if not in the building itself. The details of the clock pillar are given in Figures 1, 2 and 3.

Within 50 feet of the clock-room the station site should be selected, with an open view on the meridian, clear of buildings and trees to within an elevation of  $8^\circ$  to the north and  $15^\circ$  to the south.\* The details of the transit and collimator pillars are given in Figures 4, 5, 6, 7 and 8.

No part of the pillars should be plastered except the upper surfaces of the collimator pillars: when they are completed the insulating spaces should be filled with dry sand. A straight line touching the upper surfaces of the two collimator pillars should be horizontal, and pass  $2\frac{1}{4}$  inches above the level of the upper surface of the stone caps on the transit piers. This is important to ensure the mutual visibility of the two collimators through the hole in the cube of the transit telescope.

14. It hardly ever happens that the transit pillars occupy the exact site of a Trigonometrical Station, therefore it will be necessary to determine their geodetic position by executing a traverse or series of triangles to connect them with one or more of the Principal Stations of the Triangulation. This part of the work also generally falls to the share of an assistant or sub-surveyor. The point midway between the two pillars is taken as the exact site of the longitude station.

*Connexion with Triangulation.*

15. It is advisable on arriving at a longitude station to set up and adjust the clock first, and to obtain an approximate rate by means of a chronometer of known rate which should be always carried by a longitude party. The error of the clock can be obtained very approximately from the Telegraph Office by the 4 o'clock signal from Madras. The chief precaution to be taken in fixing the clock is the insuring its verticality. This may be done either by a rough spirit-level on the frame, or preferably by a plumb-line applied to one of the vertical edges of the clock case. Evenness of beat, which is judged by the ear only, is obtained by a regulating screw on the arm which conveys the motion of the pendulum to the clock train.

*Setting up the clock.*

16. The clock is so contrived that, by attaching to the two binding screws on the case, wires from the two poles of a galvanic battery, a continuous current passes through, broken at the commencement of each second by means of a small tilt hammer raised by one of the clock wheels. It is very necessary

*Electrical connexion of clock.*

\* These figures apply to the greater part of India. A good rule for the purpose is that the meridian on the north side should be unobstructed to a zenith distance of  $4^\circ$  more than the co-latitude.

to see that the contacts of this hammer are perfectly clean, and that it is sufficiently raised at each second to break the current.

For raising or lowering the pendulum bob and thus increasing or decreasing the clock rate, a screw at the bottom is provided with a graduated head, each division corresponding to a change of rate of one second *per diem*.

17. The collimators and transit instrument may now be mounted and adjusted, and in doing this the small transit (alluded to in Volume X, Chapter I, at the beginning of Section 3) which is made to fit the same Ys as the large one will be found of great service in adjusting the Ys by hand, both in level and azimuth, sufficiently near to be within the range of the adjusting screws of the large telescope. A description of these instruments is given in such detail in the Introductory Chapters of Volume X that it is needless to repeat it here, and it will be only necessary to describe briefly the adjustments that must be performed on the transit telescope, before an exact determination of the local time can be obtained by its means. These are six in number :—

18. *1st*—Adjustment of the eye-piece to obtain a clear view of the wires :—

This is effected by merely pushing in or drawing out the eye-glass in its socket till the eye is satisfied. This adjustment varies for different eyes.

19. *2nd*—Adjustment of the wires to the solar focus of the object glass :—

This is effected both in the collimators and in the transit telescope, by two antagonizing screws which move the eye tube in and out. When a star or any very distant object is well and clearly defined, the screws must be clamped once for all and not meddled with subsequently, as any change in this adjustment alters slightly the value of the wire intervals.

20. *3rd*—Adjustment of the line of collimation :—

The line of collimation may be defined to be the line joining the centre of the wires, and the centre of the object-glass, and unless this line is placed so as to cut the transit axis at right angles, a *correction for collimation* to the

Mounting and adjusting of the collimators and transit instrument.

Adjustment for clear vision of wires.

Adjustment to solar focus.

Collimation adjustment.



recorded time of transit becomes necessary. The movement of the line of collimation is effected by turning the micrometer head which urges a slide carrying the wires to the right or left, and its position with regard to the transit axis is ascertained by reference to the collimators. The simplest way of realizing the principle of a collimator is to consider the cross in its focus to be an infinitely distant object. Now, if we look through the south collimator at the north one, and by means of the proper adjusting screw superpose the images of the two crosses, those two images are to all intents and purposes two infinitely distant signals at the extremities of a diameter of the celestial sphere. Let the centre wire of the large transit be now placed on the cross of the north collimator and then suppose the telescope to be revolved on the transit axis through  $180^\circ$ , it is obvious that if the sight line is perpendicular to the axis it will now accurately intersect the south cross; if not, half the difference is the error of collimation which may be corrected by the proper screws, or preferably may be computed from the micrometer readings of the collimators.

21. This is the principle, but in practice the crosses of the north and south collimator are not exactly superimposed, but are separated by a small angular interval which is measured by the micrometer attached to the south collimator, and allowed for in the computation; the necessary intersections of the north and south crosses in the transit telescope are made by the micrometer, and the micrometer reading of the true line of collimation, when perpendicular to the transit axis, is deduced. The method of determining the actual value of the collimation constant  $c^*$  will be explained in the next chapter. The reader may also consult with advantage Section 2 of Chapter V of Volume X. Note, that in fitting on the cap protecting the micrometer head, care should be taken not to disturb the reading to which the micrometer is set, usually called  $C_s$ , and that when removing the cap, this reading should be checked before moving the head.

22. 4th—Adjustment of the horizontality of the transit axis:—

In small instruments this is generally performed by a striding level the feet of which stand on the pivots of the axis; but in transit instruments of the size employed on the longitude work a more delicate method is resorted to, which depends

\*  $c$  is the difference between the angle that the line joining the centre of the wires with the centre of the object-glass makes with the rotation axis, and  $90^\circ$ , and is measured in divisions of the transit telescope micrometer, one division =  $0''.0224$  or  $0'''.336$ .

for its accuracy on the horizontality of the surface assumed by mercury when at rest: a particular eye-piece called after its inventor "Bohnenberger" is required which enables light to be thrown down the telescope tube from the wires towards the object-glass, while the observer's eye is at the instrument; by this device it is possible to see at the same time the wires themselves and their image reflected from a trough of mercury to which the telescope is directed. Now suppose the central wire to be truly collimated, and then, by means of the levelling screws the axis to be so adjusted that the central wire *coincides with its own reflection*, it is evident that the line of collimation is vertical, and being also perpendicular to the transit axis by the previous adjustment in collimation, the latter must be horizontal. The actual method in use, which consists, not in truly levelling the axis, but in measuring by the micrometer its inclination, is described in Section 4 of Chapter V of Volume X.

### 23. 5th—Adjustment of the setting circles:—

These circles can be used to read either declinations or zenith distances; the latter are much the more convenient for a portable instrument, for if the former are adopted a new adjustment is necessary at every station, whereas in the latter case one adjustment once for all suffices. The best means of securing this is to set the circles to read  $0^\circ$  when the horizontal wires and their image in mercury coincide, and firmly clamp them in that position, making the final fine adjustment of the pointers by the small levels attached to them.

### 24. 6th—Approximate adjustment of the transit instrument to the meridian:—

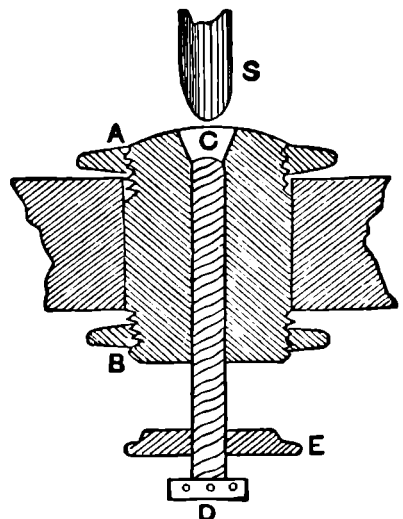
There are several well known methods of performing this adjustment, which are treated of in all text books of Astronomy. The most convenient procedure, and the one almost always adopted in the longitude parties, is by the transit of Polaris. Sidereal time having been obtained accurately, either by computation from the mean time supplied to the Telegraph Office, as explained in para. 11 of this Chapter, or by transit of a zenith star, it is only necessary to shift the whole transit telescope and axis, by means of the azimuthal screw provided for this purpose, so that when the sidereal time is equal to the right ascension of Polaris, that star may be seen on

the central wire (collimated). It is immaterial whether the tribrach fitted with the azimuthal screw is placed on the east or west pillar, but it should be noted in the observatory memo. book which pillar is used. A rough adjustment by means of the small transit is supposed to have been first made, otherwise the run of the azimuthal screw may not be sufficiently long to admit of the adjustment being completed. The line of collimation will then describe approximately the meridian of the place. The transit instrument should have been previously levelled; but if it requires to be moved much in azimuth, or if the upper surfaces of the piers are not fairly level, it may be found necessary to repeat the operation.

25. In all the three adjustments for collimation, level and azimuthal deviation, no attempt is made in practice to render them quite exact; it is far more convenient, and accordant with almost universal usage, to reduce the errors to minute quantities, and to apply corrections for them, *vide* Sections 1 to 5 of Chapter V of Volume X.

26. The Chronographs and their adjustments are fully described in Sections 4 and 5 of Chapter I of Volume X, and generally speaking, if all the teeth of the wheels are kept clean by occasional brushing with a stiff brush, and the adjustment of the fan is carefully made, these machines work satisfactorily. This latter adjustment is, however, a very delicate one, and in order to understand thoroughly how to make it, a knowledge of the mechanism of the bearing is necessary; the following diagram (not drawn to scale) sufficiently illustrates it.

27. The cup **C** in which the spindle **S** works is to be screwed up by the screws **A** and **B** until only the slightest possible play of the spindle is left, and then the lightest touch on the central steel screw **D** completes the adjustment. The screw **E** serves to clamp the screw **D**. In nine cases out of ten if the chronograph is giving trouble the fault lies at this point.



28. It has been found by experience that the most advantageous way of utilizing the telegraph is by sending through it the clock beats at either station, the same beats being thus recorded on the chronographs at both stations. Suppose the eastern clock is first made to record its beats on both chronographs, then a certain selected list of stars is observed and recorded at the east station by the east clock; and after an interval of time equal to the difference of longitude of the two stations, the same stars culminate and are recorded at the western station, still *by the east clock*. The length of time occupied by the stars in passing from the meridian of the east station to that of the west is thus known, subject only to errors in the clock rate (not in the clock correction) and to the error caused by neglecting the time occupied by the signals in passing between the two stations.

29. The errors and irregularities in the clock rate are necessarily very small, as the difference of longitude rarely exceeds 15 or 20 minutes of time, and a long programme of stars is selected, thus minimizing the error due to this cause: the length of time occupied by the signals between the two stations is cancelled very approximately by employing alternately the east clock, and west clock to transmit the beats; for it is obvious that as the current passes in opposite directions in the two cases, the resulting longitude is as much too great when the latter clock is used, as it is too small when the former one is employed. Thus, if  $\Delta LE$  represent the difference of longitude by the east clock and  $\Delta LW$  by the west clock,  $\frac{1}{2}(\Delta LE + \Delta LW)$  will be the true difference of longitude, and  $\frac{1}{2}(\Delta LW - \Delta LE)$  the retardation of the electric current in passing along the line and through the instruments.

30. The following programme (in skeleton) has been found the most convenient after many trials, and should not be departed from without sanction:—

Specimen of skeleton Programme.

- I. Both observers adjust their telescopes in collimation and level; it is supposed that the azimuthal deviation has been previously reduced to a very small quantity.
- II. The clock at the eastern station is made to graduate both chronographs, and the transits of eight stars (the same at both stations,) four to the north of the zenith and four to

the south, are recorded by each observer on his own chronograph; the leading pen marking the clock seconds and the following pen the transits over the several wires, this arrangement of the pens being usually called  $Q +$ . *Vide* Section 5, Chapter III, Volume X.

III. When these eight stars have passed both stations, after an interval of about four minutes, another similar group of eight stars is recorded on both chronographs as before, the only difference being that the leading pen now records the transits over the wires and the following pen the clock beats, this arrangement being usually called  $Q -$ . There must now follow an interval somewhat greater than the difference of the longitudes of the two stations, at the end of which

IV. The west clock being made to graduate both chronographs, the transits of eight stars at the two stations are recorded  $Q +$ .

V. Another group of eight stars is observed  $Q -$ .

VI. Both observers remeasure their collimation and level corrections. This process may also be performed with advantage between III and IV if the adjustments seem at all unsteady, the interval being made a little longer for this purpose if necessary. The zenith distance of these stars should not exceed  $25^\circ$  except under urgent necessity. It will be found convenient if one, at least, out of the sixteen stars observed with each clock, be a Nautical Almanac star, as a very approximate clock correction can be found at once by means of it.

31. Before commencing the observations of transits and again after

Pen equation.

completing them, the value of the pen equation should be recorded for at least two minutes on the chronograph drum. This is effected by the employment of certain pegs in connexion with the commutator board by which the direction of the electric current is regulated and which cause the observer's clock to actuate both pens; and thus the distance between them may be measured with the scale used in reading off the transits, which is described in the next chapter. For a description of the commutator board and the pegs appertaining to it, *vide* Section 6, Chapter III, Volume X.

32. At some time during this programme, data for computing accurately the azimuthal deviation of the transit instrument should be secured, either by observing the transits of two circumpolar stars (the nearer to the pole the better) at opposite culminations, or by transits of a high and low star: the former method being considered preferable. It is usual also for each observer to compare the two clocks at some time during the night's work. This may be done by putting in pegs 4, 6, 8, 10, 19, 22, 23, and subsequently 2, 10, 8, 18, 22, 23. In the first case the observer's clock will mark seconds with A pen, and the distant clock with B pen, and in the second the duties of the pens will be reversed.

33. During the first night's work it is convenient to ascertain the minute of the distant clock corresponding to the minutes of the observer's clock; this is done as follows:—The observer calls his colleague and telegraphs the word "minute". The latter then sends his clock signals through at least one minute break, and cuts it off a few seconds after the break, and immediately telegraphs the number of that minute. The observer by noting his own clock time when the break of the distant clock occurs secures the corresponding minutes of the two clocks. Thus suppose an observer when receiving his colleague's clock, notes the minute break at 4<sup>h</sup> 19<sup>m</sup> 27<sup>s</sup> by his own clock; and then is told by his colleague that the minute is 28, he will be in a position to fill up the minutes in the chronograph sheet by the simplest possible little computation. It is necessary for both observers to do this.

34. It has been found that practised observers have no difficulty in securing transits of stars at intervals of 1 or 1½ minutes, and in making the necessary adjustments between groups I and II and between IV and V in about 3 or 3½ minutes. Calculated on this supposition it will be seen that the duration of the programme will ordinarily not exceed 2½ hours, including the collimation and level adjustments; but inexperienced observers should certainly allow somewhat more time.

35. The wires in the transit telescopes are 25 in number (occasionally 35, but the outside groups are in this case never used) divided into 5 groups of 5 each, the equatorial intervals between the wires and between the groups being about

$2\frac{1}{4}$  and  $4\frac{1}{2}$  seconds of time respectively for an equatorial star. It has been found by experience that very little advantage is gained by noting transits over more than 11 wires. The central 11 are generally used, *viz.*, from H to R (or R to H according to the position of the illuminated pivot), the wires being called by the letters of the alphabet from A to Y. It is advantageous to use the same wires in both positions of the telescope, as any errors caused by incorrect wire intervals are thereby cancelled.

36. A well trained observatory recorder should always be attached to a longitude party. He is generally made responsible, in conjunction with the tindal of the party, for the proper packing and loading of the instruments when travelling. He should know sufficient about the setting them up to save the executive officer a considerable amount of trouble. He must be able to bring up stars' places from the catalogues approximately to date of observing, and sufficiently well trained in computations to carry on the reductions of the transits under supervision. His duties in the observatory are chiefly confined to calling out the zenith distances of the stars (north or south as the case may be) to the observer and giving him warning as the star enters the field of the telescope. He has a chronometer before him set to sidereal time, to enable him to do this; it is found that 1 minute's warning is sufficient, as the star should be then well within the field and about 40 seconds from the first wire to be observed. He also records in a book kept for the purpose, the names of the wires observed, and any other details that the observer thinks worth entering for future guidance. It has been found quite practicable for a sub-surveyor to do the recording work, as well as the accounts and correspondence of the party without being overworked.

37. Before attempting the measurement of an arc of longitude the observers must be thoroughly acquainted with the details of their commutator board, and know almost instinctively, or at all events quite by rote, the pegs to put in to produce certain effects, *vide* Section 6, Chapter III, Volume X. To have to think out the proper situation of the pegs during a programme, or even to have to refer to written rules, is objectionable, and may possibly lead to stars being missed, or at all events to hurry and nervousness which are fatal to good observing. The adjustment of the pens and relays must be thoroughly mastered, for if anything goes wrong with the chronographs during the passage of a group of stars, the time for

Duties of Observatory assistant or recorder.

Thorough knowledge of commutator board essential.

remedying it is embraced within a few seconds, otherwise stars are missed and the record rendered incomplete. The feeling that this is the case, and that the other observer is noting the transits of stars which will all have to be rejected for want of the corresponding observations at his own station, is most embarrassing to a young observer, and frequently leads him to make a casual turn of any screw that happens to be nearest to him in the hope that the fault may be thus remedied; finding this ineffective he becomes bewildered and nervous, and probably loses the whole group.

38. A few of the most usual sources of failure are here  
General causes of failure. given.

(a) Clock pen ceases to record seconds:—

If the observer's own clock is in use, the pen may cease recording either owing to its clinging to the armature, or to its failure to be attracted by it. In the first case, the spring is not strong enough to push the armature away from the electro-magnet when the current ceases, or else the clock mechanism fails to break the circuit. In the second, either the spring is too strong and the attraction of the electro-magnet to the armature is not sufficient to overcome its tension, or else the clock circuit is broken. Failures of this kind are almost always due to a derangement of the armatures and springs and not to the clock circuit; the former should therefore be first examined. If the other station clock is in use, failures of seconds may be caused in the same way, but it is in this case more often the receiving relay *F* (*vide* Plate III, Volume X) which is in fault. The adjustment of this relay, when the current from the distant station is very weak, becomes a somewhat delicate matter, and as the current constantly varies in strength the adjustment needs altering from time to time; it should be set so as to move with a very weak current, but if set too finely it is apt not to fly back at each beat: the correct setting can only be gained by experience. This relay acts directly upon the pen batteries, and the length of the signals depends upon the length of time during which the tongue of the relay *F* is jammed up against its stops.

(b) Observer's pen ceases to act:—

This is probably due to derangements of the armature and springs as above, but it may also be caused either by the



tappet failing to break the circuit, or by the circuit being permanently interrupted owing to a broken or defective wire or dirty connection somewhere: contact with other wires is a fruitful source of mishaps in this case.

(c) Distant signals cease altogether:—

This is generally irremediable and may be caused either by the distant observer having omitted to pull out peg 23, or by a break in the line, or some other station having cut in. There is no remedy but to wait till the signals recommence.

39. Always be careful when sending clock signals to withdraw  
 Cautions. peg 23. It is obvious that as long as it remains in, the clock signals, instead of passing out to the line wire, will find their way to the nearest earth through the receiving relay *F*, the observer meanwhile being quite unaware that his colleague is receiving no signals at all. Note that the tongues of relays when no current is passing should be jammed against the dummy stop. All connections must be kept very clean, and accidental contacts carefully avoided. After the night's work is over the line wire should be detached from the signalling key, and secured from contact with any part of the apparatus, otherwise there may be a hindrance to the ordinary telegraphic communication the next day. The line battery, and also the pen and sounder batteries should be "short circuited," by putting the zinc and copper poles in metallic communication with each other, if they are working feebly, otherwise there is no necessity for this precaution.

40. It will be found most desirable that observers should have a fair  
 Knowledge of signalling neces- acquaintance with the Morse Alphabet, (a copy  
 sary. of which will be found at page 85 of the Appendix to Auxiliary Tables) and be able to send messages at the rate of 6 or 7 words per minute. It has been usual to obtain the services of a Telegraphic signaller to make the necessary arrangements for joining up the wire at the beginning of a night's work, so as to be continuous between the two longitude stations, *i.e.* without the intervention of any relay, and this plan will be found the most convenient, as it is hardly likely that a Survey Officer will be sufficiently skilful at signalling or well enough up in the routine of the Telegraph Department to accomplish this for himself. But it is at the close of a night's work

that a certain amount of skill in sending and receiving messages becomes important, as it enables the two officers concerned to communicate the results of the night's work, and to make any arrangements for the next evening without having recourse to "urgent" messages the next day, which have to be paid for and are somewhat costly. The following simple code has been found useful for communicating the results of the work and is recommended for adoption. The observer at the eastern station, having called the attention of the western observer, sends the following signal — — — — to signify that he has observed all the clock stars on the Programme, and having received the signal — — 'understood,' sends — — — — to signify that the azimuth stars have also been observed, at least to such an extent that the azimuthal deviation for that particular night can be obtained. If a group is incomplete he sends a — for every unobserved star and a — — — — for each one observed. Thus — — — — — — — — — — signifies that the 1st, 4th, 5th and 6th in that group have been observed and the others missed. If the azimuth stars are lost it is signalled thus — — — — —.

It should be remembered that stars observed at one station only are absolutely useless, and therefore if one observer knows that the other for some reason is unable to observe a certain group it is quite useless for him to do so.

41. With such a programme as that sketched out in para. 30, six nights' observations, if about two-thirds of the stars are secured on an average each night, are sufficient at one station, provided there is no great disparity between the numbers of stars N and S, or between the numbers of observations with E or W clock respectively. The routine for pivot changes is generally as follows:—

|           | Eastern Observer.                     | Western Observer. |
|-----------|---------------------------------------|-------------------|
| 1st night | <i>I. P. E.</i>                       | <i>I. P. E.</i>   |
| 2nd „     | <i>I. P. W.</i>                       | <i>I. P. E.</i>   |
| 3rd „     | <i>I. P. W.</i>                       | <i>I. P. W.</i>   |
| 4th „     | <i>I. P. E.</i>                       | <i>I. P. W.</i>   |
| 5th „     | repeat one of the above combinations. |                   |
| 6th „     | „ another                             | „ „               |

selecting for repetition on the last two nights those combinations which for any reason appear to have been least satisfactory.

This succession of pivots has been arrived at as the most suitable, after many experiments. The transit instruments have given rise to much discussion on this subject, as it has been found that change of pivots gives rise to greater discrepancies in arcs of longitude than was thought probable, or can at present be accounted for. This matter is treated of in considerable detail in the Appendix to Volume X, to which the reader is referred, with a recommendation that the outlines of the subject should be mastered by any officer undertaking Electro-Telegraphic longitude work.

42. It is advisable that the relative personal equation of the observers should be determined at least three times during the season, *viz.*, once before the measurement of the first arc, again in the middle of the season, and lastly after the completion of the last arc. It is not intended that these times should be in all cases rigidly adhered to if found inconvenient, but they are so obviously the best that an effort should be made towards this end. The method of measuring the equation is fully detailed in Chapter V, Volume IX and in Chapter IV, Volume X. It is best to distribute the measures over two nights rather than do the whole together, as the comparison of the two will furnish some sort of test of trustworthiness. The transits of 40 stars, *i.e.*, 20 N and 20 S of the zenith can be conveniently observed each night. The Idiometer, described at page 35 of Volume IX, designed by Lieut-Colonel (now Major-General) Maxwell Campbell, R.E., for the purpose of measuring the *absolute* personal equation of each observer, though an exceedingly ingenious instrument, was found from some unknown causes not to agree very satisfactorily in the results it gave with those derived from divided transits, and its use has therefore been abandoned for some years past.

43. A few words are necessary as to the Star Catalogues to be employed on this work. Bearing in mind the fact that the determination of the difference of time at the two stations reduces itself in practice to the length of time occupied by a heavenly body in passing from the meridian of the eastern station to that of the western, it is obvious that great accuracy in the knowledge of star places is quite unnecessary; it suffices if their right ascensions are known within 20 or 30 seconds of time and their declinations within 3 or 4 minutes of arc. Hence the use of the B.A.C. Catalogue, one of the most extended in existence though somewhat inaccurate, is legitimate, and it may be employed with great advantage.

Besides the Nautical Almanac and the *Connaissance des Temps*, the Greenwich Catalogues of 1864, 1868 and 1872, and the Washington Catalogue should also be at hand, as they may occasionally be required when more correct places are necessary. Table LIV of the 3rd Edition of the Auxiliary Tables of the Survey of India contains a very useful list of circumpolar stars for determining azimuthal deviation which may be used with safety, so long as both observers employ the same stars: the quantities  $e, f, g, h, l$  will however have to be computed as they are not given in the table. Their places should always be brought up to the day of observation rigorously. For stars within  $3^\circ$  of the pole, observations of transits on three or five wires are sufficient. It will be found convenient if both stars of a pair are observed with same clock, and one pair with the eastern and one with the western clock.

44. If by any mischance the azimuth stars are missed, the azimuthal deviation can generally be deduced for one or even two nights with sufficient accuracy from the reading of the north collimator, its position with regard to the meridian being determined from the previous night's observations; as explained in para. 12 of the next chapter. This collimator is by its construction exceedingly stable, and its cross rarely moves more than 4 or 5 divisions of the transit micrometer from night to night and frequently much less: this movement corresponds to about 1.7 seconds of arc which is immaterial as the clock stars are always selected within a few degrees of the zenith, where the correction due to such a small error in the deviation correction is practically rejectaneous.

45. It is not always necessary that a pair of circumpolar stars at opposite culminations should be observed. In low latitudes this method may be found impracticable, as lower culminations would then occur at inconveniently low altitudes. It is then better to employ a north circumpolar star at upper culmination and a star as far south as possible. Azimuthal deviation in this case, as in the former one, has the effect of accelerating the transit of one star and retarding that of the other, thereby giving a large divisor in the formula for determining it, which is the object aimed at for securing a trustworthy value. In this system also it is most desirable that both observers should use the same stars, unless they are so well known that no uncertainty attaches to their catalogued places.

## CHAPTER II. COMPUTATION OF DIFFERENCE OF LONGITUDE.

1. It will be convenient to divide the various operations, by which the final arc of longitude is obtained from the raw material furnished by the chronographic records, into the following thirteen heads, *viz.* :—

General arrangement of computations.

- (1) Writing up the headings of the sheets, and marking them off.
- (2) Reading off the star-signals, and entering the times in the transit sheets.
- (3) Reduction to centre wire, and constants for collimation, &c.
- (4) Abstract of level, and collimation corrections.
- (5) Determination of the value of pen equation.
- (6) Reduction of circumpolar star-places to date of observation.
- (7) Computation of star-constants, A.B.C. for circumpolar stars.
- (8) „ of correction for azimuthal deviation.
- (9) „ of corrected times of transit of longitude stars, and clock-rates.
- (10) Deduction of clock-rate corrections.
- (11) Abstract of personal equation, and deduction of its final value.
- (12) „ of transits with each clock, and deduction of  $\Delta L \pm \rho$ .
- (13) Final deduction of  $\Delta L$  and  $\rho$ .

Each of these will now be explained in detail. It is not indispensable to observe this order of succession in the computations, but it will be found in the main convenient to conform to it as far as possible.

2. The first process—writing up the headings of the sheets, and marking them off—is usually undertaken on the day following the night of observation, while all the details are still fresh in the observer's mind. On each sheet should be noted in large clear round hand the following particulars :—

Writing up headings, &c.

- (a) The name of the arc, *i.e.*, of the two stations at its extremities, the eastern one being always put first.
- (b) The station of observation.

- (c) The name of the observer.
- (d) The telescope used.
- (e) Its position as regards the illuminated pivot.
- (f) The clock used.
- (g) The chronograph used.
- (h) The date.
- (i) The number of the sheet: each arc should commence with new numbering.

To "mark off" the sheet, the name of each star should be written on the record, near the place where the warning signal is found. The characteristic letters of the first and last wires observed are then inserted, and again the name of the star after the last wire. The clock minutes, and here and there the clock hour, must also be entered, as they will be found indispensable when the sheet has to be read off. The hour and minute of the other observer's clock must be obtained on the first night by the process explained in the preceding chapter: an approximate knowledge of the rate will enable the observer to fill in these details for subsequent nights. At the foot of the sheet is given an extract from the collimation and level form for the night, showing the values of  $b$  and  $c$  to be used in the computation. The sheet being then signed by the observer, is complete as a record. The reader should however be here cautioned that a somewhat different procedure with regard to the collimation and level corrections has been recently approved which is explained in para. 8 of this chapter; and it will be unnecessary in future to enter more than the values of  $C_0$ ,  $C_s$  and  $M$  at the foot of the chronograph sheets.

3. It should be remembered that these sheets are in effect the field books of the party, and must be preserved with the greatest care. After they have been subjected to the next process—reading off—they should be pasted into a file book similar to those used for filing letters, but of sufficient size to render the folding of the sheet unnecessary. At the end of this chapter will be found a specimen of a chronograph sheet from actual practice on the arc Mangalore–Bombay in 1888. It has been selected casually from the records of the astronomical parties, and contains a few omissions and imperfections such as are almost invariably met with in an ordinary night's work.

Preservation of chronograph sheets.

4. The next step is the reading off the sheets, and entering the clock times of star-transits over the wires in the form provided. The reading off is performed by means of a scale of peculiar construction. It is engraved on some transparent material, talc or glass, its length being equal to the distance between two consecutive marks representing seconds, made automatically on the chronographic sheet by the clock. As the lengths of the seconds are liable to small irregularities, the division lines of the scale are made closer at one end than at the other, so that by sliding the scale up and down a position may be gained, in which its length accurately coincides with that of a second on the sheet. The scale is divided into tenths, and half tenths are read by estimation. At the end of this chapter will be found a set of forms adapted to the whole system of longitude computation. That in which the times of transits read off, as described above, are entered, is distinguished as No. 1. To give the complete computation of one whole arc would have unduly extended these forms: they are merely intended to furnish the reader with specimens, such as have been found in practice to be convenient for the purpose. They illustrate, with the exception of the last, (No. 10), the arc Mangalore-Bombay. In Form No. 10 the arc Nagarkoil-Mangalore has been selected for illustration in consequence of its being a more complete measurement than Mangalore-Bombay, and consequently affording a better type of the method employed in deducing the final value. It is quite unnecessary (except in the case of circumpolar stars observed for azimuth, whose treatment is somewhat different throughout) to record the minutes and seconds of every wire observed; it suffices if this is done for the first and last wires only; for the intermediate ones only the decimals of the second to the nearest half tenth should be entered in the form. The hour and minute of transit over the first and centre wires are entered at the top of the form, the latter being repeated below when the mean of all the wires is deduced. It is very desirable that all the sheets appertaining to one arc *at both stations* should be read off by one and the same computer, otherwise a personality may be introduced, which would have to be determined experimentally, and applied as a correction. When two computers are employed on reading off the sheets, it will be found more expeditious that one should read and the other record the readings, than that each should work independently. It will be observed that two vertical columns are appropriated to the reduction of each star; the first having been filled up as described above, the use of the second has now to be explained.

5. The time of transit over each wire must be reduced to the centre one by means of a correction (additive for wires before the middle one and subtractive for those after it) which, for any particular star and wire, is the wire interval from the centre in equatorial seconds multiplied by the secant of the star's declination. This computation, like many others in longitude work, involves a vast quantity of arithmetic, but can be considerably abridged by the use of a judiciously constructed table. Several methods have been tried at different times, but the one now in use, has, with the help of a table given after Form 1 at the end of this chapter, proved to be the most expeditious, and is easily understood by native computers. For its employment Crelle's Tables of multiplication and division are necessary; and here it may be as well to state once for all, that these Tables are now used throughout the longitude calculations, to the almost entire exclusion of logarithms. Except in the case of close circumpolar stars for which special methods are required, the declination secants, which are taken out to four decimal places, lie between the limits of 1.0 and 1.4. The table is therefore prepared with six vertical columns, the first contains the distinguishing letter appropriated to each wire, the second is headed 1.0, and contains the value of the wire intervals multiplied by 1.0, the third is headed 1.1 and contains the same intervals multiplied by that quantity, and so on up to 1.4. The last horizontal line in the table containing the values of  $m$  is explained below. Crelle's Tables are subsequently employed in multiplying the last three figures of the secant by the first three of the wire interval, the product being added to the proper number in the reduction table. Thus suppose the equatorial interval of wire  $B$  to be 29.07 and the nat. secant of the star observed to be 1.2417, the computation would stand thus

$$\begin{array}{r} \text{By the reduction table} \quad 29.07 \times 1.2 \quad = \quad 34.88 \\ \text{By Crelle's Tables} \quad \quad *29.1 \times \quad .0417 \quad = \quad \underline{1.21} \\ \text{Total reduction for wire } B \quad = \quad 36.09 \end{array}$$

The wire intervals shown in the table were those appertaining to Telescope No. 2 during the season 1887-88: a new table would of course be necessary, if these intervals are found to vary from time to time, as in all probability they will, or if another telescope be employed. Any

\* The omission of the second decimal place in the value of the wire interval when multiplying by the last three figures of the secant, can in no case affect the second place of decimals in the total reduction to centre for any wire.



moderately good computer soon learns to perform this calculation mentally without writing down a single figure, and it is thus very quickly got through. The reduction to the centre for *all* the wires is shown in the specimen form, but in practice only those should be computed which are required: these are generally the middle eleven wires. From Form 2, in which the results are entered, the figures in the second of the two columns appropriated to each star in Form 1 are now obtained by the addition or subtraction of the reduction corrections, and their mean taken out at the bottom. The last three horizontal lines of Form 2 are filled in with the constants for collimation, level, and deviation respectively for each star. It will be noticed that the collimation constant  $C = m \sec \delta$ , and it is in order to facilitate the computation of this quantity that  $m$  has been inserted in the last line of the Reduction Table, as it can be multiplied by  $\sec \delta$  in an exactly similar manner to that employed in the case of the wire intervals.

6. Discrepancies from the mean exceeding  $\cdot 3$  of a second are not frequent; when they occur the chronograph sheet should be examined to find out if any misreading has passed unnoticed. A doubt may arise as to whether a discrepancy is sufficiently large to render the observation worthless, and to justify its rejection *in toto*. A criterion has been worked out and incorporated in the following little table as a guide to the observer. It is founded on the same principles as the well known criterion by Professor Peirce, and its construction is explained in the General Report of the Operations of the Survey of India Department for the year 1885-86. Appendix, page lxvi. It has no pretensions to theoretical accuracy but suffices for the object in view. Thus if eleven wires have been observed and the star's declination is  $25^\circ$ , an observation differing  $0^s \cdot 24$  from the mean should be rejected.

*Criterion for rejecting doubtful Observations.*

| Declination           | $0^\circ$ to $10^\circ$ | $10^\circ$ to $20^\circ$ | $20^\circ$ to $30^\circ$ | $30^\circ$ to $40^\circ$ | $40^\circ$ to $43^\circ$ |
|-----------------------|-------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| No. of wires Observed |                         |                          |                          |                          |                          |
| 5                     | $\cdot 11$              | $\cdot 13$               | $\cdot 15$               | $\cdot 16$               | $\cdot 16$               |
| 6                     | $\cdot 13$              | $\cdot 15$               | $\cdot 17$               | $\cdot 18$               | $\cdot 19$               |
| 7                     | $\cdot 14$              | $\cdot 17$               | $\cdot 19$               | $\cdot 20$               | $\cdot 21$               |
| 8                     | $\cdot 15$              | $\cdot 18$               | $\cdot 20$               | $\cdot 22$               | $\cdot 22$               |
| 9                     | $\cdot 16$              | $\cdot 20$               | $\cdot 22$               | $\cdot 24$               | $\cdot 24$               |
| 10                    | $\cdot 17$              | $\cdot 21$               | $\cdot 23$               | $\cdot 25$               | $\cdot 25$               |
| 11                    | $\cdot 18$              | $\cdot 22$               | $\cdot 24$               | $\cdot 26$               | $\cdot 27$               |

7. It not unfrequently happens that the chronograph becomes irregular in its rate, and, owing to increased friction or other causes, goes so slowly that the ordinary scale cannot be applied to the chronographic record. In this case the true distance between the recording pens must be taken off with a pair of compasses, and applied (either backwards or forwards according to which pen is leading) to the star-signals on the record, and the pricks read off by actual measurement from the preceding and succeeding second-signals; in which case the space left in Form 1, for "Correction for  $Q$ " will be left vacant, or filled in with 0. The true distance between the recording pens can always be obtained from the run of pen equation at the beginning and end of each night's work, when both pens are actuated by the observer's clock. Form 1 cannot at this stage be further proceeded with, but will be referred to again.

8. The improved method of deducing the collimation and level corrections alluded to in para. 2, requires some little explanation. The previous practice had been that the value of  $c$ , as determined by observations to the collimators each night, should be used for the correction of the transits of that night, separate values being employed *I.P.E.* and *I.P.W.* Occasionally, however, considerable discrepancies occurred between these values, and an examination into the matter showed that it would be far more satisfactory theoretically to take a mean  $C_0$  for a whole arc, rather than attempt to get nightly values: it certainly has proved more satisfactory practically also, as the test by circuits, when the arcs are computed with a mean  $C_0$ , unmistakeably shows. The procedure now adopted will be understood from the Abstract of Collimation and Level, Form 3.  $C_0$  is to be determined twice (or more) each night, preferably once at the beginning of the work, and again towards its close, and entered in the proper column of the form.  $C_s$  the micrometer head reading during the observation of transits, is also inserted for each night, and also  $M$  which is the reading of the micrometer head, when the real wires and their image reflected from mercury coincide. Up to this point no change has been made in the process which is described in Sections 2, 3 and 4, Chapter V, Volume X. The next step is to take out the mean of all the values of  $C_0$  in position *I.P.E.* and also of those in position *I.P.W.*; the mean of these two quantities gives the final value of  $C_0$ , the difference between which and  $C_s$  gives the

values of  $c_1$  for each night; and so long as  $C_s$  remains the same, so long will  $c_1$  and  $c$  (which is equal to  $c_1$  less the diurnal aberration) remain constant. Note that when the telescope is *I.P.E.*  $c_1 = C_0 - C_s$  and for *I.P.W.*  $c_1 = C_s - C_0$ . By a similar process the values of  $b$  are found: each determination being kept separate, and used for the groups to which it appertains: it is usual to determine  $b$  twice in the night's work, at the same time as  $C_0$ .

9. The next stage of computation is the determination of the value of the Pen Equation. This is measured directly from the chronograph sheets by the reading scale, and if the pens have remained unaltered during the night's work, a mean is taken for the whole. It is usual to make 20 measurements on the run of pen equation at the beginning, and 20 at the end, distributing them as evenly as possible along the whole length. There is no special form used for this process.

10. We now come to the three forms 4, 5 and 6 employed in computing the azimuthal deviation of the sight line of the telescope by means of the circumpolar star observations. Form 4 is used for bringing up to date the places of the circumpolar stars observed for deviation. It is desirable that in computing these only trustworthy catalogues should be employed. For remarks on the catalogues to be employed the reader is referred to para. 43 of the preceding chapter. When the star whose place is required is found in two or three of these catalogues, it is customary to deduce its place on the first of January of the year of observation from each of them, and take the mean as the adopted value. Its place on the day of observation is brought up from the 1st of January by means of Airy's Day Numbers or by the Independent Quantities.\* The places at Greenwich mean midnight are considered good enough, as any error incurred by neglecting a few hours one way or the other, is probably smaller than the inaccuracy of their observed places, especially as small errors in the computed places can have no appreciable effect on the final results, so long as the same stars are used by both observers.

\* The Form is adapted to Airy's Day Numbers, but very little change will be required for the Independent Quantities. These latter are called in the Nautical Almanac "Quantities for correcting the Places of Stars."

11. It will generally suffice if the place is computed for the first and last night's observation on each arc, the intermediate ones being determined by interpolation : occasionally, however, if the measurement of the arc is much protracted by bad weather, or other causes, additional intermediate values should be computed as a guide to the interpolation ; but this must be left to the computer's discretion, as no general rule can be given. In Form 5 are computed (for circumpolar stars) the star constants,  $C = m \sec \delta$  for collimation,  $B = m \sec \delta \cos \zeta$  for level, and  $A = m \sec \delta \sin \zeta$  for azimuth. These are the same constants as are computed at the bottom of Form 2 for clock stars ; but as circumpolar stars require more accuracy in their treatment, Form 5 must be used for them. For computing these the latitude of the station, and the declination of the stars observed, should be known to the nearest second. The form, with the help of the precepts regarding signs, &c., at the foot of it, is sufficiently simple to render any further explanation unnecessary : the number of decimals retained should be regulated according to the specimen given.

12. We must now return to Form 1, and taking up first the circumpolar stars only, the corrections  $Cc$ ,  $Bb$ , and  $Q$ , must be filled in with their proper signs, and added algebraically to the mean of the wires. The times thus obtained are copied into Form 6, and the deviation constant  $a$  is deduced according to the rules given in detail at pages 24, 25, 26 of Volume X. It is usual to observe two pairs of circumpolar stars each night, the first of which appertains to the first two groups, and the second to the last two groups. Unless there has been some disturbance of the telescope during the progress of the work, these two values of  $a$  should not differ by more than 10 divisions. If a high and low star are used for determining the deviation instead of a pair of circumpolars the method of computation is identically the same. It has been mentioned in para. 44 of the preceding chapter that in the event of the azimuth stars being lost on any night, the position of the telescope with regard to the meridian can be deduced from that of the North Collimator, by assuming its stability for the time elapsed. The principle of thus obtaining the deviation is as follows :—

Let  $C_0$  be the micrometer reading when the sight-line is perpendicular to the rotation axis.

Let  $D$  be the micrometer reading when the sight-line intersects the North Collimator.

Also let  $a$  and  $k$  represent respectively the deviations of the telescope and north collimator from true north, measured in divisions of the micrometer (positive to the west of north). These quantities are connected by the following formula :—

$$k = a \pm C_0 \mp D$$

the upper signs being for position *I.P.W.* and the lower for *I.P.E.* Hence it is obvious that  $k$  may be computed if  $a$  is known or  $a$  may be computed if  $k$  is known ( $C_0$  and  $D$  are always obtainable from the collimation records). If therefore  $a$  cannot be obtained on any particular night owing to the azimuth stars being lost, it is obvious that a knowledge of  $k$  from the observations of a previous (or subsequent) night, will enable us to supply the omission by the solution of the equation

$$a = k \mp C_0 \pm D.$$

On several arcs nightly values of  $k$  have been taken out, and have rarely been found to differ 5 divisions on consecutive nights; whereas  $a$  is subject to considerably larger fluctuations, especially after change of pivots.

13. We can now again return to Form 1 and with the known values  $c$ ,  $b$ , and  $a$ , form the corrections  $Cc$ ,  $Bb$ ,  $Aa$  as well as  $Q$ , which must all be carefully entered with the proper signs.\* The quantities  $C = m \sec \delta$ ,  $B = C \cos \zeta$  and  $A = C \sin \zeta$  are obtained from the last three lines of the table for reduction to centre wire. These four corrections with their proper signs being now added algebraically to the mean of the wires, the sum is entered in the line "corrected"; and the last two lines are also at the same time (after the first day) completed. Having proceeded thus far, the observer will have the satisfaction of noticing, from the agreement of his clock-rate correction as deduced from different stars, what confidence can be placed in his work.

14. The next step is to deduce the actual corrections for clock-rate for each night to be applied to the transit observations by means of Form 7. The filling up of this form seems to be too obvious to need explanation; its object

\* The signs of  $c$  and  $b$  are determined in Form 3 and of  $a$  in Form 6. The signs of  $C$ ,  $B$  and  $A$  are given in the foot-note to Form 5.

is merely to compute, by means of the known clock-rate correction for 24 hours, (or multiples of 24 hours if any nights have been missed) the proportional correction for the time elapsed during the passage of a star from the meridian of the eastern to that of the western observer: if the computer is in any doubt an explanation will be found on page 31 of Volume X.

15. Personal equation next claims the attention of the computer.

Computation of personal equation.

The stars observed in *divided transits* for this purpose must be entered and computed in Form 1 exactly like the longitude stars as far as the line "mean"; beyond this it is unnecessary to carry the calculation. Each star will of course require two pairs of columns, one for each observer. The results are then abstracted into Form 8. As the relative personal equation of the observers is generally measured three times during the field season, the selection of the exact value to be used for each arc must be partly left to their judgment. In the case of the arc Mangalore-Bombay, as the equation was measured within a few days of completion of the arc the value then obtained was used on that arc; a mean of the two days, April 12 and April 13, on which it was determined, being taken. Throughout the whole discussion of personal equation, stars of north and south aspect must be kept distinct, and two equations recognized, applicable to stars of N. and S. aspect respectively.

16. We are now in a position to fill up Form 9 and deduce the

Approximate value of  $\Delta L$ .

approximate values of the arc; they are still necessarily only approximate, for those values obtained by transits with the eastern clock are too small by the quantity  $\rho$ , (the amount of retardation of the electric current through the line wire and relays) those with the western clock being too great by the same amount. In filling up this form the most recent practice, found by experience to be the most convenient, is to enter the stars in groups of north and south aspect. The aim in selecting a programme, is to obtain four groups of longitude stars, in each of which are found 4 stars of north and 4 of south aspect; these being separated in Form 9 give rise to 8 groups, alternately north and south. The mean of the results by each group is first taken out in column 15, and then the corrections for clock-rate and personal equation are entered in columns 16 and 17; the algebraical addition of the three columns then affords the final value of  $\Delta L \pm \rho$  for that particular group.

## COMPUTATION OF DIFFERENCE OF LONGITUDE.

[PART IV.]

17. With the means of these results we now enter Form 10, and the question arises, how is the best final mean to be obtained from them. If no stars in the programme were missed, the number of north stars equal to the number of south, the number of observations with the eastern clock equal to the number with the western clock, and the pivot changes quite symmetrical, it seems obvious that the proper course would be simply to take the arithmetical mean of the whole as the best result; but when these conditions do not obtain, it becomes a matter of some difficulty to decide on the best method of proceeding. Various systems have been tried at different times, but the latest and most approved is as follows. The upper part of Form 10 gives the values of  $\Delta L \pm \rho$  for each day, keeping separate the results by north and south stars with each clock, the pivot position of each telescope being entered for each day in the first column of the form. In the next columns are shown the results by each clock, north and south stars being lumped together; in the next are given the results by north and south stars separately, the mean of two clocks being taken. The next column shows the final result for each day, being the mean of columns 7 and 8 or 9 and 10. The last column  $\rho$  is half the difference between columns 7 and 8. In the latter part of the form these results are further digested. It is obvious that only four combinations of pivot positions are possible, *viz* :—

| Telescope No. 1 | Telescope No. 2 |
|-----------------|-----------------|
| <i>I.P.E.</i>   | <i>I.P.E.</i>   |
| <i>I.P.W.</i>   | <i>I.P.E.</i>   |
| <i>I.P.W.</i>   | <i>I.P.W.</i>   |
| <i>I.P.E.</i>   | <i>I.P.W.</i>   |

These four are entered in the first column. The next ten are similar to those in the upper part of the form, differing only inasmuch as they contain (or may contain) the means of the results of several days, on which the pivot positions have been the same. The mean of the figures in the last column is the finally accepted value of the arc.

18. Wire intervals should always be determined at the beginning of the season; as, in case of an accident occurring to them, the intervals being unknown, all previous work of that season would be more or less uncertain and unsatisfactory: it is as well that a second determination be made

Method of measuring wire intervals.

towards the close, to ascertain whether any noticeable change has occurred. Experience shows that such changes are generally very minute; still it is more satisfactory to re-determine the intervals, in case the sudden jars and shocks, to which these instruments are constantly exposed in transit, should have altered them. They are most easily determined by noting the length of time occupied by a close polar—and therefore slow moving—star in passing over them; the intervals being all reckoned from the central wire. The equatorial interval is obtained simply by multiplying the time occupied in the transit, by the cosine of the star's declination; except in the case of very close circumpolar stars, when a small correction is necessary owing to the star not passing in a straight line from wire to wire, but in a slightly curved circular path. The interval obtained in the simple way must then be decreased, in the proportion that the sine of the angle described by the star in its diurnal course bears to the length of the corresponding arc. This correction is very small except in the case of stars within  $2^\circ$  of the pole, and even then it may be considered rejectaneous for intervals of less than 10 seconds: if required, it may be easily computed with the aid of the following table which is taken with slight modifications from page 148, Volume II of Chauvenet's Practical and Spherical Astronomy, second edition:—

| log $t$ | log $k$  | log $t$ | log $k$ |
|---------|----------|---------|---------|
| 1'778   | 10'00000 | 2'982   | 9'99965 |
| 2'079   | 9'99999  | 3'008   | 9'99960 |
| 2'255   | 9'99999  | 3'033   | 9'99955 |
| 2'380   | 9'99998  | 3'056   | 9'99950 |
| 2'477   | 9'99997  | 3'079   | 9'99945 |
| 2'556   | 9'99995  | 3'100   | 2'99939 |
| 2'623   | 9'99993  | 3'120   | 9'99933 |
| 2'681   | 9'99991  | 3'139   | 9'99927 |
| 2'732   | 9'99989  | 3'158   | 9'99920 |
| 2'778   | 9'99986  | 3'175   | 9'99914 |
| 2'819   | 9'99983  | 3'192   | 9'99907 |
| 2'857   | 9'99980  | 3'209   | 9'99899 |
| 2'892   | 9'99977  | 3'224   | 9'99892 |
| 2'924   | 9'99973  | 3'239   | 9'99884 |
| 2'954   | 9'99969  | 3'254   | 9'99876 |

The first column contains the logarithm of the number of seconds,  $t$ , occupied by a star in transiting the wire interval, and the second contains the value of  $\log \frac{\sin \theta}{\text{arc } \theta}$ ,  $\theta$  being the diurnal arc described by the



star during that time. The following example will sufficiently explain its use. Suppose a star whose declination is  $88^{\circ} 45'$  occupies 1623 seconds in passing from any selected wire to the central one, required the true equatorial interval.

$$\begin{array}{rcl} \text{Log} & 1623 & = 3.21032 \\ \text{Log cos} & 88^{\circ} 45' & = 8.33875 \\ \text{Log } k \text{ from the Table} & & = 9.99899 \\ \text{Log sum} & & = \underline{1.54806} \quad . . . \quad 35^s.32. \end{array}$$

At least eight or ten different stars not exceeding  $6^{\circ}$  N. P. Distance, should be observed for wire intervals; the effect however of erroneous values, should such exist, may be completely cancelled in observing longitude stars, by the simple precaution of using the same wires in both positions of the telescope, *viz.*, *I.P.E.* and *I.P.W.*

- 19. The number of decimal places to be retained in the various stages of the computation merits careful attention. In reading off the chronograph sheets record to the nearest half tenth, the reduction to centre wire is to be recorded to nearest hundredth. There is of course no object to be gained theoretically by computing the latter of these quantities more rigorously than the former, it is simply a matter of convenience as the method of calculating the reduction explained in para. 4 is certainly easier to work in its present form, than if it were imperative to enter the last figure as 0 or 5. The mean of the wires is to be entered to the nearest hundredth, A, B, C are to be computed to four places of decimals, *a*, *b*, *c* to one place, and *Q* to two places. In the Abstract of Transits, Form 9, Stars' declinations are to be entered to the nearest minute. Times of transit and corrections thereto are to be entered to two places of decimals.

20. In taking the means of the groups in Form 9, and thenceforward to the final results, including clock-rate and personal equation, the third place of decimals is to be retained. Micrometer heads are always to be read by estimation to tenths of the division engraved on the head, and it will suffice if the means of two or more readings are retained to the same order of accuracy, *viz.*, tenths of a division. In the case of close circumpolar stars, whose transits are used for determining Azimuthal deviation, it

*Continued.*

is sufficient if the times of transit over each wire, and the reduction to centre wire are given to the nearest tenth of a second. In taking the mean of the wires the hundredths should be retained.

21. The following programmes are given as specimens. It will be found convenient to paste the programme on a sheet of millboard about the size of foolscap paper, and hang it up where it can be conveniently referred to, in the clock-room. The first programme is for the eastern and the second for the western station.

| ARC MANGALORE-BOMBAY, STATION MANGALORE |                                                | $\lambda = 12^{\circ} 52'$<br>$\Delta L = 8^m 7^s$ |
|-----------------------------------------|------------------------------------------------|----------------------------------------------------|
| Local Sidereal Times                    |                                                | Chronograph pegs to be used                        |
| 11-25                                   | $\lambda$ Draconis and 1070 Gr. 72 for Azimuth | 4, 6, 8, 10, 15, 25,                               |
| 11-36                                   | Call Western station                           | 23, 24                                             |
| 11-38 to 11-51                          | 1st Group, Q +, with East Clock                | 1, 8, 13, 10, 15, 25                               |
| 12-4 to 12-17                           | 2nd Group, Q -, " "                            | 1, 10, 12, 8, 17, 20, 25                           |
| 12-26                                   | Cut off Clock and receive West Clock           |                                                    |
| 12-36 to 12-55                          | 3rd Group, Q +, with West Clock                | 8, 18, 22, 23, 10, 15, 25                          |
| 13-1 to 13-17                           | 4th Group, Q -, " "                            | 10, 19, 22, 23, 8, 17, 20, 25                      |
| 13-27                                   | Call Western station and send results          | 23, 24                                             |

| ARC MANGALORE-BOMBAY, STATION BOMBAY. |                                                      | $\lambda = 18^{\circ} 54'$<br>$\Delta L = 8^m 7^s$ |
|---------------------------------------|------------------------------------------------------|----------------------------------------------------|
| Local Sidereal Time                   |                                                      | Chronograph pegs to be used                        |
| 11-25                                 | $\lambda$ Draconis and 1070 Gr. 72 for Azimuth       | 4, 6, 8, 10, 15, 25                                |
| 11-28                                 | Receive call from Eastern station                    | 23, 24                                             |
| 11-38 to 11-51                        | 1st Group, Q +, with East Clock                      | 8, 18, 22, 23, 10, 15, 25                          |
| 12-4 to 12-17                         | 2nd Group, Q -, " "                                  | 10, 19, 22, 23, 8, 17, 20, 25                      |
| 12-18                                 | East Clock stops send West Clock at once             | 1, 9                                               |
| 12-36 to 12-55                        | 3rd Group, Q +, with West Clock                      | 1, 8, 13, 10, 15, 25                               |
| 13-1 to 13-17                         | 4th Group, Q -, " "                                  | 1, 10, 12, 8, 17, 20, 25                           |
| 13-19                                 | Receive results from Eastern station and send ditto. | 23, 24                                             |

22. For the above programme the stars were those given in the subjoined list, which is made out in the form in which it has been found convenient for the observatory recorder to have before him. The numbers refer to the British

Star lists.

## COMPUTATION OF DIFFERENCE OF LONGITUDE.

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Association Catalogue unless followed by the abbreviation Gr. 72 which signifies the Greenwich Catalogue of 1872.

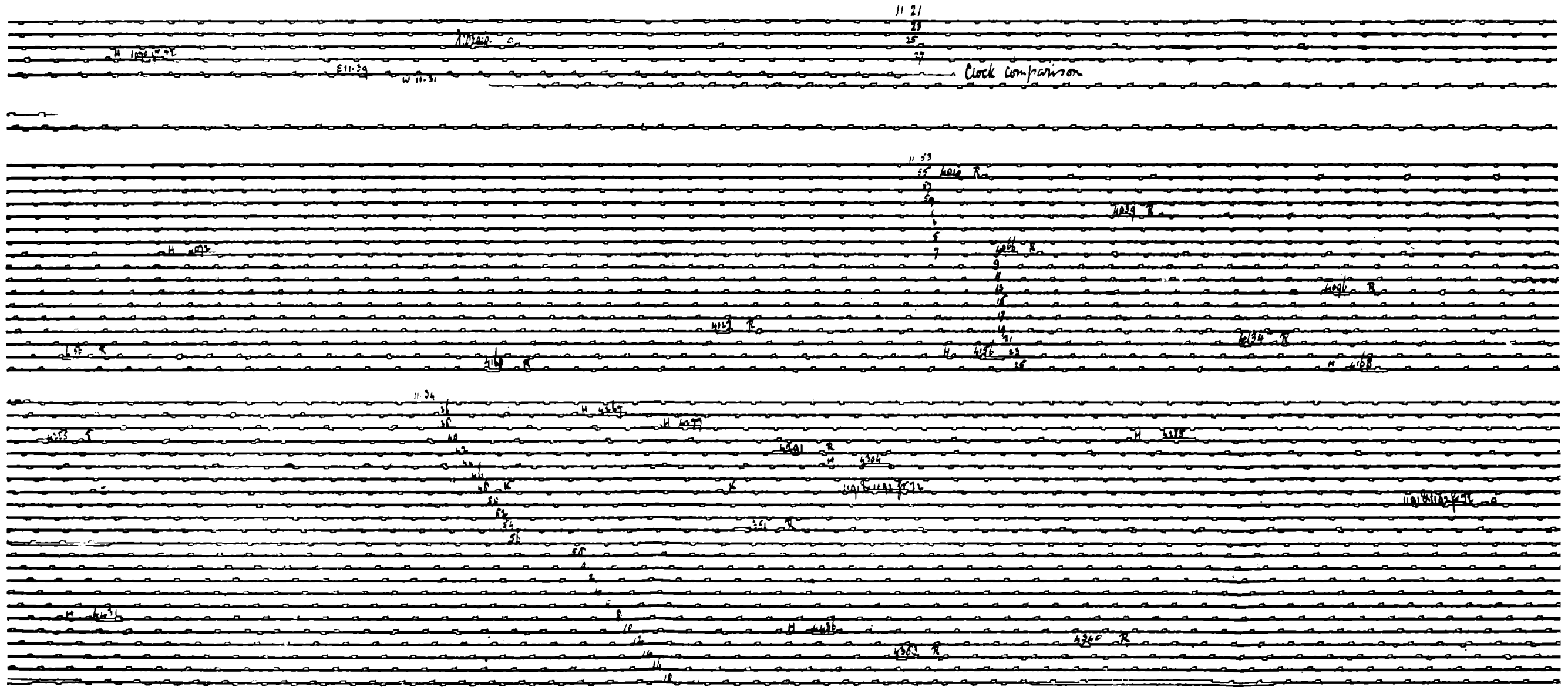
| Name of Star. |                    | Mag. | Approximate<br>R. A. |          |          | N. P. D. |          | Zenith<br>Distance.* |          | North<br>or<br>South |
|---------------|--------------------|------|----------------------|----------|----------|----------|----------|----------------------|----------|----------------------|
|               |                    |      | <i>h</i>             | <i>m</i> | <i>s</i> | <i>o</i> | <i>'</i> | <i>o</i>             | <i>'</i> |                      |
| 1st Group     | $\lambda$ Draconis | 3-4  | 11                   | 24       | 47       | 20       | 3        | 51                   | 3        | N                    |
|               | 1070 Gr. 72        | 4    |                      | 27       | 29       | 121      | 14       | 50                   | 8        | S                    |
|               | 4010               | 6-7  |                      | 46       | 33       | 51       | 29       | 19                   | 37       | N                    |
|               | 4018               | 7    |                      | 48       | 0        | 48       | 28       | 22                   | 38       | N                    |
|               | 4030               | 7    |                      | 49       | 44       | 94       | 31       | 23                   | 25       | S                    |
|               | 4038               | 7    |                      | 52       | 31       | 85       | 54       | 14                   | 48       | S                    |
|               | 4049               | 5½   |                      | 54       | 14       | 85       | 43       | 14                   | 37       | S                    |
|               | 4056               | 6    |                      | 56       | 1        | 67       | 17       | 3                    | 49       | N                    |
|               | 4066               | 6    |                      | 58       | 33       | 67       | 55       | 3                    | 11       | N                    |
|               | 4072               | 4    |                      | 59       | 31       | 80       | 39       | 9                    | 33       | S                    |
| 2nd Group     | 4096               | 6    | 12                   | 4        | 22       | 83       | 34       | 12                   | 28       | S                    |
|               | 4107               | 6    |                      | 6        | 12       | 63       | 30       | 7                    | 36       | N                    |
|               | 4114               | 6    |                      | 7        | 46       | 79       | 7        | 8                    | 1        | S                    |
|               | 4127               | 5    |                      | 10       | 42       | 65       | 26       | 5                    | 40       | N                    |
|               | 4184               | 7    |                      | 12       | 26       | 93       | 20       | 22                   | 14       | S                    |
|               | 4141               | 6    |                      | 13       | 38       | 66       | 21       | 4                    | 45       | N                    |
|               | 4156               | 5    |                      | 15       | 2        | 71       | 35       | 0                    | 29       | S                    |
|               | 4168               | 6    |                      | 16       | 52       | 84       | 4        | 12                   | 58       | S                    |
|               | 4267               | 6    |                      | 35       | 55       | 78       | 57       | 7                    | 51       | S                    |
|               | 4277               | 6    |                      | 37       | 53       | 90       | 57       | 19                   | 51       | S                    |
| 3rd Group     | 4285               | 6    |                      | 39       | 43       | 50       | 7        | 20                   | 59       | N                    |
|               | 4291               | 6    |                      | 41       | 19       | 83       | 26       | 12                   | 20       | S                    |
|               | 4304               | 6    |                      | 43       | 49       | 61       | 50       | 9                    | 16       | N                    |
|               | 4311               | 6    |                      | 44       | 53       | 51       | 52       | 19                   | 14       | N                    |
|               | 1191-92 Gr. 72     | 5-6  |                      | 48       | 8        | 5        | 58       | 65                   | 8        | N                    |
|               | 4351               | 4½   |                      | 53       | 24       | 71       | 59       | 0                    | 53       | S                    |
|               | 4358               | 6½   |                      | 54       | 51       | 92       | 46       | 21                   | 40       | S                    |
|               | 4387               | 5    | 13                   | 0        | 53       | 68       | 15       | 2                    | 51       | N                    |
|               | 4394               | 5½   |                      | 2        | 41       | 98       | 23       | 27                   | 17       | S                    |
|               | 4406               | 4½   |                      | 4        | 36       | 71       | 53       | 0                    | 47       | S                    |
| 4th Group     | 4421               | 4-5  |                      | 6        | 40       | 61       | 33       | 9                    | 33       | N                    |
|               | 4431               | 7    |                      | 8        | 17       | 87       | 57       | 16                   | 51       | S                    |
|               | 4436               | 7    |                      | 9        | 50       | 86       | 21       | 15                   | 15       | S                    |
|               | 4440               | 6    |                      | 11       | 14       | 80       | 0        | 8                    | 54       | S                    |
|               | 4453               | 6    |                      | 13       | 17       | 55       | 19       | 15                   | 47       | N                    |
|               | Polaris S.P.       | 1½   |                      | 17       | 1        | 1        | 17       | 72                   | 23       | N                    |

\* This form was made out for the observer at Bombay, the zenith distances therefore refer to that station.

23. This programme has been selected at random from the records, and is one that was carried out in the season 1887-88 by Colonels Strahan and Heaviside. The intervals between the stars and between the groups will be found too small for inexperienced hands, but quite practicable for experienced observers. The facsimile of the Chronograph record given at the end of the chapter is that of one night's work on this particular programme, and will be found of great assistance to the novice as a standard of reference on the proper method of writing up the record, &c.

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# Arc Mangalore Bombay - Station Bombay. Power Strain



marked up by ST  
Read off WJK

|           | $c_0$  | $c_s$  | $m$    | $e_1$ | $e$  | $h$  |                         |
|-----------|--------|--------|--------|-------|------|------|-------------------------|
| Oct 11-10 | 2499.8 | 2500.0 | 2498.0 | -0.2  | -1.1 | +1.4 | for first two groups    |
| 12 25     | 2499.0 |        | 2499.2 | -1.0  | -1.9 | +0.2 | for last " "            |
| mean      | 2499.4 |        |        |       |      |      | mean -1.5 for all pumps |

re Mangalore Bombay - Station Bombay. Observer Strahan. Telescope No. 2. I. P. E. Clock 1134. Chronog. A. April 8/88

7. Dracons & 1070 ft. 72. for Azim<sup>n</sup> - W. Clock R +

11 21

Clock comparison

Prin. elongation

1<sup>st</sup> group E. Clock R +

11 53

2<sup>nd</sup> group E. Clock R +

3<sup>rd</sup> group W. Clock R +

4<sup>th</sup> group W. Clock R +

|       |        |  |  |  |  |  |  |  |  |
|-------|--------|--|--|--|--|--|--|--|--|
| 11 21 |        |  |  |  |  |  |  |  |  |
| 23    |        |  |  |  |  |  |  |  |  |
| 25    |        |  |  |  |  |  |  |  |  |
| 27    |        |  |  |  |  |  |  |  |  |
| 29    |        |  |  |  |  |  |  |  |  |
| 31    |        |  |  |  |  |  |  |  |  |
| 11 53 |        |  |  |  |  |  |  |  |  |
| 55    | 4016 R |  |  |  |  |  |  |  |  |
| 57    |        |  |  |  |  |  |  |  |  |
| 59    |        |  |  |  |  |  |  |  |  |
| 61    |        |  |  |  |  |  |  |  |  |
| 63    |        |  |  |  |  |  |  |  |  |
| 65    |        |  |  |  |  |  |  |  |  |
| 67    |        |  |  |  |  |  |  |  |  |
| 69    |        |  |  |  |  |  |  |  |  |
| 71    |        |  |  |  |  |  |  |  |  |
| 73    |        |  |  |  |  |  |  |  |  |
| 75    |        |  |  |  |  |  |  |  |  |
| 77    |        |  |  |  |  |  |  |  |  |
| 79    |        |  |  |  |  |  |  |  |  |
| 81    |        |  |  |  |  |  |  |  |  |
| 83    |        |  |  |  |  |  |  |  |  |
| 85    |        |  |  |  |  |  |  |  |  |
| 87    |        |  |  |  |  |  |  |  |  |
| 89    |        |  |  |  |  |  |  |  |  |
| 91    |        |  |  |  |  |  |  |  |  |
| 93    |        |  |  |  |  |  |  |  |  |
| 95    |        |  |  |  |  |  |  |  |  |
| 97    |        |  |  |  |  |  |  |  |  |
| 99    |        |  |  |  |  |  |  |  |  |
| 11 53 |        |  |  |  |  |  |  |  |  |
| 55    |        |  |  |  |  |  |  |  |  |
| 57    |        |  |  |  |  |  |  |  |  |
| 59    |        |  |  |  |  |  |  |  |  |
| 61    |        |  |  |  |  |  |  |  |  |
| 63    |        |  |  |  |  |  |  |  |  |
| 65    |        |  |  |  |  |  |  |  |  |
| 67    |        |  |  |  |  |  |  |  |  |
| 69    |        |  |  |  |  |  |  |  |  |
| 71    |        |  |  |  |  |  |  |  |  |
| 73    |        |  |  |  |  |  |  |  |  |
| 75    |        |  |  |  |  |  |  |  |  |
| 77    |        |  |  |  |  |  |  |  |  |
| 79    |        |  |  |  |  |  |  |  |  |
| 81    |        |  |  |  |  |  |  |  |  |
| 83    |        |  |  |  |  |  |  |  |  |
| 85    |        |  |  |  |  |  |  |  |  |
| 87    |        |  |  |  |  |  |  |  |  |
| 89    |        |  |  |  |  |  |  |  |  |
| 91    |        |  |  |  |  |  |  |  |  |
| 93    |        |  |  |  |  |  |  |  |  |
| 95    |        |  |  |  |  |  |  |  |  |
| 97    |        |  |  |  |  |  |  |  |  |
| 99    |        |  |  |  |  |  |  |  |  |

$\begin{matrix} 7 & e_1 & e & L \\ 10 & -0.2 & -1.1 & +1.4 \end{matrix}$  for first two groups  
 $\begin{matrix} 12 & -1.0 & -1.9 & +0.2 \end{matrix}$  for last " "  
 mean - 1.5 for all groups

Strahan. Telescope No. 2. I. P. E. Clock 1134. Chronog. A. April 8/88

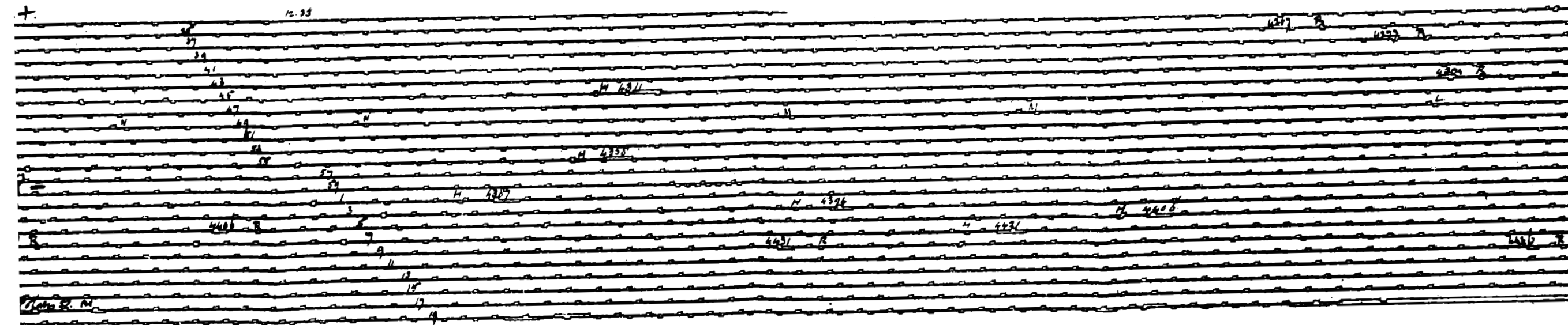
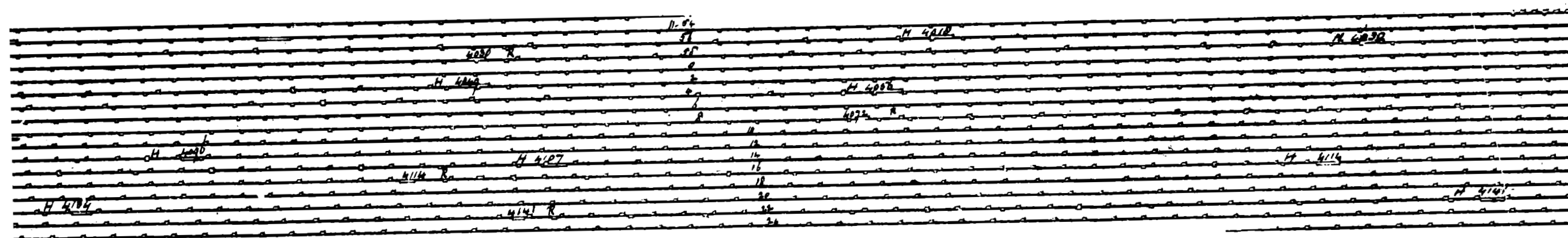
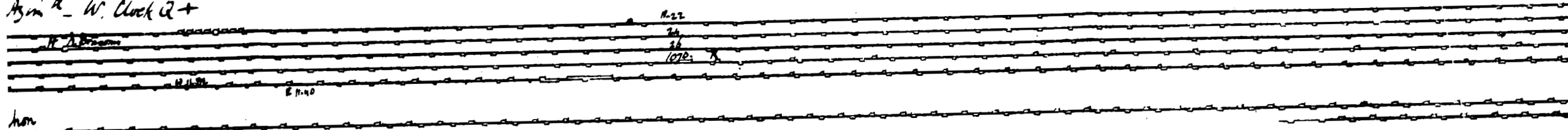
7. Dracons & 1070 pr. 72. for Azim<sup>n</sup> - W. Clock Q+

| H. A. Dracons |        | H. 1134 |        | H. 1070 |        | H. 72  |        |
|---------------|--------|---------|--------|---------|--------|--------|--------|
| 4049 R        | 4049 R | 4049 R  | 4049 R | 4049 R  | 4049 R | 4049 R | 4049 R |
| 4050 R        | 4050 R | 4050 R  | 4050 R | 4050 R  | 4050 R | 4050 R | 4050 R |
| 4051 R        | 4051 R | 4051 R  | 4051 R | 4051 R  | 4051 R | 4051 R | 4051 R |
| 4052 R        | 4052 R | 4052 R  | 4052 R | 4052 R  | 4052 R | 4052 R | 4052 R |
| 4053 R        | 4053 R | 4053 R  | 4053 R | 4053 R  | 4053 R | 4053 R | 4053 R |
| 4054 R        | 4054 R | 4054 R  | 4054 R | 4054 R  | 4054 R | 4054 R | 4054 R |
| 4055 R        | 4055 R | 4055 R  | 4055 R | 4055 R  | 4055 R | 4055 R | 4055 R |
| 4056 R        | 4056 R | 4056 R  | 4056 R | 4056 R  | 4056 R | 4056 R | 4056 R |
| 4057 R        | 4057 R | 4057 R  | 4057 R | 4057 R  | 4057 R | 4057 R | 4057 R |
| 4058 R        | 4058 R | 4058 R  | 4058 R | 4058 R  | 4058 R | 4058 R | 4058 R |
| 4059 R        | 4059 R | 4059 R  | 4059 R | 4059 R  | 4059 R | 4059 R | 4059 R |
| 4060 R        | 4060 R | 4060 R  | 4060 R | 4060 R  | 4060 R | 4060 R | 4060 R |
| 4061 R        | 4061 R | 4061 R  | 4061 R | 4061 R  | 4061 R | 4061 R | 4061 R |
| 4062 R        | 4062 R | 4062 R  | 4062 R | 4062 R  | 4062 R | 4062 R | 4062 R |
| 4063 R        | 4063 R | 4063 R  | 4063 R | 4063 R  | 4063 R | 4063 R | 4063 R |
| 4064 R        | 4064 R | 4064 R  | 4064 R | 4064 R  | 4064 R | 4064 R | 4064 R |
| 4065 R        | 4065 R | 4065 R  | 4065 R | 4065 R  | 4065 R | 4065 R | 4065 R |
| 4066 R        | 4066 R | 4066 R  | 4066 R | 4066 R  | 4066 R | 4066 R | 4066 R |
| 4067 R        | 4067 R | 4067 R  | 4067 R | 4067 R  | 4067 R | 4067 R | 4067 R |
| 4068 R        | 4068 R | 4068 R  | 4068 R | 4068 R  | 4068 R | 4068 R | 4068 R |
| 4069 R        | 4069 R | 4069 R  | 4069 R | 4069 R  | 4069 R | 4069 R | 4069 R |
| 4070 R        | 4070 R | 4070 R  | 4070 R | 4070 R  | 4070 R | 4070 R | 4070 R |
| 4071 R        | 4071 R | 4071 R  | 4071 R | 4071 R  | 4071 R | 4071 R | 4071 R |
| 4072 R        | 4072 R | 4072 R  | 4072 R | 4072 R  | 4072 R | 4072 R | 4072 R |
| 4073 R        | 4073 R | 4073 R  | 4073 R | 4073 R  | 4073 R | 4073 R | 4073 R |
| 4074 R        | 4074 R | 4074 R  | 4074 R | 4074 R  | 4074 R | 4074 R | 4074 R |
| 4075 R        | 4075 R | 4075 R  | 4075 R | 4075 R  | 4075 R | 4075 R | 4075 R |
| 4076 R        | 4076 R | 4076 R  | 4076 R | 4076 R  | 4076 R | 4076 R | 4076 R |
| 4077 R        | 4077 R | 4077 R  | 4077 R | 4077 R  | 4077 R | 4077 R | 4077 R |
| 4078 R        | 4078 R | 4078 R  | 4078 R | 4078 R  | 4078 R | 4078 R | 4078 R |
| 4079 R        | 4079 R | 4079 R  | 4079 R | 4079 R  | 4079 R | 4079 R | 4079 R |
| 4080 R        | 4080 R | 4080 R  | 4080 R | 4080 R  | 4080 R | 4080 R | 4080 R |
| 4081 R        | 4081 R | 4081 R  | 4081 R | 4081 R  | 4081 R | 4081 R | 4081 R |
| 4082 R        | 4082 R | 4082 R  | 4082 R | 4082 R  | 4082 R | 4082 R | 4082 R |
| 4083 R        | 4083 R | 4083 R  | 4083 R | 4083 R  | 4083 R | 4083 R | 4083 R |
| 4084 R        | 4084 R | 4084 R  | 4084 R | 4084 R  | 4084 R | 4084 R | 4084 R |
| 4085 R        | 4085 R | 4085 R  | 4085 R | 4085 R  | 4085 R | 4085 R | 4085 R |
| 4086 R        | 4086 R | 4086 R  | 4086 R | 4086 R  | 4086 R | 4086 R | 4086 R |
| 4087 R        | 4087 R | 4087 R  | 4087 R | 4087 R  | 4087 R | 4087 R | 4087 R |
| 4088 R        | 4088 R | 4088 R  | 4088 R | 4088 R  | 4088 R | 4088 R | 4088 R |
| 4089 R        | 4089 R | 4089 R  | 4089 R | 4089 R  | 4089 R | 4089 R | 4089 R |
| 4090 R        | 4090 R | 4090 R  | 4090 R | 4090 R  | 4090 R | 4090 R | 4090 R |
| 4091 R        | 4091 R | 4091 R  | 4091 R | 4091 R  | 4091 R | 4091 R | 4091 R |
| 4092 R        | 4092 R | 4092 R  | 4092 R | 4092 R  | 4092 R | 4092 R | 4092 R |
| 4093 R        | 4093 R | 4093 R  | 4093 R | 4093 R  | 4093 R | 4093 R | 4093 R |
| 4094 R        | 4094 R | 4094 R  | 4094 R | 4094 R  | 4094 R | 4094 R | 4094 R |
| 4095 R        | 4095 R | 4095 R  | 4095 R | 4095 R  | 4095 R | 4095 R | 4095 R |
| 4096 R        | 4096 R | 4096 R  | 4096 R | 4096 R  | 4096 R | 4096 R | 4096 R |
| 4097 R        | 4097 R | 4097 R  | 4097 R | 4097 R  | 4097 R | 4097 R | 4097 R |
| 4098 R        | 4098 R | 4098 R  | 4098 R | 4098 R  | 4098 R | 4098 R | 4098 R |
| 4099 R        | 4099 R | 4099 R  | 4099 R | 4099 R  | 4099 R | 4099 R | 4099 R |
| 4100 R        | 4100 R | 4100 R  | 4100 R | 4100 R  | 4100 R | 4100 R | 4100 R |

Chronog. A. April 8/88

Mang. Bowl.  
Bowl. No 3  
Ab 8/88

Asim R - W. Clock Q +







Form No. 1. 8th April 1888, Observer Colonel G. Strahan, Transit Telescope No. 2,  
I.P.E. East Clock, Station Bombay, Arc Mangalore-Bombay.

| Star ...                                                    | 4267                |                     | 4277                |                     | 4285                |                     |
|-------------------------------------------------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| Declination and Aspect*                                     | + 11° 3' S          |                     | - 0° 57' S          |                     | + 39° 53' N         |                     |
| Observed Times of Wires<br>and<br>Reduction to Central Wire | <i>h m</i><br>12 35 | <i>h m</i><br>12 36 | <i>h m</i><br>12 37 | <i>h m</i><br>12 38 | <i>h m</i><br>12 39 | <i>h m</i><br>12 39 |
|                                                             | H 55°50             | 9°19                | H 52°90             | 6°34                | H 37°60             | 55°11               |
|                                                             | °83                 | °23                 | °13                 | °32                 | °55                 | °14                 |
|                                                             | °20                 | °27                 | °35                 | °25                 | °45                 | °05                 |
|                                                             | °70                 | °19                 | °90                 | °31                 | °35                 | °09                 |
|                                                             | °00                 | °25                 | °20                 | °41                 | °25                 | °13                 |
|                                                             | °27                 | °27                 | °43                 | °43                 | °10                 | °10                 |
|                                                             | °57                 | °24                 | °73                 | °44                 | °15                 | °16                 |
|                                                             | °83                 | °19                 | °97                 | °42                 | °10                 | °17                 |
|                                                             | °55                 | °33                 | °40                 | °35                 | °90                 | °10                 |
|                                                             | °75                 | °29                 | °63                 | °38                 | °93                 | °26                 |
|                                                             | R 23°07             | 9°33                | R 19°90             | 6°42                | R 12°80             | 55°24               |
| Mean                                                        | <i>h m</i><br>12 36 | <i>s</i><br>9°25    | <i>h m</i><br>12 38 | <i>s</i><br>6°37    | <i>h m</i><br>12 39 | <i>s</i><br>55°14   |
| Corrections                                                 | C c } c = -2°9      |                     | ...                 | - °06               | ...                 | - °08               |
|                                                             | B b } b = -0°6      |                     | ...                 | - °01               | ...                 | - °02               |
|                                                             | A a } a = +8°3      |                     | ...                 | + °06               | ...                 | - °09               |
|                                                             | Q } Q = +1°61       |                     | ...                 | + 1°61              | ...                 | + 1°61              |
| Corrected Mean                                              | 12 36               | 10°82               | 12 38               | 7°97                | 12 39               | 56°56               |
| Do. of Day Preceding                                        | ...                 | 6°20                | ...                 | 3°44                | ...                 | 52°03               |
| Apparent Clock Rate Correction                              | ...                 | - 4°62              | ...                 | - 4°53              | ...                 | - 4°53              |

\* For explanation of the term "aspect" consult Volume X, Chapter IV, Section I.

Transcribed by \_\_\_\_\_ Computed and Compared by \_\_\_\_\_

Table for Reduction to Centre Wire, Telescope No. 2. 1887-88.

| Wire | Equatorial<br>Intervals<br>=<br>I | I        | I        | I        | I        |
|------|-----------------------------------|----------|----------|----------|----------|
|      |                                   | x<br>1'1 | x<br>1'2 | x<br>1'3 | x<br>1'4 |
| A    | 31'36                             | 34'50    | 37'63    | 40'77    | 43'90    |
| B    | 29'07                             | 31'98    | 34'88    | 37'79    | 40'70    |
| C    | 26'88                             | 29'57    | 32'26    | 34'94    | 37'63    |
| D    | 24'50                             | 26'95    | 29'40    | 31'85    | 34'30    |
| E    | 22'34                             | 24'57    | 26'81    | 29'04    | 31'28    |
| F    | 17'89                             | 19'68    | 21'48    | 23'26    | 25'05    |
| G    | 15'63                             | 17'19    | 18'76    | 20'32    | 21'88    |
| H    | 13'44                             | 14'78    | 16'13    | 17'47    | 18'81    |
| I    | 11'19                             | 12'31    | 13'43    | 14'55    | 15'67    |
| J    | 8'90                              | 9'79     | 10'68    | 11'57    | 12'46    |
| K    | 4'41                              | 4'85     | 5'29     | 5'73     | 6'17     |
| L    | 2'21                              | 2'43     | 2'65     | 2'87     | 3'09     |
| M    | ...                               | ...      | ...      | ...      | ...      |
| N    | 2'29                              | 2'52     | 2'75     | 2'98     | 3'21     |
| O    | 4'55                              | 5'01     | 5'46     | 5'92     | 6'37     |
| P    | 9'05                              | 9'96     | 10'86    | 11'77    | 12'67    |
| Q    | 11'25                             | 12'38    | 13'50    | 14'63    | 15'75    |
| R    | 13'48                             | 14'83    | 16'18    | 17'52    | 18'87    |
| S    | 15'74                             | 17'31    | 18'89    | 20'46    | 22'04    |
| T    | 18'01                             | 19'81    | 21'61    | 23'41    | 25'21    |
| U    | 22'47                             | 24'72    | 26'96    | 29'21    | 31'46    |
| V    | 24'68                             | 27'15    | 29'62    | 32'08    | 34'55    |
| W    | 26'94                             | 29'63    | 32'33    | 35'02    | 37'72    |
| X    | 29'17                             | 32'09    | 35'00    | 37'92    | 40'84    |
| Y    | 31'43                             | 34'57    | 37'72    | 40'86    | 44'00    |
| m*   | 0'0224                            | 0'0246   | 0'0269   | 0'0291   | 0'0314   |

\* m = value of Telescope Micrometer.

Form No. 2. Reduction to Centre Wire, and Constants for Collimation, Level, and Azimuth, C.B.A., for Telescope No. 2, at Bombay, Lat. = 18° 54'.

| Star ...                        | 4267     | 4277     | 4285      |  |  |  |
|---------------------------------|----------|----------|-----------|--|--|--|
| Declination = $\delta$ ...      | + 11° 3' | - 0° 57' | + 39° 53' |  |  |  |
| Zenith Distance = $\zeta^*$ ... | 7 51     | 19 51    | 20 59     |  |  |  |
| Nat. sec $\delta$ ...           | 1'0189   | 1'0001   | 1'3032    |  |  |  |
| Nat. cos $\zeta$ ...            | '9906    | '9406    | '9337     |  |  |  |
| Nat. sin $\zeta$ ...            | '1366    | '3396    | '3581     |  |  |  |
| A ...                           | 31'95    | 31'36    | 40'87     |  |  |  |
| B ...                           | 29'62    | 29'07    | 37'88     |  |  |  |
| C ...                           | 27'39    | 26'88    | 35'03     |  |  |  |
| D ...                           | 24'96    | 24'50    | 31'93     |  |  |  |
| E ...                           | 22'76    | 22'34    | 29'11     |  |  |  |
| F ...                           | 18'23    | 17'89    | 23'32     |  |  |  |
| G ...                           | 15'92    | 15'63    | 20'37     |  |  |  |
| H ...                           | 13'69    | 13'44    | 17'51     |  |  |  |
| I ...                           | 11'40    | 11'19    | 14'59     |  |  |  |
| J ...                           | 9'07     | 8'90     | 11'60     |  |  |  |
| K ...                           | 4'49     | 4'41     | 5'74      |  |  |  |
| L ...                           | 2'25     | 2'21     | 2'88      |  |  |  |
| M ...                           | ...      | ...      | ...       |  |  |  |
| N ...                           | 2'33     | 2'29     | 2'99      |  |  |  |
| O ...                           | 4'64     | 4'55     | 5'93      |  |  |  |
| P ...                           | 9'22     | 9'05     | 11'80     |  |  |  |
| Q ...                           | 11'46    | 11'25    | 14'67     |  |  |  |
| R ...                           | 13'74    | 13'48    | 17'56     |  |  |  |
| S ...                           | 16'04    | 15'74    | 20'51     |  |  |  |
| T ...                           | 18'35    | 18'01    | 23'47     |  |  |  |
| U ...                           | 22'89    | 22'47    | 29'28     |  |  |  |
| V ...                           | 25'15    | 24'68    | 32'16     |  |  |  |
| W ...                           | 27'45    | 26'94    | 35'11     |  |  |  |
| X ...                           | 29'72    | 29'17    | 38'01     |  |  |  |
| Y ...                           | 32'02    | 31'43    | 40'96     |  |  |  |
| $m \sec \delta = C$ ...         | 0'0228   | 0'0224   | 0'0292    |  |  |  |
| $C \cos \zeta = B$ ...          | 0'0226   | 0'0211   | 0'0273    |  |  |  |
| $C \sin \zeta = A$ ...          | 0'0031   | 0'0076   | 0'0105    |  |  |  |

\* If declination ( $\delta$ ) is North and greater than ( $\lambda$ ) the Lat.  $\zeta = \delta - \lambda$ .  
 " " " " less "  $\zeta = \lambda - \delta$ .  
 " " " " South  $\zeta = \lambda + \delta$ .  
 In the first case the star is north of the zenith, and south in the last two.

*Abstract of Determination of Collimation and Level Correction Constants.*

**Form No. 3.**

| Astro-<br>nomical<br>Date | Station and<br>Telescope    | Instrumental<br>Position | Collimation           |        |       |      | Level  |      | Mean $C_0$            | Station and<br>Telescope | Instrumental<br>Position | Collimation |        |        |      | Level |                 | Mean $C_0$ |
|---------------------------|-----------------------------|--------------------------|-----------------------|--------|-------|------|--------|------|-----------------------|--------------------------|--------------------------|-------------|--------|--------|------|-------|-----------------|------------|
|                           |                             |                          | $C_0$                 | $C_s$  | $c_1$ | $c$  | M      | b    |                       |                          |                          | $C_0$       | $C_s$  | $c_1$  | $c$  | M     | b               |            |
| 1888<br>April 4           | MANGALORE (Telescope No. 1) | I. P. W.                 | $d$                   | $d$    | $d$   | $d$  | $d$    | $d$  | I. P. W. 1742.7       | I. P. W.                 | $d$                      | $d$         | $d$    | $d$    | $d$  | $d$   | I. P. W. 2497.8 |            |
|                           |                             |                          | 1742.1                | 1745.0 | +3.2  | +2.3 | 1742.6 | +0.9 |                       |                          | 1741.0                   | -0.2        | 2496.8 | 2498.0 | 0.0  | -0.9  |                 | 2495.2     |
| "                         | MANGALORE (Telescope No. 1) | I. P. W.                 | $d$                   | $d$    | $d$   | $d$  | $d$    | $d$  | I. P. W. 1741.3       | I. P. W.                 | $d$                      | $d$         | $d$    | $d$    | $d$  | $d$   | I. P. W. 2495.9 |            |
|                           |                             |                          | 1742.1                | 1745.0 | +3.2  | +2.3 | 1742.6 | +0.9 |                       |                          | 1741.0                   | -0.2        | 2496.8 | 2497.0 | -1.0 | -1.9  |                 | 2500.8     |
| "                         | MANGALORE (Telescope No. 1) | I. P. E.                 | $d$                   | $d$    | $d$   | $d$  | $d$    | $d$  | I. P. E. 1741.4       | I. P. E.                 | $d$                      | $d$         | $d$    | $d$    | $d$  | $d$   | I. P. E. 2499.8 |            |
|                           |                             |                          | 1741.1                | 1740.0 | +1.8  | +0.9 | 1739.2 | +2.6 |                       |                          | 1739.2                   | -2.9        | 2500.0 | 2500.0 | -2.0 | -2.9  |                 | 2498.0     |
|                           |                             |                          | General Mean = 1741.8 |        |       |      |        |      | General Mean = 2498.0 |                          |                          |             |        |        |      |       |                 |            |
|                           |                             |                          | I. P. W. = 1742.2     |        |       |      |        |      | I. P. E. = 2499.4     |                          |                          |             |        |        |      |       |                 |            |
|                           |                             |                          | I. P. E. = 1741.3     |        |       |      |        |      | I. P. W. = 2496.6     |                          |                          |             |        |        |      |       |                 |            |

|                                                                      |                                        |                   |
|----------------------------------------------------------------------|----------------------------------------|-------------------|
| Name of Star ... ..                                                  | 1191, Gr. 72.                          |                   |
| Date for which required ...                                          | April 8, 1888.                         |                   |
| Epoch of Catalogue ... ..                                            | Jan. 1, 1872.                          |                   |
| $y$ = Number of years elapsed to Jan. 1 of year of observation = 16. |                                        |                   |
|                                                                      | R. A.                                  | N. P. D.          |
|                                                                      | "                                      | "                 |
| Secular variation ... ..                                             | + 0'221                                | - 0'02            |
| Do. $\times y$ ... ..                                                | + 3'536                                | - 0'32            |
| Do. $\times y \div 200$ ... ..                                       | + 0'018                                | 0'00              |
| Annual precession ... ..                                             | + 0'371                                | + 19'62           |
| Proper motion ... ..                                                 | - 0'006                                | - 0'02            |
| Sum ... ..                                                           | + 0'383                                | + 19'60           |
| Sum $\times y$ ... ..                                                | + 6'128                                | + 5 13'6          |
| Place at epoch of Catalogue ...                                      | 12 <sup>h</sup> 48 <sup>m</sup> 4'787  | 5° 53' 10'1       |
| Reduced place January 1, 1888 ...                                    | 12 48 10'915                           | 5 58 23'7         |
| <b>e</b> ... ..                                                      | 9'75118                                | 9'97695           |
| <b>E</b> ... ..                                                      | 0'86503                                | 0'86503           |
|                                                                      | 0'61621      4'132                     | 0'84198      6'9  |
| <b>f</b> ... ..                                                      | 0'02716                                | 0'33705           |
| <b>F</b> ... ..                                                      | 1'25887                                | 1'25887           |
|                                                                      | 1'28603      19'321                    | 1'59592      39'4 |
| <b>g</b> ... ..                                                      | 1'40432                                | 1'64948           |
| <b>G</b> ... ..                                                      | 0'06985                                | 0'06985           |
|                                                                      | 1'47417      29'797                    | 1'71933      52'4 |
| <b>h</b> ... ..                                                      | 9'75376                                | 9'99639           |
| <b>H</b> ... ..                                                      | 1'47667                                | 1'47667           |
|                                                                      | 1'23043      16'999                    | 1'47306      29'7 |
| <b>l</b> ... ..                                                      | 124'665                                | 53'6              |
| <b>L</b> ... ..                                                      | 114'104                                | 114'1             |
| Sum ...                                                              | + 309'018                              | + 296'1           |
| Constant ...                                                         | - 300'000                              | - 300'0           |
| Proper motion for fraction of year ...                               | - 0'002                                | 0                 |
| Reduction to day of observation ...                                  | + 9'016                                | - 3'9             |
| Reduced place ... ..                                                 | 12 <sup>h</sup> 48 <sup>m</sup> 19'931 | 5° 58' 19'8       |

Form No. 5. Computation of Star-Constants A. B. C. for reduction of transits of Circumpolar Stars.

| Name of Star ...                               | 1191 Gr. 72              |                          | Polaris (sub-pole)       |                          |
|------------------------------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| Date ...                                       | April 8, 1888            |                          | April 8, 1888            |                          |
|                                                | E. Mangalore             | W. Bombay                | E. Mangalore             | W. Bombay                |
|                                                | ° ' "                    | ° ' "                    | ° ' "                    | ° ' "                    |
| Lat = $\lambda$ ...                            | 12 52 14                 | 18 53 49                 | 12 52 14                 | 18 53 49                 |
| Declination = $\delta$ or $\delta^*$           | 84 1 40                  | 84 1 40                  | 91 17 22                 | 91 17 22                 |
| $Z = \lambda - \delta$ or $\lambda - \delta^*$ | -71 9 26                 | -65 7 51                 | -78 25 8                 | -72 23 33                |
| Constant log † ...                             | $\bar{2}.35025$          | $\bar{2}.35025$          | $\bar{2}.35025$          | $\bar{2}.35025$          |
| log sec $\delta$ or $\delta^*$ ...             | 0.98277                  | 0.98277                  | 1.64776 <i>n</i>         | 1.64776 <i>n</i>         |
| Sum = log O (1) ...                            | $\bar{1}.33302$          | $\bar{1}.33302$          | $\bar{1}.99801$ <i>n</i> | $\bar{1}.99801$ <i>n</i> |
| log sin Z (2) ...                              | $\bar{1}.97608$ <i>n</i> | $\bar{1}.95774$ <i>n</i> | $\bar{1}.99107$ <i>n</i> | $\bar{1}.97916$ <i>n</i> |
| log cos Z (3) ...                              | $\bar{1}.50917$          | $\bar{1}.62382$          | $\bar{1}.30267$          | $\bar{1}.48072$          |
| log A = (1) + (2) ...                          | $\bar{1}.30910$ <i>n</i> | $\bar{1}.29076$ <i>n</i> | $\bar{1}.98908$          | $\bar{1}.97717$          |
| log B = (1) + (3) ...                          | $\bar{2}.84219$          | $\bar{2}.95684$          | $\bar{1}.30068$ <i>n</i> | $\bar{1}.47873$ <i>n</i> |
| A ...                                          | -0.2038                  | -0.1953                  | +0.9752                  | +0.9488                  |
| B ...                                          | +0.0695                  | +0.0905                  | -0.1998                  | -0.3011                  |
| C ...                                          | +0.2153                  | +0.2153                  | -0.9954                  | -0.9954                  |

\* $\delta^*$  = (180 -  $\delta$ ) is to be used instead of  $\delta$  for stars below the Pole.

† This is the logarithm of *m*, the value of the micrometer screw of the Telescope taken at 0°.0224. The letter *n* after a logarithm signifies that it is the logarithm of a negative quantity.

N.B.—Attend to signs throughout, South Declination is -. As a check on the proper application of the signs, note that for stars between

- North horizon and pole, A is +, B -, C -.
- Pole and zenith, A is -, B +, C +.
- Zenith and south horizon, A is +, B +, C +.

| Arc              | Station | Astro-<br>nomical<br>date | Instrumental<br>Position | Clock in use       | Star's<br>Name     | Culmination | No. of Wires<br>Observed | Devia-<br>tion<br>Constant<br>A | Observed<br>Time of<br>Transit | Correction for   |            |                        |               | Seconds of<br>Corrected Time<br>of Transit | Right<br>Ascension<br>(increased<br>by 12 hours<br>for lower<br>Culmina-<br>tions) | Appa-<br>rent Clock<br>Correction | Deducted Value<br>of Deviation<br>Correction <i>a</i> |
|------------------|---------|---------------------------|--------------------------|--------------------|--------------------|-------------|--------------------------|---------------------------------|--------------------------------|------------------|------------|------------------------|---------------|--------------------------------------------|------------------------------------------------------------------------------------|-----------------------------------|-------------------------------------------------------|
|                  |         |                           |                          |                    |                    |             |                          |                                 |                                | Colli-<br>mation | Level      | Pen<br>Equa-<br>tion Q | Clock<br>Rate |                                            |                                                                                    |                                   |                                                       |
| MANGALORE-BOMBAY | Bombay  | April 4                   | <i>I. P. W.</i>          | W                  | $\lambda$ Draconis | U           | 8                        | -0.0508                         | h m s<br>11 24 43.72           | s<br>-0.06       | s<br>-0.15 | s<br>+1.67             | s<br>-0.09    | s<br>45.18                                 | h m s<br>11 24 47.08                                                               | s<br>+ 1.90                       | d<br>+38.1                                            |
|                  |         | "                         | "                        | "                  | 1070 Gr. 72        | "           | 11                       | +0.0201                         | 11 27 24.93                    | -0.03            | -0.06      | +1.67                  |               | 26.51                                      | 11 27 31.11                                                                        | + 4.60                            |                                                       |
|                  |         | "                         | "                        | "                  | 1191 "             | "           | 5                        | -0.1957                         | 12 48 21.61                    | -0.19            | -0.31      | +1.67                  |               | 22.78                                      | 12 48 20.00                                                                        | - 2.78                            |                                                       |
|                  |         | "                         | "                        | "                  | 1192 "             | "           | 5                        | -0.1955                         | 12 48 29.63                    | -0.19            | -0.31      | +1.67                  |               | 30.80                                      | 12 48 27.68                                                                        | - 3.12                            |                                                       |
|                  |         | "                         | "                        | "                  | Polaris            | L           | 2                        | +0.9488                         | 13 16 22.81                    | +0.90            | +1.02      | -1.67                  | -0.09         | 22.97                                      | 13 16 59.67                                                                        | +36.70                            | +34.7                                                 |
|                  |         | April 7                   | <i>I. P. W.</i>          | W                  | $\lambda$ Draconis | U           | 8                        | -0.0508                         | 11 24 56.78                    | -0.07            | +0.12      | +1.66                  |               | 58.49                                      | 11 24 47.02                                                                        | -11.47                            | +44.6                                                 |
|                  |         | "                         | "                        | "                  | 1070 Gr. 72        | "           | 11                       | +0.0201                         | 11 27 37.73                    | -0.03            | +0.05      | +1.66                  |               | 39.41                                      | 11 27 31.10                                                                        | - 8.31                            |                                                       |
|                  |         | "                         | "                        | "                  | 1191 "             | "           | 5                        | -0.1957                         | 12 48 35.50                    | -0.21            | +0.25      | +1.66                  |               | 37.20                                      | 12 48 19.95                                                                        | -17.25                            |                                                       |
|                  |         | "                         | "                        | "                  | 1192 "             | "           | 5                        | -0.1955                         | 12 48 42.52                    | -0.21            | +0.25      | +1.66                  |               | 44.22                                      | 12 48 27.62                                                                        | -16.60                            |                                                       |
|                  |         | "                         | "                        | "                  | Polaris            | L           | 2                        | +0.9488                         | 13 16 32.91                    | +1.00            | -0.83      | -1.66                  | -0.09         | 31.33                                      | 13 16 59.31                                                                        | +27.98                            | +39.2                                                 |
|                  | April 8 | <i>I. P. E.</i>           | W                        | $\lambda$ Draconis | U                  | 8           | -0.0508                  | 11 24 59.96                     | -0.20                          | -0.03            | +1.61      |                        | 61.34         | 11 24 46.99                                | -14.35                                                                             |                                   |                                                       |
|                  | "       | "                         | "                        | 1070 Gr. 72        | "                  | 11          | +0.0201                  | 11 27 43.03                     | -0.08                          | -0.01            | +1.61      |                        | 44.55         | 11 27 31.09                                | -13.46                                                                             | +12.6                             |                                                       |
|                  | "       | "                         | "                        | 1191 "             | "                  | 5           | -0.1957                  | 12 48 34.54                     | -0.62                          | -0.05            | +1.61      |                        | 35.48         | 12 48 19.93                                | -15.55                                                                             |                                   |                                                       |
|                  | "       | "                         | "                        | 1192 "             | "                  | 5           | -0.1955                  | 12 48 42.49                     | -0.62                          | -0.05            | +1.61      |                        | 43.43         | 12 48 27.60                                | -15.83                                                                             |                                   |                                                       |
|                  | "       | "                         | "                        | Polaris            | L                  | 2           | +0.9488                  | 13 17 9.20                      | +2.89                          | +0.18            | -1.61      | -0.09                  | 10.57         | 13 16 59.26                                | -11.31                                                                             | + 3.9                             |                                                       |



## Deduction of Clock Rate Corrections.

| Arc              | Approximate Difference of Longitude | Intervals between nights of observations | $\alpha$ = Correction for intervals between nights of observations.<br>$\beta$ = Hourly corrections for nights of observations, interpolated by means of $\alpha$ . |                |                           |                |                                    |                |                |                                                           |                |
|------------------|-------------------------------------|------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------|---------------------------|----------------|------------------------------------|----------------|----------------|-----------------------------------------------------------|----------------|
|                  |                                     |                                          | $\alpha$ at E Station for                                                                                                                                           |                | $\alpha$ at W Station for |                | Astronomical Dates of Observations | $\beta$ for    |                | Correction to Observed Difference of Times of Transit for |                |
|                  |                                     |                                          | E Clock                                                                                                                                                             | W Clock        | E Clock                   | W Clock        |                                    | E Clock        | W Clock        | E Clock                                                   | W Clock        |
| MANGALORE-BOMBAY | $8^m 7^s = 0^h 135$                 | 1888<br>April 4 to 7                     | $s$<br>+ 1'29                                                                                                                                                       | $s$<br>- 13'03 | $s$<br>+ 1'21             | $s$<br>- 12'99 | 1888<br>April 4                    | $s$<br>+ 0'017 | $s$<br>- 0'181 | $s$<br>+ 0'002                                            | $s$<br>- 0'024 |
|                  |                                     | „ 7 to 8                                 | + 0'61                                                                                                                                                              | - 4'45         | + 0'44                    | - 4'66         | „ 7                                | + 0'020        | - 0'185        | + 0'003                                                   | - 0'025        |
|                  |                                     |                                          |                                                                                                                                                                     |                |                           | „ 8            | + 0'022                            | - 0'190        | + 0'003        | - 0'026                                                   |                |

Form No. 9. *Abstract of Observed Values of Personal Equation between Colonels Strahan and Heaviside.*

| Observed with Telescope No. 2                  |                |             |                                                |                |             |                   |
|------------------------------------------------|----------------|-------------|------------------------------------------------|----------------|-------------|-------------------|
| At BOMBAY Lat. = 18° 54'                       |                |             |                                                |                |             |                   |
| By Stars of                                    | April 12, 1888 |             |                                                | April 13, 1888 |             |                   |
|                                                | Star           | Declination | Equation<br>S - H                              | Star           | Declination | Equation<br>S - H |
| NORTH ASPECT                                   |                | ° '<br>s    |                                                |                | ° '<br>s    |                   |
|                                                | 3710           | 28 34       | - 0.02                                         | 3710           | 28 34       | - 0.02            |
|                                                | 3735           | 26 5        | - 0.01                                         | 3735           | 26 5        | - 0.05            |
|                                                | 3751           | 26 6        | - 0.01                                         | 3751           | 26 6        | - 0.09            |
|                                                | 3776           | 20 48       | - 0.08                                         | 3776           | 20 59       | - 0.06            |
|                                                | 3797           | 26 9        | - 0.04                                         | 3797           | 26 9        | - 0.03            |
|                                                | 3809           | 25 16       | - 0.05                                         | 3809           | 25 16       | - 0.02            |
|                                                | 3842           | 23 42       | - 0.08                                         | 3851           | 32 10       | - 0.04            |
|                                                | 3851           | 32 10       | - 0.03                                         | 3915           | 19 2        | - 0.08            |
|                                                | 3915           | 19 2        | - 0.13                                         | 3937           | 28 24       | - 0.04            |
|                                                | 3937           | 28 24       | - 0.08                                         | 3952           | 44 15       | + 0.04            |
|                                                | 3952           | 44 15       | - 0.03                                         | 3964           | 21 59       | - 0.05            |
|                                                | 3964           | 21 59       | - 0.05                                         | 3990           | 20 51       | - 0.03            |
|                                                | 3990           | 20 51       | - 0.01                                         | 3998           | 35 33       | - 0.01            |
|                                                | 3998           | 35 33       | - 0.08                                         | 4010           | 38 31       | - 0.02            |
| 4010                                           | 38 31          | - 0.02      | 4018                                           | 41 45          | + 0.05      |                   |
| Mean S <sub>N</sub> - H <sub>N</sub> = - 0.048 |                |             | Mean S <sub>N</sub> - H <sub>N</sub> = - 0.030 |                |             |                   |
| SOUTH ASPECT                                   | 3720           | 4 11        | - 0.04                                         | 3696           | 7 8         | - 0.03            |
|                                                | 3761           | 12 18       | - 0.04                                         | 3720           | 4 11        | + 0.02            |
|                                                | 3785           | 4 14        | - 0.01                                         | 3761           | 12 18       | + 0.02            |
|                                                | 3824           | 15 0        | - 0.08                                         | 3785           | 4 14        | - 0.03            |
|                                                | 3832           | 0 32        | - 0.07                                         | 3824           | 15 0        | - 0.07            |
|                                                | 3862           | 6 39        | - 0.07                                         | 3832           | 0 32        | - 0.03            |
|                                                | 3873           | 0 45        | 0.00                                           | 3862           | 6 39        | 0.00              |
|                                                | 3886           | 17 4        | - 0.13                                         | 3873           | 0 45        | + 0.04            |
|                                                | 3900           | 3 28        | 0.00                                           | 3886           | 17 4        | - 0.11            |
|                                                | 3932           | 17 25       | - 0.05                                         | 3900           | 3 28        | + 0.04            |
|                                                | 3971           | 5 22        | - 0.01                                         | 3932           | 17 25       | - 0.05            |
|                                                | 3975           | - 6 3       | - 0.04                                         | 3971           | 5 22        | + 0.04            |
|                                                | 4030           | - 4 31      | - 0.05                                         | 3975           | - 6 3       | - 0.02            |
|                                                | 4039           | 4 6         | + 0.03                                         | 3979           | 9 5         | + 0.03            |
|                                                | 4049           | 4 17        | - 0.01                                         | 4030           | - 4 31      | - 0.01            |
| 4063                                           | - 4 51         | + 0.03      | 4039                                           | 4 6            | - 0.01      |                   |
| 4077                                           | - 2 30         | - 0.03      | 4049                                           | 4 17           | - 0.09      |                   |
| Mean S <sub>S</sub> - H <sub>S</sub> = - 0.034 |                |             | Mean S <sub>S</sub> - H <sub>S</sub> = - 0.015 |                |             |                   |

**Form No. 9.** *Observations of Transits with E Clock, and Deduction of the Apparent Difference of Longitudes,  $\Delta L - \rho$ .*

MANGALORE (E) Lat.  $12^{\circ} 52' 14''$ . 14, Long.  $74^{\circ} 53' 9''$ . 89 : AND BOMBAY (W) Lat.  $18^{\circ} 53' 49''$ . 49, Long.  $72^{\circ} 51' 16''$ . 27.

| Astronomical Date | STAR |         | TRANSITS OBSERVED AT E                    |                               |               |                                                |                    | TRANSITS OBSERVED AT W                   |                               |               |                                                |                    | Difference of Corrected Times (W - E) |      | Correction for Rate of E Clock |  | Cor. for Per. Equations |  | $\Delta L - \rho$ |                  |                           |
|-------------------|------|---------|-------------------------------------------|-------------------------------|---------------|------------------------------------------------|--------------------|------------------------------------------|-------------------------------|---------------|------------------------------------------------|--------------------|---------------------------------------|------|--------------------------------|--|-------------------------|--|-------------------|------------------|---------------------------|
|                   |      |         | By <i>Hearside</i> , with Telescope No. 1 |                               |               |                                                |                    | By <i>Strahan</i> , with Telescope No. 2 |                               |               |                                                |                    |                                       |      |                                |  |                         |  |                   |                  |                           |
|                   |      |         | B.A.C. Number                             | Declination                   | Star's Aspect | Instrumental Position and Correction Constants | Mean Observed Time | Total Correction                         | Seconds of Corrected Time     | Star's Aspect | Instrumental Position and Correction Constants | Mean Observed Time |                                       |      |                                |  |                         |  |                   | Total Correction | Seconds of Corrected Time |
| 1888              |      |         |                                           |                               |               |                                                |                    |                                          |                               |               |                                                |                    |                                       |      |                                |  |                         |  |                   |                  |                           |
| Apr. 8            | 4018 | + 41 32 | N                                         | I. P. E.                      | 11 48 3.42    | + 1.90                                         | 5.32               | N                                        | I. P. E.                      | 11 56 11.06   | + 1.42                                         | 12.48              | 8                                     | 7.16 |                                |  |                         |  |                   |                  |                           |
|                   | 4056 | + 22 43 | N                                         | d                             | 56 1.95       | + 1.84                                         | 3.79               | N                                        | d                             | 12 4 9.50     | + 1.49                                         | 10.99              |                                       | 7.20 |                                |  |                         |  |                   |                  |                           |
|                   | 4066 | + 22 5  | N                                         | c + 0.9<br>b + 2.6<br>a - 6.0 | 58 34.40      | + 1.83                                         | 36.23              | N                                        | c - 2.9<br>b - 0.6<br>a + 8.3 | 6 41.85       | + 1.50                                         | 43.35              |                                       | 7.12 |                                |  |                         |  |                   |                  |                           |
| "                 | 4049 | + 4 17  | S                                         | s                             | 11 54 14.66   | + 1.77                                         | 16.43              | S                                        | s                             | 12 2 22.05    | + 1.58                                         | 23.63              | 8                                     | 7.20 |                                |  |                         |  |                   |                  |                           |
|                   | 4072 | + 9 21  | S                                         | Q + 1.71                      | 59 32.21      | + 1.80                                         | 34.01              | S                                        | Q + 1.61                      | 7 39.61       | + 1.57                                         | 41.18              |                                       | 7.17 |                                |  |                         |  |                   |                  |                           |
| "                 | 4107 | + 26 30 | N                                         | -                             | 12 6 15.53    | - 1.57                                         | 13.96              | N                                        | -                             | 12 14 22.94   | - 1.74                                         | 21.20              | 8                                     | 7.24 |                                |  |                         |  |                   |                  |                           |
|                   | 4127 | + 24 34 | N                                         |                               | 10 45.84      | - 1.58                                         | 44.26              | N                                        |                               | 18 53.25      | - 1.73                                         | 51.52              |                                       | 7.26 |                                |  |                         |  |                   |                  |                           |
|                   | 4156 | + 18 25 | N                                         |                               | 15 8.83       | - 1.60                                         | 7.23               | N                                        |                               | 23 16.21      | - 1.71                                         | 14.50              |                                       | 7.27 |                                |  |                         |  |                   |                  |                           |
| "                 | 4096 | + 6 26  | S                                         |                               | 12 4 26.30    | - 1.65                                         | 24.65              | S                                        |                               | 12 12 33.59   | - 1.65                                         | 31.94              | 8                                     | 7.29 |                                |  |                         |  |                   |                  |                           |
|                   | 4134 | - 3 20  | S                                         |                               | 12 29.91      | - 1.67                                         | 28.24              | S                                        |                               | 20 37.16      | - 1.62                                         | 35.54              |                                       | 7.30 |                                |  |                         |  |                   |                  |                           |
|                   | 4168 | + 5 56  | S                                         |                               | 16 55.74      | - 1.63                                         | 54.11              | S                                        |                               | 25 3.03       | - 1.65                                         | 1.38               |                                       | 7.27 |                                |  |                         |  |                   |                  |                           |

NOTE.—Transcribing Equation *mil*, all records having been transcribed by the same person.  
\*  $\rho$  is the retardation of an electric signal between the stations.

Form No. 10. ARC NAGARKOIL-MANGALORE. Deduction of  $\Delta L$  the difference of Longitude and  $\rho$  the retardation of Signals.

(Telescope No. 2) at NAGARKOIL.

(Telescope No. 1) at MANGALORE.

| Telescope and Pivot Position | Astro-nomical Date | E. Clock   |            | W. Clock   |            | All Stars  |            | Both Clocks |            | Both Clocks | $\rho$   |
|------------------------------|--------------------|------------|------------|------------|------------|------------|------------|-------------|------------|-------------|----------|
|                              |                    | N. Stars   | S. Stars   | N. Stars   | S. Stars   | E. Clock   | W. Clock   | N. Stars    | S. Stars   | All Stars   |          |
| Tel. No. 1, I.P.E.           | 1888               | <i>m s</i> | <i>m s</i> | <i>m s</i> | <i>m s</i> | <i>m s</i> | <i>m s</i> | <i>m s</i>  | <i>m s</i> | <i>m s</i>  | <i>s</i> |
| „ 2, I.P.E.                  | Feb. 18            | 10 21' 121 | 10 21' 093 | 10 21' 195 | 10 21' 187 | 10 21' 107 | 10 21' 191 | 10 21' 158  | 10 21' 140 | 10 21' 149  | 0' 042   |
| Tel. No. 1, I.P.E.           | Feb. 19            | 10 21' 026 | 10 21' 028 | 10 21' 094 | 10 21' 101 | 10 21' 027 | 10 21' 097 | 10 21' 060  | 10 21' 065 | 10 21' 062  | 0' 035   |
| „ 2, I.P.W.                  | Feb. 19            | 10 21' 026 | 10 21' 028 | 10 21' 094 | 10 21' 101 | 10 21' 027 | 10 21' 097 | 10 21' 060  | 10 21' 065 | 10 21' 062  | 0' 035   |
| Tel. No. 1, I.P.W.           | Feb. 20            | ...        | ...        | 10 21' 189 | 10 21' 205 | ...        | 10 21' 197 | ...         | ...        | 10 21' 197  | ...      |
| „ 2, I.P.W.                  | Feb. 20            | ...        | ...        | 10 21' 189 | 10 21' 205 | ...        | 10 21' 197 | ...         | ...        | 10 21' 197  | ...      |
| Tel. No. 1, I.P.W.           | Feb. 21            | ...        | ...        | 10 21' 252 | 10 21' 252 | ...        | 10 21' 252 | ...         | ...        | 10 21' 252  | ...      |
| „ 2, I.P.E.                  | Feb. 21            | ...        | ...        | 10 21' 252 | 10 21' 252 | ...        | 10 21' 252 | ...         | ...        | 10 21' 252  | ...      |
| Tel. No. 1, I.P.W.           | Feb. 22            | 10 21' 136 | 10 21' 186 | 10 21' 256 | 10 21' 259 | 10 21' 161 | 10 21' 257 | 10 21' 196  | 10 21' 223 | 10 21' 209  | 0' 048   |
| „ 2, I.P.E.                  | Feb. 22            | 10 21' 136 | 10 21' 186 | 10 21' 256 | 10 21' 259 | 10 21' 161 | 10 21' 257 | 10 21' 196  | 10 21' 223 | 10 21' 209  | 0' 048   |
| Tel. No. 1, I.P.W.           | Feb. 24            | 10 21' 090 | 10 21' 112 | 10 21' 180 | 10 21' 185 | 10 21' 101 | 10 21' 183 | 10 21' 135  | 10 21' 149 | 10 21' 142  | 0' 041   |
| „ 2, I.P.W.                  | Feb. 24            | 10 21' 090 | 10 21' 112 | 10 21' 180 | 10 21' 185 | 10 21' 101 | 10 21' 183 | 10 21' 135  | 10 21' 149 | 10 21' 142  | 0' 041   |

Form No. 10—(Continued). **ARC NAGARKOIL-MANGALORE.** *Synopsis showing the results in the various Pivot positions and the Final Value of the Arc.*

| Telescope and Pivot Position | Astronomical Date | E. Clock   |            | W. Clock   |            | All Stars  |            | Both Clocks |            | Both Clocks |   |          |   |
|------------------------------|-------------------|------------|------------|------------|------------|------------|------------|-------------|------------|-------------|---|----------|---|
|                              |                   | N. Stars   |            | S. Stars   |            | E. Clock   |            | W. Clock    |            | N. Stars    |   | S. Stars |   |
|                              |                   | m          | s          | m          | s          | m          | s          | m           | s          | m           | s | m        | s |
| Tel. No. 1, <i>I.P.E.</i>    | 1888              | m          | s          | m          | s          | m          | s          | m           | s          | m           | s | m        | s |
| " 2, <i>I.P.E.</i>           | Feb. 18           | 10 21' 121 | 10 21' 093 | 10 21' 195 | 10 21' 187 | 10 21' 107 | 10 21' 191 | 10 21' 158  | 10 21' 140 | 10 21' 149  |   |          |   |
| Tel. No. 1, <i>I.P.E.</i>    | Feb. 19           | 10 21' 026 | 10 21' 028 | 10 21' 094 | 10 21' 101 | 10 21' 027 | 10 21' 097 | 10 21' 060  | 10 21' 065 | 10 21' 062  |   |          |   |
| " 2, <i>I.P.W.</i>           | Feb. 20 & 24      | 10 21' 090 | 10 21' 112 | 10 21' 185 | 10 21' 195 | 10 21' 101 | 10 21' 190 | 10 21' 138  | 10 21' 154 | 10 21' 146  |   |          |   |
| Tel. No. 1, <i>I.P.W.</i>    | Feb. 21 & 22      | 10 21' 136 | 10 21' 187 | 10 21' 254 | 10 21' 256 | 10 21' 161 | 10 21' 255 | 10 21' 195  | 10 21' 222 | 10 21' 208  |   |          |   |
| " 2, <i>I.P.E.</i>           | Means ...         | 10 21' 093 | 10 21' 105 | 10 21' 182 | 10 21' 185 | 10 21' 099 | 10 21' 183 | 10 21' 139  | 10 21' 145 | 10 21' 141  |   |          |   |

## PART V.

**Astronomical Latitudes.**

## CHAPTER I. GENERAL OBSERVATIONS.

1. When surveying is carried on with great accuracy and over large areas, it becomes necessary to take into account the deviation of the figure of the earth from the spherical form, and consequently one of the primary objects of a trigonometrical survey—and one that should be never lost sight of—is the determination of the absolute lengths of the equatorial and polar axes of the globe. It has been proved that the earth is only approximately spherical, but whether it is spheroidal or ellipsoidal or of some other form still has yet to be decided. If it is found to be either spheroidal or ellipsoidal the eccentricity of the meridian ellipse—and in the latter case that of the equator also—will have to be determined with the greatest precision. Hitherto in the calculations of the Great Trigonometrical Survey of India the earth has been assumed to be a spheroid, and the values of the axes that have been employed in the formulæ, by which the latitudes, longitudes and azimuths of the stations have been computed from the triangulation, are those determined by Colonel Everest in 1830 and known as “Everest’s Constants, 1st Set.” These values were deduced from a comparatively small amount of reliable data and are now believed to be sensibly erroneous.

Were the earth a perfect sphere composed of homogeneous matter, its size could be easily found by measuring the distance between two points on the same meridian, and finding their difference of latitude: this geodetic operation, which is usually called the “measurement of an arc of meridian,” gives the number of feet on the circumference of a great circle of the globe, that subtend a known angle at the centre. Were the earth a perfect homogeneous spheroid, two “arcs of meridian” would have to be measured in different latitudes to establish its size and form, and were it a perfect homogeneous ellipsoid three such arcs would be necessary, two at least of which must be separated by a known difference of longitude.

Simple, however, as these methods appear in theory, the determination of the figure in practice is greatly complicated by a force known as local

attraction, which is due to irregularities on the external surface and to variations of density of the material composing the crust, and which causes the direction of gravity to deviate from the normal to the mathematical form.\* Owing to its presence very discordant values are obtained from different combinations of measured arcs for the ellipticity of the surface, and in order to eliminate error in the final result the stations of comparison have to be largely multiplied, arcs of parallel have to be measured in addition to arcs of meridian, and the astronomical azimuth determined at several of the latitude stations. The differences between the observed and geodetic latitudes, longitudes and azimuths can then be treated by the method of minimum squares, and the figure to which the mathematical surface of the earth most nearly conforms can be determined.

The following distinctive definitions are now given to emphasize and explain the special effect that local attraction has on direct determinations of latitude, as it is with them alone that this part of the Handbook has to deal:—

The *true* horizon is that great circle of the celestial sphere, whose plane touches the earth at the observer.

The *true* zenith is the elevated pole of the *true* horizon.

The *true* or *geodetic* latitude of a place is the angle that the normal to the mathematical surface makes with the plane of the equator; it cannot be determined by direct observation, as it is dependent on the position of the true zenith, and no means exist of finding the latter.

The *observed* or *astronomical* latitude is the angle made with the plane of the equator by the direction of gravity, as indicated by the plumb-line.

Were it not for local attraction† the plumb-line would coincide with the normal, and the difference between the geodetic and observed latitude would be solely due to errors in the adopted value of the ellipticity.

\* By the "mathematical" form is meant the geometrical solid, whether spheroid or ellipsoid, to which the figure of the earth most nearly conforms.

† Local attraction affects every observed latitude directly and every geodetic latitude indirectly in having caused error in the adopted values of the initial azimuth and latitude at Kaliānpūr, on which all the Great Trigonometrical Survey computations are based.

Throughout this part of the Handbook wherever the word "latitude" occurs, the astronomical latitude is to be understood: similarly by the term "zenith" is meant that point in which the direction of the plumb-line infinitely produced above the horizon meets the celestial sphere, and by the term "horizon" (or the plane from which altitudes are measured) is meant the great circle perpendicular to the direction of the plumb-line.

2. It follows directly from the definitions of the words employed that the latitude is equal both to the declination of the zenith and to the altitude of the elevated pole, and every known method of determination is based on one or the other of these equalities.

*Method of determination.*

In India the determination of latitudes for geodetic purposes is at present effected with Strange's Zenith Sectors by measuring the zenith distances of stars on the meridian,\* and very excellent results have been obtained from this method. There are several instances where 30 stars have been observed successively in one night and the resulting values of latitude have all fallen within a range of 2". At many of Colonel Herschel's stations the probable error of a single determination was less than  $\pm 0'' \cdot 3$ , and at Kátpálayam where he observed 50 stars, the probable error of the final value of latitude was  $\pm 0'' \cdot 042$ . No observer however runs the slightest chance of obtaining such results as these, unless in choosing his stars, in handling his instrument, in taking his observations and in computing out his results he pays the closest attention to every detail.

3. When an officer is deputed to take latitude observations for geodetic purposes the series of principal triangulation on which he is to operate is notified, and the first duty that devolves on him is the selection of the particular stations of that series. The choice of the latitude stations should be governed by the following rules:—

*Selection of latitude stations.*

(a) Every latitude station must be a principal station of the specified series of triangulation.

(b) Every principal station at which an astronomical azimuth has been observed for geodetic purposes must be a latitude station.

\* The meridional zenith distance of a star is the difference between its declination and that of the zenith, and therefore the latitude is equal to the declination of the star  $\pm$  its zenith distance, the upper sign being used for south stars the lower for north.



(c) On meridional series, latitude stations should be  $\frac{1}{2}^\circ$  in latitude apart, and on longitudinal series the same amount in longitude.

(d) Stations are to be avoided at which a deflection of the plumb-line is to be anticipated in the direction of the meridian, a deflection in the prime vertical being of no consequence.

4. The strength of an Astronomical Party when employed on latitude operations is as follows:—

- (1) The observer, who is generally a Deputy Superintendent.
- (2) An Assistant-Surveyor or Sub-Surveyor whose duty is to build the pillars and who is always some ten days in advance of the main-body of the party.
- (3) An observatory recorder, who also acts the part of writer.
- (4) Eight burkundazes, who act as sentries over the treasure, instruments, baggage, &c.
- (5) Twenty kahars or bearers, who have to carry the zenith sector on the march. During one field season the zenith sector was carried about in a hand-cart provided with springs: this method is inexpensive and answered admirably, but it is only feasible when the operations lie in a flat country, and when all the stations to be visited are plain stations.
- (6) Forty khalasies. Thirty of these remain with the main-body: during a halt they are employed as dâk bearers and on ordinary observatory work, whilst on the march they carry the barometers, chronometers, &c., in doolies and pitch the tents. The remaining ten khalasies accompany the Assistant-Surveyor in charge of the advanced party and assist him in building the pillars and in laying out the meridian lines.
- (7) It is advisable in many districts to engage a mason permanently for the season to accompany the advanced party.
- (8) Sixteen country carts are required for the baggage of the main-body, and two for that of the advanced party.

CHAPTER II. DUTIES IN THE FIELD.

1. The advanced party should take the field ten days before the main body: the duties of the assistant in charge are:—(1) To select a site for the latitude pillar in the vicinity of the trigonometrical station; (2) To lay down the meridian by means of azimuth observations; (3) To build the latitude and collimator pillars.

The position for the latitude station should be chosen where a space 15 feet square can be levelled for the observatory tent, where the collimator pillar can be built without much expense, and where the pole star can be seen at any hour of the night clear above all trees and buildings: in selecting it the advantage of minimising the effect of local attraction should not be lost sight of. In order to avoid the necessity of having to connect the latitude and trigonometrical stations by triangulation, the latitude station should be built if not over the trigonometrical station itself at any rate in its prime vertical, the direction of which is determined and laid down by means of the azimuth observations. If the trigonometrical station itself is not in a suitable position, and some site has to be found on the prime vertical, the *nearest* favorable place should be chosen, as the greater the distance the more effect will an error in the azimuth determination have in placing the latitude station off the prime vertical. If the error in the azimuth is 1', the latitude of the trigonometrical station will differ from that of the latitude station, when 100 yards distant, by 0''·085, when 500 yards distant by 0''·43: an error in the azimuth of 2' will double these differences. In Indian latitudes the prime vertical of a station and its latitude parallel are at the end of the first mile not one foot apart.

2. The assistant, who is provided with a small theodolite, has to mark out on the ground the direction of the meridian of the latitude station: this is necessary to enable him to find the correct position for the collimator pillar, or if collimators are dispensed with, to serve as a guide to the observer in choosing a natural meridian mark: it is also of use in laying down the direction of the prime vertical, when this is required. In order to mark out the direction of the meridian, the assistant must determine the azimuth of a referring mark from the trigonometrical station: if one of the neighbouring trigonometrical stations can be made easily

visible, it will serve as the referring mark and its azimuth can be taken out from the G. T. S. Synoptical Volume of the Series.

If no trigonometrical station can be seen without ray-cutting, the azimuth of a lamp set up a mile off in any direction must be determined by astronomical observations. The most convenient method of doing this, when the time is not known is, to observe the angle between the referring mark and a star and the star's altitude simultaneously: the star should be near the prime vertical and about  $30^\circ$  above the horizon. This process should be gone through with two different stars, one east the other west, and the results from the two sets of observations should not differ by more than  $1'$ .

A simple method of checking the azimuth and one that should always be followed is to find the correct time by observing the altitudes of three or four stars and to then measure the horizontal angle between the referring mark and the pole star, a few minutes before and after the time that the latter is calculated to transit. The time may also be found by noting the time of transit of any known star near the zenith over the centre wire of the theodolite previously placed near the meridian.

The telescope of the theodolite should now, by means of the known azimuth of the referring mark, be pointed in the direction of the meridian, and a peg driven into the ground in the exact alignment about half a mile to the north of the station: as a single pointing of a telescope is affected by numerous sources of error, the angle between the peg and the referring mark should then be measured on one pair of zeros, and the position of the peg corrected in accordance with the result. The peg having eventually been placed exactly due north of the station, three more should be driven in to the east, south, and west by moving the telescope through arcs of  $90^\circ$ ,  $180^\circ$  and  $270^\circ$  respectively: the positions of these latter should then be carefully checked by measuring a round of angles on both faces.

It is of the utmost importance that the latitude pillar be erected at some point on the line joining the east and west pegs, and its position in the true alignment should be carefully checked from both ends. When the actual site of the latitude station has been decided on, the direction of its meridian should be marked out on the ground with pegs as before: this direction cuts the known prime vertical line at right angles and can be at once obtained with a theodolite.

3. The latitude pillar should be built of burnt bricks in mortar, attention being paid to the bond. The foundation should be 2 feet square and from 18 inches to 2 feet below the ground level: it should stand in a pit  $2\frac{1}{2}$  feet square faced with brick; the sand for filling the intervening space of 3 inches all round should be collected and stored *in situ* by the assistant, but not placed round the pillar until the observer has inspected the foundation. Above the ground level the pillar should be made circular in horizontal section and 2 feet in diameter: its height without the stone cap should be from  $2\frac{1}{4}$  feet to  $2\frac{3}{4}$  feet according to the height of the observer. As the stone cap is of special construction, one is carried about with the main body of the party and used successively, for every station.

4. A collimator pillar should be built on the meridian line some 15 or 20 feet north or south of the latitude pillar: its horizontal section should be rectangular in shape and 3 feet by  $2\frac{1}{2}$  feet in size, the shorter sides being parallel to the meridian line and equidistant from it, and its height should be greater than that of the brickwork (not including the stone cap of the latitude pillar) by 2 feet 7 inches. The foundations of the collimator pillar must depend on the nature of the soil and the weight of the pillar. A stone cap is unnecessary. A meridian mark consisting of a thin but well-defined line one foot long, must be drawn vertically down the centre of the side that faces the latitude station.

The collimator pillar is intended to carry two collimators, by means of which the deviation of the instrument from the meridian can be seen at any moment. The zenith sector however has no tendency to move in azimuth, and when once it has been adjusted to the meridian nothing short of a severe jar will disturb it. There is consequently no necessity to test the adjustment in azimuth by repeated references to collimators, and the possession of these instruments is rather a needless luxury. It is almost always feasible, by means of the zenith sector when first adjusted in azimuth, to find a substitute for them in some natural feature such as a distant tree or peak, which may be used as a meridian mark every evening at sunset, and as the actual deviation error of the instrument during work is determined from star observations nightly, and may be safely considered constant throughout a whole programme, the meridian mark need not be observed again.

As the greater number of latitude stations are near hill tops, the collimator pillars have often to be of considerable height, and a large saving of expense would be effected by their abolition: it might happen however that the latitude station was so shut in by trees that no distant meridian mark was obtainable, and in this case collimators would be indispensable. The decision therefore of whether to build a collimator pillar or not must rest with the assistant.

The natural meridian mark when employed should be as distant as possible, for there will be a parallax in the apparent position of any terrestrial point as observed with the telescope, since the line of collimation is 9·6 inches laterally distant from the meridian passing through the centre of the latitude pillar.

5. The permanent establishment of the Astronomical Parties is only maintained at a strength sufficient for electro-  
Preliminary work for the observer. telegraph longitude work, and is considerably below that required for latitude operations. At the commencement of a "latitude" field season all the kahars, bullocks and carts and a large number of khalasies have to be raised and engaged for six months. It is therefore essential to start operations from some large city on the railway. Whilst the new hands are being engaged, a latitude pillar should be built at any suitable spot and the zenith sector\* set up and overhauled. It would be useless to take a regular set of observations unless a principal station were in the vicinity, but the instrument should be thoroughly cleaned, tested and put into working order, before the party leaves for the field, as no artificer capable of executing the simplest repair is likely to be met with in the districts. The permanent adjustments also should be made, and the values of the instrumental constants determined before any regular observations are taken.

6. When the zenith sector has been brought into working order,  
Permanent adjustments, the following adjustments, which will be found to remain invariable throughout the season, should be made once for all:—

- (i) Stellar focus.
- (ii) Horizontality of the horizontal wire.
- (iii) Collimation in azimuth.
- (iv) The zero of the setting-circle.

\* For a description of this instrument see Volume XI of the "Account of the Operations of the Great Trigonometrical Survey of India."

7. This adjustment consists in placing the wires of the telescope in the principal focus of the object-glass, and its correctness can be tested as follows: obtain distinct vision of the wires by pushing in and drawing out the eye-piece from its cell, and direct the telescope upon some distant object; now move the eye first to one side and then to the other: if the image of the object retains a fixed position with regard to the wires, the latter are truly in the focus of the object-glass. If, however, on moving the eye to one side, the image of the object appears to move with the eye, then the principal focus of the telescope lies *beyond* the wires, and the object-glass and wires must be approximated. If on the other hand the image is between the eye and wires, it will move in the contrary direction to the eye.

Stellar focus.

The distance between the object-glass and wires can be altered by means of two antagonizing screws that work in the direction of the optical axis against a brass nut.

8. The horizontal wire can be made horizontal by means of two antagonizing screws that work against the same brass nut alluded to in the preceding paragraph, but in a direction at right angles to the screws mentioned there. By working against the nut they cause the eye-end to revolve in the telescope. To verify the horizontality of the horizontal wire, level the instrument and having set the telescope to a distant steady defined object near the horizon, or to a collimator, move it in azimuth so that the object may appear to move along the wire from one extremity of the field to the other. If the object does not remain intersected at all parts of the field, the error ought to be rectified by moving the wire-plate in the appropriate direction.

Horizontality of horizontal wire.

9. The adjustment for collimation in azimuth consists in making the line of collimation intersect the horizontal axis of rotation at right angles approximately; and can best be made by Gauss's method. Two small theodolites serve as collimators and are set up on their stands, one to the north and the other to the south, their crosses being superimposed: the wire-plate of the telescope is then moved laterally by means of two antagonistic screws placed behind the micrometer box, until the centre vertical wire intersects the

Collimation in azimuth.

crosses of both theodolites, when collimation error has been approximately eliminated.

This approximate adjustment of the line of collimation in azimuth when once made will remain undisturbed throughout the season: there will of course be a small remaining collimation error, which will probably vary within narrow limits from station to station; *this has to be determined nightly from star observations.*

It should be noted that the zenith sector in its working position obstructs the view of one theodolite through the other, and that it has to be revolved through  $45^\circ$  in azimuth and pointed some  $15^\circ$  from the zenith before this view can be obtained.

10. When the telescope is pointed in any direction, and the bubble of the setting-circle is at the centre of its run, The zero of the setting-circle. the vernier of the setting-circle should read zenith distances to within half a minute: unless it does this, there will be difficulties in identifying the correct star, when two or more are in the field together.

The unit of measurement into which the field of view of the telescope is divided is one division of the micrometer, which works the horizontal wire. The head of this micrometer is divided into 100 divisions, and a comb with 100 teeth runs vertically down one side of the field: one revolution of the micrometer-head causes the horizontal wire to pass exactly over one whole tooth, the breadth of which is accordingly equal to 100 divisions. The whole field of view is therefore divided up into 10,000 divisions. To prevent confusion in counting the teeth, a small round hole has been made in the comb at the 10th tooth from one end, two such holes at the 20th, three such at the 30th, and so on up to the other extremity of the comb, except that at the 50th tooth and after, one larger hole has been made to represent five of the small ones. These marks at every 10th tooth divide the field of view into sections of 1,000 divisions each, and as the mark that denotes the 50th tooth or 5,000 divisions is near the centre of the field, it has been assumed as *the zero of vertical measurements.* When the horizontal wire is exactly in the centre of this mark, and the micrometer-head is reading 0·0 division, the point of intersection of the horizontal wire and the centre vertical wire is in the line of collimation.

To make the setting-circle read correctly, the following steps should be gone through in the order here given :—

- (i) Place the horizontal wire by turning the micrometer-screw exactly in the centre of the 5,000 division mark on the comb, and then make the micrometer-head, which is only kept in position by friction, read 0·0 division by revolving it on its axis.
- (ii) Observe the zenith distance, as recorded on the limb of the setting-circle, of any distant terrestrial object in both telescopic positions,\* intersecting it by means of the tangent screw that belongs to the vernier of the setting-circle and not touching the micrometer-screw.
- (iii) Take out the mean of the two zenith distances, and set the vernier of the setting-circle to read it in either telescopic position ; bring the object again into the field.
- (iv) Now tap the telescope until the object is intersected by the horizontal wire, taking care not to touch the tangent-screw of the setting-circle.
- (v) Bring the bubble to the centre of its run by raising or lowering one end of the level itself by means of the capstan-headed screws provided for the purpose: the setting-circle will be then adjusted to read zenith distances.

II. Before leaving for the field, the values of the following instrumental constants should be determined :—

Determination of the instrumental constants.

- (i) The equatorial intervals of the vertical wires.
- (ii) The divisions of the head of the eye-piece micrometer.
- (iii) The divisions of the scales attached to the levels.

\* In a theodolite the two telescopic positions are distinguished by the position of the vertical circle and are known as "face-right" and "face-left": in a transit instrument they are denoted *I.P.E.* and *I.P.W.* according to the position of the illuminated pivot. In a zenith sector the telescope is at one end of the horizontal axis and the two positions have been named *T.E.* and *T.W.* according as to whether the telescope is at the east or west end of the horizontal axis.



If the determination of these constants is postponed till the middle or end of the season, the risk is run of some accident happening to the wires, levels, or micrometer which might render all previous observations useless.

12. There are seven vertical wires known respectively as I, II, III, IV, V, VI and VII and so numbered that in the position telescope east, a zenith star transits the wires in the order I to VII, whilst in the position telescope west, wire VII is the first wire reached and wire I the last. In both telescopic positions wire IV is the centre wire, from which the intervals of all the others are measured. Wire I is liable to be confused with wire VII, as they are symmetrically situated with regard to wire IV, only on different sides of it, so that in one telescopic position wire I is to the apparent left and wire VII to the apparent right, whilst in the other the contrary holds. Similarly wires II and VI may be mistaken for each other, as also wires III and V. When an observer is timing the transit of a star, and has to inform the recorder of the particular wire crossed, he may easily fall into error, for in naming the wire he has to first note the telescopic position of the instrument and to then recollect the order of the wires in that position. He is more likely to confuse the wires in a zenith sector than in a transit instrument, as the telescopic position of the former is changed at every star. He will avoid, however, all possibility of mistake if he recollects that wire I is situated *nearest* to the comb.

When the transit of a star is being timed, the telescopic position must be recorded, or otherwise the observation will be useless. Suppose an equatorial star transits wire V at  $7^{\text{h}} 31^{\text{m}} 11^{\text{s}}$  and that the equatorial interval of wire V is  $7^{\text{s}}$ : then if the instrumental position was "telescope east," the star would have crossed the line of collimation at  $7^{\text{h}} 31^{\text{m}} 4^{\text{s}}$ , whereas if the instrumental position had been "west" the correct time of transit would have been  $7^{\text{h}} 31^{\text{m}} 18^{\text{s}}$ .

The equatorial wire intervals are required to be known to the nearest tenth of a second of time. They can best be determined by noting the time of transit of slow-moving circumpolar stars over each of the vertical wires. The cosecant of the declination of the star, multiplied by the observed interval between the times of transit over any wire and the centre, is the equatorial interval of that wire.

If the star observed is very near the pole, the radius of curvature of its path will be so small, that it may not travel at right angles to all the vertical wires: a correction for curvature will then have to be applied to the wire intervals, *vide* Chauvenet's Astronomy, Volume 2, Section 131, page 147, and page 111 of this Handbook.

13. The value of a division of the eye-piece micrometer, though it never practically varies, had best be determined at the commencement of every field season.

One division of the eye-piece micrometer.

To enable the limb to be read by the microscopes, when the telescope is horizontal, the sectors should be unscrewed and revolved on their axis through  $90^\circ$ : their central diameter will then be parallel to the telescope instead of perpendicular, as it is in ordinary work. Now intersect the cross of a collimator with the horizontal wire of the telescope: clamp the telescope, and take the readings of the four microscopes and of the eye-piece micrometer. Move the telescope in altitude some 5 or 10 minutes by means of the tangent-screw, and again read the four microscopes. Now by working the micrometer intersect the collimator cross again with the horizontal wire and take the new reading of the micrometer-head. The angle in seconds of arc that the telescope was moved can be ascertained by comparing the two mean readings of the limb by the four microscopes, whilst the difference between the two readings of the micrometer gives it in micrometric divisions. The value of the angle is now therefore known both in seconds of arc and in micrometric divisions, and by equating the two measures, the relative proportion of the units can be found. Before the value of a division of the micrometer can be satisfactorily accepted as final, the above process should be gone through several times, as many different parts of the thread of the micrometer screw and as many different graduations of the limb being brought into play as possible.

14. In all there are three levels employed with the zenith sector: two, mounted on the cradle, are to shew the inclination of the vertical axis in the meridian and should, if accurately made and mounted, always give identical results. The third is a striding level, and is used to find the dislevelment of the transit axis. In addition to these three it is advisable, as levels are easily broken, to subject two or three spare ones to the tests given hereafter, and to determine the values of their scales before leaving for the field.

The values of the level scales.

The values of the divisions of the level scales are found in practice to vary both with temperature and age, and should consequently be determined annually. The most accurate and at the same time most simple method of determination is by means of the bubble tester, an instrument that can be had temporarily on loan either from the Mathematical Instrument Office, Calcutta, or the Dehra Office.

If no bubble tester can be obtained, the values of the divisions can easily be determined as follows:—The levels should be attached to the telescope with their axes parallel to the line of collimation: the telescope should be brought to a horizontal position, a collimator cross intersected with the horizontal wire, and the readings of both ends of each bubble and of the micrometer recorded. The telescope should now be moved in altitude by a slight tap, and the readings of the levels again taken: the collimator cross should then be intersected by means of the micrometer with the horizontal wire, and the new reading of the micrometer recorded. The number of divisions that each bubble has moved is equal to the difference between the two readings of the micrometer, and the value of a division of each level scale is therefore obtainable in terms of micrometric divisions, and thence in seconds of arc. The number of divisions that a bubble moves, when the telescope is tapped, is

$$\frac{N_1 + S_1 - N_2 - S_2}{2},$$

where

$N_1$  = first reading of northern extremity,

$N_2$  = second „ „ „

$S_1$  = first „ southern „

$S_2$  = second „ „ „

$\frac{N_1 + S_1}{2}$  being the position of the centre of the bubble before it is disturbed, and  $\frac{N_2 + S_2}{2}$  the position of the centre after the disturbance when it has again come to rest.

There is another method of determining the values of the level scales, which, owing perhaps to some difficulty in attaching the levels to the telescope as directed above, has been occasionally followed. The levels

are mounted in their proper places on the instrument, and the readings of their bubbles taken, the position of the sectors at the moment is found from the four microscopes. The verticality of the vertical axis is now altered by means of the thumb-screws at its foot: the change in its inclination is then found in divisions from the levels, and in seconds of arc from the microscopes, and the value of one division deduced. This method is cumbersome and inaccurate, and is not recommended.

15. The zenith sector has been made with such delicacy that it is very difficult to obtain levels at all worthy of being employed with it, and no *new* level should ever be brought into use without being thoroughly *tested* first. In testing the fitness of a level the following points should be examined by means of the bubble tester:—

Testing a new level.

- (i) If the inclination of the level be altered either *gradually* or *suddenly* and then brought back to what it was originally, the readings of the bubble before and after the disturbance should be identical.
- (ii) A bubble should rapidly come to rest, after motion however violent has been imparted to it.
- (iii) The uniformity of the curvature of the tube should be such, that no two individual divisions of its scale should differ by  $0'' \cdot 1$ .

A great number of levels will be found, whose position at rest depends to a certain extent on the direction of motion last imparted to the bubble: they apparently overshoot their mark and have not the power of returning. They should never be employed with the zenith sector.

Many also will be found which after disturbance take a long time such as five minutes to settle, and there are a few to be met with that will not come to rest under twenty minutes. The accuracy of work with the zenith sector depends on the ability of the observer to measure and read several quantities *without* change of conditions, and unless a bubble settles in half a minute the level should be discarded. This point should be carefully decided on, as in many levels the bubble will go on moving a long time after it has apparently come to rest: its motion is so slow as to be invisible, but will be found to be real if its position be noted at the end of each minute. The values of the division scales of new and

untried levels should be determined both by long sweeps extending from one end of the scale to the other, and by short ones of two or three divisions. Short sweeps up and down the whole length of the scale in both directions are most essential, as the value of every individual division should be determined.

It is useless to adopt the *mean* value of a division of the whole scale, unless the values of all the individual divisions agree with it very closely, as the ordinary amount of dislevelment to be corrected for in the zenith sector is under two divisions. In a really good level\* a change of inclination of 1" should make the bubble move lineally through  $\frac{1}{20}$ th of an inch *at least* and the value of no individual division† should differ from the mean by more than  $0''\cdot05$ .

In all levels the extreme 10 or 15 divisions at the two ends will be found to fail, although the centre portion may give excellent results: in determining therefore the value of the divisions, the two limits of reliable curvature on either side of the centre should be found and noted: in subsequent work no dislevelment should be accepted, if either end of the bubble has overstepped these limits.

The values of the divisions should be determined to  $0''\cdot001$  at different temperatures and on several different days: at times a slow and gradual motion should be imparted, at others a sudden and turbulent one. The zero also of the micrometer of the bubble tester should be frequently changed by placing coins under the point of the screw, or the same portion of the thread of the screw will always be in play.

16. When the divisions of the level scales have been accurately determined, the levels that shew the inclination of the vertical axis should be placed in their working position on the instrument, where they should be allowed to remain on the march. In placing them on the instrument, the following points should be attended to:—

Precautions necessary in mounting the vertical axis levels on the cradle of the sector.

- (i) A strip of the interior surface of the glass tube  $\frac{1}{4}$  inch broad is ground to something like curvature from end to end: no

\* If a change of inclination of  $1''\cdot00$  makes the bubble move through  $\frac{1}{20}$ th of an inch, the curvature of the tube has been struck with a radius of 550 yards.

† The value of an individual division must not be taken from a single determination.

attempt is made to give curvature to the remaining portion of the interior surface, which is therefore no more suited for use as a level than an ordinary bottle. In only a single line of the ground strip running from end to end do the best makers pretend that the curvature is uniform: it is consequently essential that in mounting the level this particular line should be placed uppermost and in the same *vertical* plane as the axis of the level.

On most good levels there are two small opaque dots, one at each end: when the level is in its correct position, the longitudinal diameter of its bubble should coincide with the line joining the two dots, a condition that can be tested by running the bubble up to the two dots in succession.

In some levels instead of opaque dots, there are two broad arrows pointing inwards towards one another, about  $\frac{1}{4}$  of an inch apart, near the middle of the tube, the bubble should then be made to float exactly between them. In other levels the tube has graduations cut on the glass along its whole length, in which case the centre of the bubble must be made to run down the line joining the centre of the graduation-cuts. The absence of opaque dots generally denotes an inferior level.

- (ii) The glass tube of the level should be rigidly attached to the brass caps by wedging pieces of cork between the two, a very difficult operation. The pieces of cork that are fixed in the brass caps must first be scooped out on one side so as to fit the twisted extremities of the glass tube: one piece must then be placed in the centre of a brass cap, and one end of the level inserted in it: the glass cylinder must then be brought down round the level, until it takes up its proper position in the brass cap. Lastly, the second brass cap must be fitted on to the other end of the cylinder, the piece of cork at its centre receiving the free extremity of the level. If the pieces of cork employed are very thin, they will not support the level at all: if they are very thick, the risk is run of compressing the tube too forcibly and smashing it: if they are sufficiently thick to support the

tube without breaking it, there is still a fear that the different portions of the level may not be quite rigidly connected, and that the tube may have slight longitudinal motion open to it in its brass case. The interposition of the glass cylinder prevents the tube being felt with the hand, and the rigidity of the connection cannot therefore be tested by touch. If the tube has play in its case, no level reading is reliable: in regular work the rigid attachment of the two is subjected to a severe test by the constant rapid rotation of the instrument in azimuth, and it has therefore to be carried out with great thoroughness.

The level should occasionally be taken to pieces and the connection of its parts examined, as in hot weather the pieces of cork are apt to shrink, and to lose their elasticity.

- (iii) The brass case of the level must be rigidly attached to the  $V_s$ : this is done by means of springs, which are screwed down to the  $V_s$  and exert pressure on the pivots of the case. If this attachment be not rigid, the level will change its position in its  $V_s$  during the revolution of the instrument in azimuth.
- (iv) The two levels when mounted on the cradle are parallel to one another, but are situated at unequal distances from the vertical axis, on which the instrument rotates in azimuth. As the rapidity with which they move varies with the square of their distance from the axis, a more violent motion is imparted during rotation to the distant one than to the near. The *more* sensitive level of the two should therefore be always mounted in the place of least disturbance.

17. Before leaving for the field the sidereal chronometer to be used during the latitude observations should be tested for uniformity of rate, and the index errors of the barometer and thermometers should be determined against the departmental standard. For actual work one mercurial barometer and two thermometers are required, but spare ones whose index errors are known should also be taken. The determination of the index errors must not be postponed till after the field season, as the instruments may get broken during work.

Chronometer, barometer and thermometers.

When the necessary preliminary operations that have been described have been carried out, the party should take the field under the observer and commence regular observations for latitude.

18. On arrival at the latitude station set up a small theodolite on the pillar and measure the angle between the meridian mark and the trigonometrical station : if it prove equal to  $90^\circ$  and if the meridian mark is afterwards found from observations with the zenith sector itself to be truly on the meridian of the latitude station, the latter has been correctly placed on the prime vertical of the trigonometrical station.

Examine the *material*, the bond and the isolation of the pillar : fill in the 3-inch space that has been left round the foundation with the sand collected for the purpose by the assistant. Fix the stone-cap to the pillar with cement, taking care that it is in its correct position\* with regard to the meridian and that its upper surface is approximately level. The correct position for the stone-cap can be found by stretching a piece of string across the pillar in the direction of the meridian line, and by revolving the cap on the pillar until the line (that has been cut across its upper surface for the purpose of the adjustment) coincides with the string.

Pitch the observatory tent over the pillar, leaving the side towards the sun open until the new-laid cement is dry. Suspend the two thermometers to tent-ropes outside the observatory, one on the north side, the other on the south about 4 feet from the ground and out of the range of possible contact with other bodies. Hang the barometer inside the observatory to the frame-work of the tent with its cistern about 18 inches from the ground.

Make two pencil marks on the collimator pillar 9.6† inches on either side of the meridian line cut on the brick-work by the assistant : set up the collimators on the pillar, plumb the centres of their object glasses accurately over the pencil marks and place their optical axes by eye parallel to the meridian line. If the chronometer has stopped, or its travelling rate is unreliable, observe altitudes of the sun for time with a small theodolite.

\* Holes have been bored for the foot-bolts of the sector in the stone-cap and the latter has therefore to be so placed in the horizontal plane that the holes will be vertically under the feet of the instrument when adjusted to the meridian.

† The line of collimation of the telescope is 9.6 inches to the east of the vertical axis with "telescope east" and the same amount west with "telescope west."



When the cement attaching the stone-cap to the pillar is dry, set up the zenith sector.

19. Place the cast-iron pillar on the stone-cap with the azimuthal stud to the north, and see that all the three feet are resting firmly on it: place the hemispherical steel button under the base-plate screw, and test the rotation of the instrument by revolving the pillar several times on it. Bring the pillar to rest with the azimuthal stud to the south, and again see that the three feet are resting firmly on the stone-cap in this new position. It is advisable to commence work at a station with the azimuthal stud south, as the original adjustment in azimuth is facilitated by the observer being able to work the stud screw and intersect a star with the telescope simultaneously.

Loosen the base-plate screw: bring the screw that works in the azimuthal stud, and the four levelling thumb screws to the centres of their threads. Put in and *loosely* screw up the three foot-bolts that hold the pillar to the stone: if they all three go in easily without touching the sides of the holes in the stone, the instrument is roughly adjusted to the meridian. Find the dislevelment of the cast-iron pillar by means of an ordinary spirit-level, and make the pillar approximately level by placing tin washers of various thickness under the feet.

Thoroughly clean the cup that receives the point of the vertical axis, and the three hemispheres that are fixed in the neck of the pillar. Fill the cup with oil, and oil the hemispheres. (The hemispheres can be revolved by means of brass-heads, situated on the outside of the pillar near the top: these heads should be so screwed on, that the diameters cut across their surfaces are parallel to the grooves of the hemispheres: when the hemispheres are exerting pressure on the vertical axis, the grooves should be vertical, but as they are out of sight, their position can only be told by means of the diameters on the heads). Having screwed on the heads correctly, turn the hemispheres till their grooves are vertical.

Clean the vertical axis and oil it near its junction with the table: insert the axis into the cast-iron pillar, taking care that the flying stop is properly situated between the two fixed stops. Let down the dust-cap over the point of the axis: test the rotation of the axis in the cast-iron

pillar by giving it a semi-revolution in azimuth, making the flying stop come into contact with the two fixed stops successively: glance at the brass heads, that turn the hemispheres, and see by the position of the diametrical cuts, whether the verticality of the grooves was disturbed by the insertion of the axis.

Revolve the whole pillar, which has been as yet only loosely attached to the stone-caps with bolts, in azimuth, until the centre line through the two fixed Vs of the table passes 3·2 inches to the east or west\* of the meridian mark: by this means the adjustment of the instrument to the meridian can be somewhat improved. Place a spirit-level on the table and re-level the pillar approximately by means of washers under the feet. Tighten the three foot-bolts.

Put on the cradle; in raising the cradle *be careful not to lift by the microscopes*, which are hollow; when the cradle is in position, secure it *at once* to the table with the large bolt. See that the back studs of the cradle rest on their bearings: if one is resting and the other not, the box containing the spindle and the two bearing shafts must be opened, the endless screw must be put out of gear and the lower shaft raised to the same level as the higher by hand: to do this the cradle has to be removed. The dislevelment of the vertical axis is now shewn by the two vertical-axis levels: by means of the four thumb-screws make the axis perfectly vertical both in the meridian and the prime vertical: clamp the base-plate screw.

Put on the cylindrical brass counterpoise at the back of the table, and place ready to hand the two other counterpoises. Raise the telescope and sectors out of the box: *do not touch* the sectors in lifting, and remember too that the two wooden-tipped handles of the sectors are hollow and weak. So place the telescope that the horizontal axis points upwards and the brass cap, which shuts the window opposite the reflector in the telescope, rests on the truncated pyramid of the lid of the box. Remove the four capstan-headed screws situated round the telescope, and unclamp the sector-handles.

Consider whether  $360^\circ$  or  $180^\circ$  is to be under the index microscope; this is purely a matter of choice for the observer. Whilst two men hold the telescope steady, turn the sectors through  $90^\circ$ : on the direction, in which they are rotated depends, whether  $180^\circ$  or  $360^\circ$  comes under

\* East or west, according to which of the fixed stops is in contact with the flying stop.

the index. When the sectors are in position, screw in again the four capstan-headed screws.

Screw on the clamping arm, taking care that it points towards the eye-end of the telescope. Tie up the spring of the clamping arm tight with string, and loosen the clamping cam and the tangent-screw of the arm attached to the cradle. Examine the friction rollers, situated at the junction of the telescope and the horizontal axis. Clean the horizontal axis; oil its collars and its steel bearings in the front wall and at the back of the cradle. Affix the wooden guides and conical cap to the horizontal axis; and clamp the handles of the sectors at right angles to the telescope. The telescope and sectors are now ready to be lifted.

Before lifting, see how the telescope should be moved so as to make the two clamping arms on the sectors and cradle face each other. So arrange the revolving iron guides that the two large relieving wheels will be *under* the telescope, when the latter is in position. Make three men raise the telescope, sectors and axis vertically, and when clear of the box turn the axis horizontal, keeping the eye-end micrometer *up*. Place the axis so far into its hole in the cradle, that the iron guides are just free, but on no account allow the axis to *rest* on the cradle. Direct the iron guides through their holes in the front wall of the cradle, making certain that the relieving wheels are *under* the telescope. Run the axis home through the cradle, and see that the two clamping arms meet and fit into each other. Two men must still support the telescope, or the whole instrument will over-balance.

Take off the wooden guides and conical cap; fix on the mushroom counterpoise: connect the iron guides with the brass yoke, and into the latter screw the black counterpoise. Screw on the dew-cap: put on the bridge-piece with screws towards the counterpoises: loosen the screws of the bridge-piece, to enable its springs to act. If the bridge-piece does not go on easily, turn the telescope slowly in altitude, while putting it on: if it does not go on then, the horizontal axis is not properly home. Screw on the oblique eye-piece and the brass handle for rotating the instrument in azimuth.

Until the bridge-piece is fixed, two men should support the telescope. When the bridge-piece is in position, the observer may direct the men to let go their hold.

20. When the sector has been put up, and been left for three or four hours to settle, the following adjustments should be made in the order given :—

To adjust the sector for work.

- (1) Adjustment of the lamps.
- (2) The telescope micrometer.
- (3) The four microscopes.
- (4) Elimination of the zero of the sectors.
- (5) Verticality of the vertical axis in the meridian.
- (6) Horizontality of the horizontal axis.
- (7) Adjustment of the sector in azimuth.

21. Two lamps are provided for the general illumination of the instrument, each of which has one central and four surrounding lenses. They are set up east and west of the instrument on firm adjustable supports, and afford a perfect and constant illumination if properly placed. The centre lens of one lamp illuminates the reflector at the window and thence the interior of the telescope, while *at the same time* the four surrounding lenses of the other lamp illuminate the portions of the limb under the microscopes, the light of the latter being focused by the lenses attached to the microscopes: when the instrument is revolved in azimuth, the lamps simply interchange duties.

Adjustment of the lamps.

*Perfect* illumination though difficult to attain is all important; and though hours may be spent in adjusting the lamps the trouble is amply repaid afterwards.

The first step necessary is to bring the reflecting plate of the telescope to its mean position by means of the illuminating rod: if this is not done, the illumination of the wires may be obtained with the reflector in one or the other of its extreme positions, and the observer will not thus have the power of regulating the amount of light in the telescope at will.

Open the lamps and see that the heart of the flame in each is at the point where the principal axes of the five lenses meet: the flame

may be too high, too low, or too far away from the lenses. It can be lowered by turning down the wick and raised by placing washers underneath the vessel containing the oil: if, when advanced as far as it will go, it is too distant from the lenses, either the vessel or the outer case has become distorted by wear or heat, and they must be made to fit each other by bending and filing.

Set up the wooden supports due east and west of the pillar, and place the lamps on them: the flames should be five feet from the vertical axis and the same height as the horizontal axis. Turn the central lens of each lamp towards the instrument, and cover up the eight surrounding lenses: then with the light from the central lenses only, obtain good illumination of the field in both telescopic positions: this can only be done by revolving the lamps slightly in azimuth backwards and forwards, by placing wedges under the back and front edges of the lamp and thus altering its inclination to the vertical, and by moving the whole lamp laterally parallel to itself.

When good illumination of the field has been obtained, the line joining the heart of the flame and the centre of the lens passes through the centre of the telescopic reflector; the distance of the lamp from the telescope may, however, still be wrong, and if it is so, the surrounding lenses will fail to answer their purpose.

When the results by the central lenses are satisfactory, uncover the surrounding lenses, and try whether the portions of the limb under the microscopes can be illuminated by means of the lenses *on* the microscopes and without touching the lamps. If the illumination of the limb cannot be obtained by means of the microscope lenses, try the effect of moving the lamp towards the instrument or away from it. This movement will probably disturb the adjustment of the central lens, which will have to be made again.

When both central lenses are illuminating the field and the surrounding lenses of one lamp also have been got to give good results, those of the other lamp have to be adjusted: the difficulties of doing this are now enhanced through the lenses of the microscopes having been placed to suit the former lamp and being therefore no longer available for the adjustment. As the lenses of the microscopes in one fixed position have to illuminate the limb with both lamps, they should never be placed in any abnormal position: it will often be seen that

by inclining these lenses at extraordinary angles, illumination of the limb can be attained, but under these circumstances the position of the lamp is really at fault, and on revolving the instrument the lenses will be found to act no longer. When the lamps have been once adjusted, mark on the ground the exact position of the supports, and on the supports the exact position of the lamps, so as to have no difficulty in replacing them at any time.

22. With the oblique eye-piece get the *horizontal\** wire into focus: examine Polaris for parallax, and whilst it is in the field, make use of it to test the horizontality of the intersecting wire. Revolve the micrometer-head on its axis, until it reads 0·0 division, when the horizontal wire is exactly midway between two teeth.

Adjustment of the telescope micrometer.

23. Loosen all the screws that fasten the microscopes to the arms of the cradle, and see that the microscopes can be revolved on their axes quite freely and moved longitudinally without any strain being set up. Get good focus of the wires by moving the eye-pieces in their cells, and then get distinct vision of the limb by moving the whole microscope either towards it or away from it: see that there is no parallax. By revolving the microscopes, make the wires accurately parallel to the longest divisions of the limb. Good illumination of the limb is all important: to have one side of a division light and the other dark is fatal.

Adjustment of the four microscopes.

By means of the small screw at the end of the micrometer box, bring the zero of the comb of A microscope as near the centre of the field as possible, and then with the tangent-screw of the sectors make a division of the limb exactly bisect that zero. Now intersect the division with the wires by turning the micrometer screw, when the head of the micrometer should read 0·0 division; if it does not do so correct it by revolving the graduated head on its axis. When A microscope has been thus adjusted, intersect the corresponding divisions of the limb with B, C, D microscopes, and by revolving their graduated heads on their axes, make them all in the position of intersection read 10·0 division. To guard against possible mistakes in the number of the minutes, the B, C, D microscopes should always read 10 divisions

\* Focus is not the same for the horizontal and vertical wires.

more than A, but their combs need not be touched, as they are never referred to. The graduated heads of microscopes are held in position by friction only, and can easily be rotated, if they are first drawn a slight distance out from the micrometer box: it will, however, be found that this process often disturbs the intersection of the limb, which has consequently to be made again.

It is a peculiar feature of the zenith sector microscopes, that they cannot be adjusted for "run." The divisions of their heads therefore cannot be made to equal seconds of arc, when referred to the graduations of the limb. The mean value of the error of the "run" is about  $0''\cdot264$  too small in five minutes, so that  $299\cdot736$  micrometer divisions are equal to 300 seconds of arc: at different stations the alteration of the focussing adjustment may decrease the error of "run" to  $0''\cdot100$  or increase it to  $0''\cdot400$ ; variations of temperature are believed too to cause changes in its value.

The "run" of the microscopes has to be determined some 15 or 16 times a night, (but never twice over the same 5' space) as follows:—Two consecutive divisions of the limb are intersected successively with each microscope, and the eight readings recorded: the mean of the four "low" readings is then deducted from the mean of the four "high" readings, the resulting difference being the value of the run correction. If the divisions of the micrometer heads were equal to seconds of arc, the mean of the "low" readings would equal that of the "high."

Owing to differences in the make of the four microscopes, an exceptional amount of care is required to avoid confusing in one or the other of them the "high" reading with the "low": the "high" reading can only be distinguished from the "low" by observing the direction in which the graduations of the head are moving: if the micrometer is being so turned, that its reading is constantly *increasing*, (that is to say the graduations of the head are passing the index in the order 0, 10, 20, 30, 40, 50 and 60), the wires *are approaching* the "high" division of the limb: if on the other hand the revolution of the micrometer is causing its readings to decrease and to pass the index in the order 60, 50, 40 . . . . ., the wires are approaching the "low" division of the limb. It is a matter of indifference whether the "low" or "high" reading is taken first.

In taking a "run" over any 5' space, say from  $358^{\circ} 10'$  to  $358^{\circ} 15'$ , one is apt at first to imagine that the reading of the 15' division, which is the *higher* division on the limb, must be the "high" reading: this is quite wrong, as the terms "high" and "low" when used in reference to the run have nothing whatever to do with the limb. One is liable too, if in a hurry, to assume that the "high" reading must be approached, if the micrometer head is turned with the "screwing-up" motion, this is also wrong, as in two of the microscopes, when the "screwing-up" motion is employed, the wires are receding from the "high" reading and approaching the "low." A confusion of the "high" and "low" readings can only be avoided by a strict observance of the following rule:—*Never fail, when a run is being taken, to watch whether the reading of each micrometer head is increasing or decreasing.*

An observer when reading the four microscopes must take care *what* graduation of the limb he intersects, or he will not afterwards be able to discover from the record the sign to be given to the run correction. Suppose the A microscope reads  $176^{\circ} 32' 30''$  approximately: the zero of the microscope will be situated nearly midway between the  $30'$  and  $35'$  graduations of the limb: if the  $30'$  graduation be intersected, the micrometer reading will be seen to increase as the screw is turned, and the "run" correction will be consequently +; if the  $35'$  graduation be taken, the micrometer reading will decrease as the screw is turned, and the correction will be -. The two consecutive graduations will not read the same, but will differ by the amount of the "run" correction for a 5' space. If the value of the "run" correction were  $0'' \cdot 264$ , the micrometer on intersecting the  $30'$  and  $35'$  graduations would read respectively  $29 \cdot 87$  and  $30 \cdot 13$  divisions; the correction for  $2' 30''$  is half that for 5' and is therefore  $\cdot 13$ : in the first case the correction would be +, and the reading  $29 \cdot 87 + \cdot 13 = 30'' \cdot 00$ : in the second case the correction would be -, and the reading  $30 \cdot 13 - \cdot 13 = 30'' \cdot 00$ . If the observer will adhere to the three following rules, no error can creep into the sign of a run correction:—

- (i) When the zero of A microscope is *conspicuously nearer* to one division of the limb than to any other, intersect that division.
- (ii) If there is the slightest doubt as to which division of the limb is the nearer, intersect either, and tell the recorder to note which: the recorder will then write in the margin



“referred to 30’” or “referred to 35’” as the case may be, meaning that the 30’ or 35’ division has been intersected.

- (iii) Invariably intersect that division of the limb with the B, C, D microscopes, which corresponds with the one intersected with the A microscope.

The sign of the run correction is +, if the division intersected is less than the reading of A microscope, and – if greater. When the A microscope reads any thing between 0’ and 2’, the 0’ division being conspicuously nearer will be intersected, and the correction will be + : if it reads between 5’ and 7’, the 5’ will be intersected and the correction be + again : if it reads between 3’ and 5’ or between 8’ and 10’, the 5’ and 10’ being the nearer will be intersected and the correction be –. If it reads between 2’ and 3’ or between 7’ and 8’, the division intersected should be noted.

An officer, who is to be entrusted with the conduct of latitude operations, need not be told that on every possible occasion, when a division of the limb is being intersected with the wires of a microscope, the final motion given to the micrometer head must be in the “screwing-up” direction ; if a division is being approached for the purpose of intersection with an “unscrewing” motion, whether it is to give an ordinary reading or a “high” or “low” one for run, the wires must be made to clearly pass the division and then brought back to it for intersection. In the zenith sector more discrimination is required than in other instruments to recognize instinctively, which is the “screwing-up” motion and which not, because two of the microscopes A and D possess a left-handed screw, and the other two B and C a right-handed one.

One more caution is necessary to anyone working with these microscopes, and that is to take care that no mistake arises in *reading* off the graduations from the heads, after the intersections have been made : the graduations in two microscopes increase from left to right, and in the other two from right to left : it may happen and often does, that one number only, say 40, can be seen by the observer, those on either side, *viz.*, 30 and 50 being invisible, unless he changes his position for the purpose of noting them. There is generally no time to spare, and the observer simply sees the number 40.

Suppose the index is pointing to the second division to the *left* of 40: in two microscopes this would mean that the reading was 42, and in the other two 38.

It will be seen from what has been said above, that the manipulation of the microscopes, without hesitation or flurry, requires considerable practice, and practice alone will enable a new observer to master their idiosyncracies: the following maxims however may at first be found useful:—

- (i) In all the four microscopes the “screwing-up” motion is *inwards*; consequently when a division is to be intersected the final motion given to *all* the micrometer heads must be *towards* the mushroom counterpoise.
- (ii) A’s and D’s micrometers read from right to left, B’s and C’s from left to right.
- (iii) A and B read *downwards* on the limb, C and D *upwards*; that is to say, if a 30’ division of the limb is visible in all the microscopes, the division apparently\* below it in A and B will be the 35’ division, and in C and D the 25’.
- (iv) In taking runs the “high” reading with A and B is the apparent\* *upper* division on the limb, with C and D the apparent *lower*.

In reading the four microscopes it will be found advisable to take them in the order A, B, D, C, for whilst intersecting with C microscope, one generally breathes on the eye-piece of D which prevents it being used for some minutes: if D is read before C, no such difficulty arises.

24. Test the adjustment of the setting-circle to read zenith distances

Elimination of the zero error of the sectors.

by observing to some terrestrial object on both faces: if there is any facial difference, eliminate it by altering the bubble. When the setting-circle is in good adjustment, set it to read accurately  $0^{\circ} 0'$ : move the sectors in altitude until the index microscope reads  $180^{\circ}$  or  $360^{\circ}$ , and the A microscope  $0'$ , and clamp. If now the bubble of the level of the setting-circle is at the centre of its run, the sectors have been placed in their correct position with regard to the telescope, and no zero error exists: if the bubble is not at the centre of its run, it must be brought there by changing the relative

\* The word “apparent” is used to denote the direction of the limb graduations as they appear through the inverting microscopes.

positions of the sectors and telescope. To do this loosen the four large capstan-headed screws attaching the telescope to the sectors, and tap the telescope without moving the sectors, until the bubble comes to the centre. It will be found very difficult to move the telescope without moving the sectors too: after every tap therefore the reading of A microscope should be taken, and brought back to  $o'$  if disturbed. Continue tapping the telescope and re-adjusting the reading of A microscope, until at one and the same time the setting-circle bubble is at the centre of its run, the setting-circle vernier is pointing to  $o^\circ o'$ , the index microscope to  $180^\circ$  or  $360^\circ$ , and the A microscope is reading  $o'$ : when these four conditions all hold simultaneously, screw in and tighten the four large screws.\* To guard against mistakes, test the zero error by observing a star on both faces. A zero error of  $60''$  or under may be accepted: a larger might entail several stars being lost. When the zero error of the sectors has been eliminated, examine the microscope adjustments, which will probably have been slightly disturbed.

25. Examine the two vertical-axis levels, and see that they are properly mounted on the cradle in accordance with the instructions given before. Take the readings of one of the levels in the instrumental position telescope east: reverse the instrument and take them again with telescope west. If the vertical axis is vertical in the meridian, the reading of the north end in the first position will be the same as that of the south end in the second. If the reading of the north end in one position differs by more than two divisions from the reading of the south end in the other position, the axis is not sufficiently vertical: its verticality can be improved by unclamping the base-plate screw with its spanner, and then moving the lower end of the axis in the meridian by means of the north and south thumb-screws.

This adjustment will often get disturbed and have to be re-made during actual work, when there is but little time to spare for the operation: in these cases always *remember to clamp the base-plate screw*, after the verticality of the axis has been secured: in the hurry of the moment *it is likely to be forgotten*, and if the base-plate screw is not clamped, the inclination of the axis will be always varying.

\* These four screws should be put in loosely, and gradually screwed home together, one or two turns being given to each in rotation. In dealing with delicate instruments this rule is universal. If the first screw be screwed home before the second is inserted, and then the second made tight before the insertion of the third, and so on, torsion may possibly result.

26. The readings of the transit axis level depend on:—(i) The position of zero of the level\* itself. (ii) The inclination of the vertical axis in the prime vertical. (iii) The inclination of the horizontal axis to the vertical axis. The first step necessary when levelling the horizontal axis is to bring the zero of the level to the centre of its run: if this is not done, the bubble, which is a very long one, will generally be found to settle too much to one side to be read, and on reversal too much to the other, rendering it impossible to decide what the dislevelment really is.

It is *useless* to attempt to level the horizontal axis, until the vertical axis has been made vertical in the prime vertical. Suppose in the position telescope east, the vertical axis has an inclination  $\theta$  to the east: if the horizontal axis is now made level by lowering the bearing shafts in the back of the cradle, the two axes will be inclined to each other at an angle of  $(90^\circ \pm \theta)$ . The result of thus making errors compensate instead of eliminating them will be, that on changing the instrumental position to telescope west, the horizontal axis instead of remaining level will assume an inclination of  $2\theta$ .

To make the vertical axis vertical in the prime vertical, first make it so in the meridian, and then note what the *right* end of one of the vertical-axis levels reads, when the telescope is in the meridian: rotate the instrument through  $90^\circ$  in azimuth, and then by means of the east and west thumb-screws, the base-plate screw being unclamped, make the right end of the same level read as it did before. The two fixed stops limit the motion of the telescope in azimuth to  $180^\circ$ , and it cannot therefore be brought both to the north and south of the instrument: the levels consequently can only be read in one position in the prime vertical and cannot be reversed end for end, as is practically done when the instrumental position is changed. When the reading of the end of a level remains the same throughout the rotation of the telescope in azimuth from its eastern position to its western, the vertical axis is vertical both in the meridian and the prime vertical, and the base-plate screw may be clamped.

The striding level can now be applied to the horizontal axis, and the dislevelment of the latter corrected by means of the capstan-headed

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\* The zero of the level is the reading of the centre of the bubble in the position of no dislevelment, *i.e.*, when the level is applied to a truly horizontal surface.

spindle, which raises and lowers the two bearing shafts: the spindle should not be turned, until the large screw, that attaches the cradle to the table, has been loosened. On the completion of the adjustment this screw should be tightened at once, or the safety of the instrument may be endangered.

From what has been said above, it will be seen that the dislevelment of the transit axis, arising as it does from two independent sources, will not be the same in the two instrumental positions, and two distinct values must always be deduced for it. In determining their amounts apply the level to the horizontal axis in the position telescope east, and take its readings: reverse the level end for end and take its readings again. Now change the instrument to telescope west and read the level a third time: reverse it end for end and read it a fourth time.

27. The "flying stop," affixed underneath the table, arrests the rotation of the instrument in azimuth in both directions by coming in contact with one or the other of the fixed stops. If the instrument be adjusted to the meridian when the flying stop is in contact with one of the fixed stops, there may still be a large deviation error when the instrument is rotated, and the flying stop brought into contact with the other fixed stop. The two directions of the line of collimation in the two telescopic positions are not necessarily parallel, but have to be made so. An adjustment to the meridian is therefore necessary in *both* telescopic positions.

One of the faces of the flying stop is a vertical plane, and the other carries a horizontal screw. In one\* telescopic position the plane face is in contact with a fixed stop and in the other the screw face. When adjusting the instrument in azimuth, it is always necessary *to begin* with that telescopic position, in which the *plane face* of the flying stop is in contact with a fixed stop: the reason is, that when the plane face is in contact, the only method of correcting the deviation error is to rotate the whole instrument by means of the azimuthal stud screw, whilst when the screw face is in contact, the deviation of the telescope can be altered by making the screw protrude more or less from the stop: the adjustment therefore in the former case would disturb that

\* Whether the eastern or western depends on the position of the azimuthal stud.

in the latter, if it had been performed first. If the telescope be first placed in the meridian with the plane face in contact, the horizontal screw of the flying stop enables the range of the instrument between the two fixed stops to be adjusted to exactly  $180^\circ$ , and the elimination of deviation error in the second telescopic position does not disturb the adjustment in the first.

If the directions for putting up the sector have been complied with, it has already been approximately adjusted in azimuth by means of the meridian line that has been cut on the collimator pillar. The error of the chronometer too by this time should have been discovered to within a minute or two.

Find the exact chronometer error by timing the transit of a Nautical Almanac star, selecting one as near the zenith as possible and setting for it with the setting-circle. Now compute out the chronometer times of transit of a circumpolar star over all the wires in both instrumental positions: recollect that in one telescopic position it will transit the wires in the order I to VII and in the other from VII to I. The time of transit of wire IV should be the same in both positions: the time of transit of wire I with telescope east will, as stated before, be approximately the same as that of wire VII with telescope west, but not, however, sufficiently near to allow of the same time being adopted for both events.

Bring the plane face of the "flying" stop into contact with one of the fixed stops: about half an hour before the time of transit of the circumpolar star, set the telescope to the correct zenith distance. If the circumpolar is not in the field, slowly *sweep* with the telescope to the right and left: as the fixed stop arrests the rotation of the instrument in azimuth, the observer should take care to so arrange the deviation error, that the flying stop is able to travel a little *past* the prime vertical, and is not arrested before it; unless this precaution is attended to, the observer will be only able to sweep with the telescope on one side of the meridian.

In sweeping with the telescope, be careful that it is set to the exact zenith distance, corrected for refraction: a great number of circumpolars are often met with in sweeping, and one is very liable to observe a wrong one. In deciding whether it is the correct star in the telescope, three tests can be applied, *viz.*, zenith distance, rate of motion, and magnitude. The rate with which a star passes from one wire to

another is an excellent test: the equatorial intervals of the wires are known, and it is easy to compute the time, that a circumpolar of given declination takes to pass. In November and May Polaris gives ample time to get the meridian in both telescopic positions, and this is a star that no one can mistake.

When the correct circumpolar is in the field, bring it amongst the wires by sweeping the telescope: it is not necessary at this period to time it over any particular wire: the times of transit over all the wires are known, and the observer can judge from them fairly accurately, whereabouts the circumpolar should be at any given moment. When the circumpolar is approximately in its correct position amongst the wires at a given moment, the telescope is in the meridian, but as the flying and fixed stops are not now in contact, the adjustment in azimuth will be lost immediately the telescope is moved. Of the two stops it is the "flying" one that is correctly placed in azimuth, and the fixed one has to be brought up into contact with it. To do this, unscrew the nuts of the six steel bolts, that connect the upper portion of the cast-iron pillar to its base-plate, and rotate the pillar by means of the azimuthal stud-screw. The telescope and vertical axis must be held steady with one hand, whilst the azimuthal stud-screw is being turned, or they will revolve with the cast-iron pillar, and the whole essence of the adjustment is to rotate the latter independently of the former. The pillar should be revolved by means of the screw, until the "fixed" stop has been brought round into contact with the "flying" one.

The slotting of the six bolt holes will only permit the upper portion of the pillar being revolved on the base-plate through about  $2^\circ$ , so that if the azimuthal stud-screw is brought to the centre of its run, when the instrument is first put up,  $1^\circ$  is the extreme deviation error, that can be corrected by means of it: if too  $1^\circ$  even were corrected for, the bolts would be hard down against one side of their slots, the screw would be at the very end of its run, and any small deviation error that might arise in the future could only be eliminated by rotating the whole instrument on the stone-cap. As it is very inadvisable to commence work with no room on the screw for further azimuthal adjustments,  $30'$  may be considered the maximum allowable angle in both directions through which the iron-pillar should be rotated when *first* adjusting the instrument. If the meridian has not been laid down accurately, or if the zenith sector has been carelessly put up, the observer

may find on placing a circumpolar amongst the wires, that the deviation error exceeds  $30'$ : if this is the case, (and he must judge whether it is or not by the distance between the fixed and flying stops, when the telescope is in the meridian) he will have to loosen the three foot-bolts, that connect the instrument to the stone cap, and revolve the whole sector by means of a crowbar placed under the base-plate.

When the circumpolar is approximately in its correct position amongst the wires, and the fixed stop has been brought by means of the azimuthal stud-screw into contact with the plane face of the flying stop, take another N. A. Star, and if the chronometer error now differs from its earlier value, correct all the computed times of transit of the circumpolar over the several wires in accordance. Re-level the vertical and horizontal axes. The adjustment in azimuth must now be perfected by making the circumpolar star transit some particular wire at the computed time. One minute before the time, the recorder begins to count seconds, and the observer brings the star on to the wire by means of the stud-screw: as the star moves off the wire, the observer constantly brings it back, and keeps it there, until the recorder reaches the computed time of transit. When a northern circumpolar is being taken, and the azimuthal stud is north, the observer cannot turn the screw himself; this makes the adjustment more difficult, as a third person, the tindal, is introduced: the observer, recorder and tindal should, however, with a little practice work well together and perform the adjustment without difficulty. Be most careful, when the stud screw is being turned at this final phase of the adjustment, to see that the two stops *are* in contact; *they are very apt not to be*.

Do not eliminate deviation error by moving the fixed stop in its channel round the neck of the pillar.

When the circumpolar transits a wire at the right moment, the meridian has been found and the nuts of the six steel bolts may be tightly screwed down. In screwing the nuts down to the bolts, the azimuth adjustment will be slightly and inevitably disturbed, but as it is always by the same amount and in the same direction, an experienced observer soon learns how to allow for it by placing the circumpolar at the computed time of transit not exactly on the wire.

When the cast-iron pillar is rotated by means of the stud-screw, the vertical and horizontal axes will often become disleveled: unless



however both are nearly level, no accurate adjustment to the meridian is possible, and consequently it is desirable to frequently take the readings of the levels throughout the process. Not only does the azimuthal adjustment disturb the levels, but the act of levelling alters the deviation of the telescope.

When the instrument has been accurately adjusted in azimuth with the plane face of the flying stop in contact with a fixed stop, change the telescopic position and again observe the same circumpolar star. Bring the screw head of the flying stop into contact with the fixed stop, and decide whether the deviation error in the new position is to the east or west, and whether the stop-screw must be made to protrude more or less in order to bring the telescope to the meridian. As the time of transit over a wire draws near, make the recorder count, and gradually bring the star nearer to the wire by turning the screw of the stop. When the time of transit is reached, the screw should have been made to so protrude from the stop, that when it is in contact with the fixed stop, the circumpolar is transiting the right wire. When the stop-screw protrudes the correct amount, clamp it in its position by the second screw situated on the outer side of the stop. If this precaution to clamp is omitted, the constant concussion, that takes place in regular work between the flying and fixed stops, gradually drives the screw into the flying stop and increases the deviation. The sector has been now adjusted to the meridian in *both* telescopic positions and is ready for work.

28. When the sector has for the first time been accurately adjusted in azimuth, bring the telescope horizontal, and move the two collimators, until their crosses are cut by the centre wire, one in the position telescope east, the other in the position telescope west: these collimators, which are found in practice to steadily retain one position, will give the direction of the meridian in both telescopic positions on any occasion, that a circumpolar star owing to clouds or other causes cannot be observed. If collimators have not been taken into the field, point the telescope, when adjusted in azimuth, to the horizon, and select some terrestrial meridian mark to serve for future reference. As a prominent natural object is not likely to exist exactly on the meridian, the observer will find it advisable to draw on paper, what he sees through the telescope, marking the position of the centre wire on his drawing: if he omits to do this, he may fail to recognise the view again.

Selection of a meridian mark.

29. Every night *before* work the vertical and transit axes must be accurately levelled, the microscopes examined for focus and parallax, the illumination of the limb tested, and the deviation of the telescope in both its positions from the meridian corrected by reference to the meridian mark or collimators. Before commencing work too, *always* see that the black counterpoise hangs freely and exerts its whole force: its suspension bar is apt to jam against the table.

30. The limb of the sectors extends through such a small arc, that a greater zenith distance than  $17\frac{1}{2}^{\circ}$  cannot be read: all stars, whose zenith distances are to be observed, must therefore be situated near the zenith.

It is a fundamental principle in working with the zenith sector that no observation is to be accepted as complete, unless the star has been intersected in both telescopic positions: when a star has been intersected in one position only, the observation *must be rejected*. Intersection of the same star in both positions eliminates the zero error of the sectors: it is wholly inadmissible to take a single intersection of a star and correct it for zero error, the amount of the correction being determined from a double intersection of another star. The zero error of the sectors is not a constant quantity, but varies with the dislevelment of the vertical axis.

The procedure for observing a star for latitude is as follows:—

- (i) Set the telescope to the correct zenith distance of the star by means of the Index microscope of the sectors; complete the setting to greater accuracy with the A microscope, and clamp the sectors.
- (ii) Make the setting-circle read the same zenith distance, but the reverse way; that is, set the vernier to the proper reading *below* the zero of the setting-circle limb. The bubble will then be at the centre of its run when the telescope is inclined at the same angle to the vertical as it is in its present position, but on the opposite side of the zenith.
- (iii) Bring the flying stop into contact with one of the fixed stops.

- (iv)  $2\frac{1}{2}$  minutes before the star is due, read the four microscopes, and then the two levels.
- (v) At one minute before transit the recorder gives warning and the observer seats himself.
- (vi) When the star is exactly *midway* between the first two vertical wires, intersect it with the horizontal wire and take the reading of the micrometer.
- (vii) Rise up, unclamp the sectors, pass the telescope through the zenith, bring the bubble of the setting-circle to the centre of its run by tapping the telescope to the correct altitude, clamp the sectors, and rapidly rotate the instrument  $180^\circ$  in azimuth, bringing the flying stop up into contact with the second fixed stop *without jerk or jar*.
- (viii) Again sit down and bring the star close to the horizontal wire by moving the telescope with the tangent screw of the sectors.
- (ix) *Unclamp* and *re-clamp* the sectors.
- (x) Intersect the star again with the horizontal wire, and take the second reading of the micrometer.
- (xi) Read the two levels, and *lastly* the four microscopes. The observation is then completed.

31. During an observation, take care that the flying stop *is* in contact with a fixed stop: it is apt not to be. Beware of the flying stop being too in the hurry of the moment of reading the levels, *before* the bubbles are at rest.

Precantions to be taken when observing.

Between the first reading of the microscopes and the first intersection of the star, no tangent-screw nor any other part of the instrument may be touched. The second intersection may be approximately made with the tangent-screw of the sectors, but when completed with the micrometer, the tangent-screw must not be touched again, till the final reading of the microscopes has been taken. The reason of the second intersection being first made approximately with the tangent-screw is, that the two intersections are then taken at equal

distances from the zero of the telescope comb, and as the two readings of a *complete* observation are supplements of one another, error in the value of the telescope micrometer is cancelled.

Colonel Herschel, who knew the zenith sector well, laid great stress on the importance of invariably unclamping and re-clamping the sectors, after their tangent-screw had been used; "this step," he wrote, "should become an acquired habit and not be left to memory: the observer's hand after quitting the tangent-screw should at once move by instinct to the milled heads of the clamping cam."

The equatorial interval between wires I and II is about 14 seconds of time: between each of the pairs II and III, III and IV, IV and V, V and VI it is about 7 seconds: between VI and VII it is again about 14 seconds. The time that elapses between the two intersections of one star is consequently 42 seconds, the first intersection being made 21 seconds *before* the centre wire, and the second 21 seconds *after*. For an observer who knows his work, 42 seconds is ample time to do all that has to be done.

It may often happen, owing to accidents, mistakes or clouds, that the first intersection or the second or even both are not made exactly midway between the two first or two last wires, and consequently not at 21 seconds from the meridian. In these *exceptional* cases the observer must cause the recorder to note on the record the distance in time from the centre wire that the intersection was made, so that in the computations the correction for "intersection off the centre" may be applied for the actual amount and not for the usual 21 seconds. Knowing the intervals between the inner consecutive wires to be 7 seconds each, and the two outer intervals to be 14 seconds, the observer can estimate fairly accurately at what distance from any particular wire he has made the intersection. Suppose, for instance, he makes the intersection, not as he ought midway between wires VI and VII, but just before the star transits wire VII: he says to the recorder "intersection made at VI + 13<sup>s</sup>." Suppose again that he fails to intersect the star until it has passed wire VII by a distance slightly less than half one of the inner wire intervals, he informs the recorder "intersection at VII + 3<sup>s</sup>." In the first of these hypothetical cases the correction for "intersection off the centre" would be for 27 seconds instead of 21, and in the second for 31 seconds.

When stars have been observed once, their correct readings on the sector should be taken out, and these latter used when setting on future occasions: it is desirable to *set* as accurately as possible, so as to keep the readings of the telescope micrometer within 30 or 40 divisions of its zero.

32. Suppose the instrument to be in position and in perfect adjustment with its azimuthal stud to the north. If there is no prominent meridian mark easily recognisable through the telescope, bring the latter down horizontal, and make a careful drawing of the distant terrestrial view as seen through it, marking the position of the centre wire on the drawing. If collimators are being used, and their crosses are not exactly on the centre wire of the telescope, when pointing in the meridian, do not trouble to move the collimators, but make a drawing of the wires of the telescope in both positions and mark on each the exact situation of the collimator crosses. These drawings are necessary to enable the meridian to be re-found after reversal, as during that process all adjustments are upset: it is safer to trust to a drawing than to memory.

Take out the three foot-bolts: place a crowbar under the foot *nearest* the telescope, and raise that side of the instrument: insert a wooden wedge under the foot, so as to raise the latter  $\frac{3}{4}$  of an inch from the stone, and remove the crowbar. Insert the hemispherical steel button under the brass-nut in the centre of the base-plate. Remove the wedge. Rotate the instrument on the button through  $180^\circ$ . Again lift with the crowbar and insert the wedge. Remove the button, and then the wedge, lowering the instrument very gently with the crowbar on to the stone.

Bring the plane face of the flying stop into contact with a fixed stop: level the instrument fairly accurately by placing washers under the feet, and then shift it approximately into the meridian with the crowbar. Screw up the foot-bolts tight, and level again with the *four* thumb-screws. Point the telescope to the distant horizon or collimator, unscrew the nuts of the six steel-bolts and rotate the instrument in azimuth by means of the stud-screw, until its centre wire has taken up the same position in regard to external objects, as it occupies in the drawing made before reversal, when deviation error will have been

eliminated. Screw down the nuts of the six bolts: re-level with the *four* thumb-screws: re-level the horizontal axis by raising or lowering the bearing shafts.

The steel button is of such a height, that when the zenith sector is balanced on it, the feet of the latter are raised about  $\frac{1}{20}$ th of an inch off the stone-cap. Care should be taken to place the button well in the centre of the brass nut, for if placed near the edge, the instrument might be rotated off it, and badly jarred.

33. To dismantle the sector unscrew the eye-piece and screw on the eye-end cover. Remove the dew-cap and screw on the telescope cap. Tie up the spring of the clamping arm with string. Loosen the clamping cam and tangent-screw of the arm attached to the cradle. Turn the telescope horizontal and have it supported by two men. Remove the bridge-piece. Tie on the wooden guides marked (1). Have the telescope-box brought near: see in which direction the telescope has to point when in the box, and arrange the latter accordingly: place the lid with its truncated pyramid on the box but without screwing it down. Remove the black counterpoise, and tie on the wooden guides marked (2). Remove the mushroom counterpoise and affix the conical wooden cap.

To dismantle the sector.

Three men must now pull at the telescope, whilst a fourth supports the outer end of the horizontal axis by means of the wooden cone: when the telescope has been pulled out some 4 or 5 inches, the horizontal axis must be supported between the cradle and sectors. The telescope and horizontal axis may now be entirely removed from the cradle, no one being allowed to touch the sectors.

Rest the telescope on the truncated pyramid of the box, and remove its clamping arm; wipe all oil from the horizontal axis; take out the four large screws that attach the sectors to the telescope, and rotate the former until they are parallel to the latter: replace the four screws. Turn the wooden-tipped handles of the sectors, so that they may lie in one of the diagonals of the box, when the telescope is packed. Place the telescope, sectors, and horizontal axis in the box, lowering it with every care.

Before packing the remaining parts of the instrument, wipe the oil off and thoroughly clean the cradle-bearings, vertical axis, hemispheres, and base-plate cup.

The stone-cap of the pillar must be taken with the instrument to the next station.

34. The geodetic latitude of every station is known to the observer, and as it seldom differs by more than a few seconds from the astronomical latitude, the distance at transit of any star from the zenith can always be calculated with sufficient accuracy to allow of its being found with the telescope.

Rules by which the selection of stars for latitude observations ought to be governed.

To observe 40 zenith stars for latitude is a hard night's work: to observe 30 is easy. The number of stars, included in a programme of work for *one* night on latitude operations, should therefore be about 36.

In selecting these 36 stars the following conditions should be fulfilled:—

- (i) All stars must be taken from the latest Greenwich Catalogue.
- (ii) No star is to be considered sufficiently trustworthy for observation unless its north polar distance is shewn in that catalogue as determined by at least six observations.
- (iii) No star is to be included that has not a proper motion in north polar distance assigned to it in the Greenwich Catalogue.
- (iv) Double stars and stars of the 1st and 2nd magnitude should be avoided.
- (v) No star should be included that has a greater zenith distance than  $13^{\circ}$ .
- (vi) The number of north stars must be the same as the number of south.
- (vii) The mean zenith distance of all the north stars must not differ from the mean zenith distance of all the south by more than half a degree.
- (viii) Stars that are  $8^{\circ}$  from the zenith or more should be paired as nearly as possible.

- (ix) The minimum difference in right ascension between two consecutive stars should be six minutes.
- (x) Two to four Nautical Almanac stars equally distributed north and south of the zenith should be included to enable the chronometer error to be determined.
- (xi) Two to four stars within  $1^\circ$  of the zenith should be included to enable the collimation error in azimuth to be determined\*.

35. The programme of work for one night is made up as Programme of work for one night. follows :—

- (i) The zenith distance of about 36 zenith stars must be measured. The stars being selected in accordance with the above rules, and each observed in both telescopic positions.
- (ii) The time of transit of a circumpolar star, whose right ascension has been well-determined, must be taken over two wires in each telescopic position: from this is deduced the deviation error of the instrument.
- (iii) The transit-axis level must be read before and after work in both telescopic positions, being reversed end for end in each position.
- (iv) The two thermometers outside the tent should be read every 15 or 20 minutes.
- (v) The barometer should be read every hour, the mercury in the cistern being lowered and raised again to the zero pointer each time.
- (vi) The microscope run must be determined 15 times, the same  $5'$  space being never utilised twice; the high reading should be always recorded *above* the low, whether read first or not.
- (vii) It is of importance, that the temperature of the interior of the observatory tent should not differ from that of the

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\* These eleven rules for governing the selection of the stars were laid down after much consideration by Colonel Heaviside, and have been closely adhered to by subsequent observers. Any departure from them will be surely followed by discordance in the results.



outside air by more than  $1^{\circ}$  or  $2^{\circ}$  Fahrenheit: the thermometer attached to the barometer should therefore be *occasionally* glanced at, but its readings need not be recorded.

- (viii) The error of the chronometer and the collimation error of the telescope in azimuth have to be found nightly, but no special observations are needed for their determination, if in the selection of the stars attention has been paid to rules x and xi given in paragraph 34.

The observer is *again* reminded here of the necessity of noting on the record, *which* graduation of the sector limb he intersects with his microscopes, (*vide* para. 23) and *how far* from the centre wire, he intersects his stars with the telescope micrometer, (*vide* para. 31).

36. It must be borne in mind that the object of these observations is the determination of astronomical latitudes and not merely of zenith distances, and that therefore the accuracy of the results is affected by errors in north polar distance as well as by those of observation. The mean of *two* zenith distance observations with the sector is, as a rule, a more reliable measure than the north polar distance of a star as computed from the Greenwich Catalogue, a fact, that speaks very highly of the sector as an astronomical instrument.

Supposing an observer to devote four nights to one station and to take 40 observations per night, it has been found that the probable error of his final value of latitude will be

$\pm 0''.063$ , if he observes the same 40 stars every night.

$\pm 0''.049$ , if he observes 40 stars the first two nights and 40 different ones the last two nights.

$\pm 0''.039$ , if he observes 40 different stars every night.

In each of these three cases the number of observations taken on the whole and the total amount of work carried out will be the same. The result attained by observing 40 fresh stars on each of the four nights will evidently be very much better than if 80 stars are observed twice each. It is moreover easy of proof that but slight advantage accrues at all from taking a third zenith distance *per* star, and that *a fourth such observation is absolutely waste of time.*

It has been found that the zenith distance of a star when observed E to W is not the same as when observed W to E;\* the difference between the two values is the same for every star, but varies with different observers. The existence of such a difference is probably due to the observer's personality, and necessitates every star being observed twice, once from E to W, and once from W to E: though these two observations may differ by  $0''.5$ , their mean deserves the same weight as if they had been identical, personal error being in all probability entirely cancelled in it.

In Northern India as many suitable stars as can possibly be needed at any latitude station may be obtained; but south of the 18th parallel of latitude, it is difficult to get more than 35 to 40 well-placed south stars from the Greenwich Catalogue. An observer cannot of course work all night, but by commencing on one night shortly after sunset, on another at 12 midnight, and on a third at three o'clock in morning, he will be able to observe almost all the suitable stars, that transit the meridian, near the zenith, between sunset and sunrise.

Taking all these facts into consideration, the following may be laid down as the proper programme of work for a latitude station:—

(i) From 70 to 100 stars should be observed, 100 if possible.

(ii) Each star should be observed once E to W and once W to E.

In order that this should be done without confusion, commence a programme of stars on its first night by taking the first star E to W, the second W to E, and so on alternately: on the second night take the first star W to E, the second E to W, and so on. The only reason for describing such an obvious method is to remind the observer, that when he misses a star on either night, he must change the telescopic position before taking the next, for unless he does so, every star after the miss, that was observed E to W the first night, will be observed E to W the second.

(iii) Four or six nights must be devoted to each station, an even number being advisable: four would suffice, if there was a paucity of stars and no misses occurred; but six are better,

\* "Observed W to E" means that the first intersection of the star was made with the telescope west, and the second intersection with the telescope east.

as they allow of stars missed once being observed in their second direction E to W or W to E.

- (iv) When half the observations have been secured, the instrument should be reversed, *i.e.*, if the azimuthal stud has been originally placed north, it should now be brought round to south.
- (v) If only 70 stars are to be observed, they should be divided into two programmes of 35 each: during the first night the first programme should be worked through, during the second the second programme, and during the third those stars of both programmes, that have been missed, should be observed: on the morning of the fourth day the instrument should be reversed, and in the evening the first programme again worked through, those stars taken E to W on the first and third days being now taken W to E and *vice versa*: on the fifth night the second programme should be again taken up, the direction E to W and W to E being changed for each star from what it was on the second night: on the last night, those stars of both programmes, that had been missed during the fourth and fifth nights, should be observed.
- (vi) If 100 stars are to be taken they should be divided into three programmes: each programme should be worked through once with the azimuthal stud north, and once with it south: those stars observed E to W on the first occasion of each must be taken W to E on the second and *vice versa*. As three programmes have to be got through in six nights, no spare nights are available for picking up misses, but by judicious interchanges of stars between the three a star missed from one programme can often be observed again in one of the other two.
- (vii) The number of observations per station need not exceed 200, and should not be less than 140.\*

The errors of the chronometer, collimation, inclination, and deviation during any night's work ought to be computed out before the next night's observations are commenced; the methods of computing them

\* It will be found as a rule that all the observations taken on any one particular night will be burdened with some small constant error, running throughout; the more nights therefore that the observations are extended over, the better will be the final result.

have been therefore included under the heading "duties in the field" and not "duties in the recess," and are now described.

37. The chronometer error is only required for computing the deviation of the instrument from the meridian. It is found as follows:—Suppose the instrument is so placed for the first intersection that wire VII is the wire first reached by the star: wire VII will then be the last wire crossed by the star after the second intersection, the telescopic position having been changed between the two. When the observer seated at the telescope sees the star come into the field, he makes the recorder count seconds from the chronometer: as the star crosses wire VII, he notes the nearest half-second of time. Half-way between VII and VI he observes the zenith distance by intersecting the star with the horizontal wire and reading the micrometer-head. He then changes the telescopic position of the instrument and intersects the star again midway between VI and VII. The recorder again counts seconds, and the observer notes the half-second of time, that the star again transits wire VII: after this he reads off the micrometer-head, which should not have been touched since the last intersection. The mean of the two recorded times of transit gives the chronometer time, at which the star crossed the centre wire of the telescope. The difference between this mean and the right ascension of the star is the chronometer error: if the right ascension is the larger, the chronometer is slow.

The right ascension of a star, that is to be used for finding the chronometer error, should be accurately known: it is therefore advisable to always select one from the Nautical Almanac. If a star that is suitable for latitude observations is selected from the Nautical Almanac, it can be observed for zenith distance and time simultaneously as described above. If no Nautical Almanac star is included in the programme of latitude stars, special observations must be taken for the purpose of determining the chronometer error.

In timing the transit of a star over wire VII in both telescopic positions, error of collimation cancels out. Error arising from inclination of the transit axis does *not* cancel owing to the inclination being different in the two telescopic positions\*; it should, however, be kept so

\* *Vide* description of method of levelling the transit-axis, (para. 26).

small as to exercise no appreciable effect on a time of transit. This fact of non-cancelment should be borne in mind, for if the inclination were large, and the same end of the axis happened to be the higher in both telescopic positions, a correction would have to be applied to the two chronometer times of transit. To cancel the effect of deviation error on the times of transit, the chronometer error should be determined nightly from the observation of both a north and south star, or better still of two north and two south stars.

38. A knowledge of the inclination of the transit-axis is necessary  
Inclination of the transit-axis. (i) for determining during the field season both the collimation and deviation errors of the instrument, and (ii) for computing during the recess the distances from the meridian, at which stars were intersected and the resulting corrections to be applied on that account.

To determine the inclination of the transit-axis, the striding level is read both with "telescope east" and "telescope west" and reversed in each position.

There are consequently eight readings, *viz.*,

|                     |   |                  |   |                               |
|---------------------|---|------------------|---|-------------------------------|
| With telescope east | { | Cross-level east | { | East-end of bubble = $E_1$ ,  |
|                     |   |                  | { | West-end     ,,     = $W_1$ , |
|                     |   | ,,     west      | { | East-end     ,,     = $E_2$ , |
|                     |   |                  | { | West-end     ,,     = $W_2$ , |
| With telescope west | { | Cross-level east | { | East-end of bubble = $E_3$ ,  |
|                     |   |                  | { | West-end     ,,     = $W_3$ , |
|                     |   | ,,     west      | { | East-end     ,,     = $E_4$ , |
|                     |   |                  | { | West-end     ,,     = $W_4$ . |

If  $n$  = value of one division of the level-scale in seconds of arc, the dislevelment will be

$$\frac{(W_1 + W_2) - (E_1 + E_2)}{4} \times n \text{ with telescope east,}$$

and

$$\frac{(W_3 + W_4) - (E_3 + E_4)}{4} \times n \text{ with telescope west,}$$

in each case a positive sign denoting that the western end of the axis was the higher. This dislevelment has to be calculated every morning and reduced if large.

The *correction*\* for inclination to the time of transit of a star would be

$$+ \frac{(W_1 + W_2 - E_1 - E_2) \times n \cos \zeta \sec \delta}{4 \times 15}$$

with telescope east, and

$$+ \frac{(W_3 + W_4 - E_3 - E_4) \times n \cos \zeta \sec \delta}{4 \times 15}$$

with telescope west, where  $\zeta$  and  $\delta$  denote the zenith distance and declination of the star.

39. The collimation error has to be taken account of (i) in determining during the field season the deviation of the instrument from the meridian, and (ii) in computing during the recess the distances from the meridian, at which stars were intersected and the resulting corrections to be applied on that account.

The collimation error is found by timing the transit of a star, whose zenith distance is less than  $1^\circ$ , over the same wire in both telescopic positions: an ordinary latitude star should be used for the determination, as special observations for the purpose are then saved. The method of both timing the transit and intersecting the star in the two telescopic positions, that has already been described under the heading "Chronometer error," should be followed in finding the collimation error also. The only difference between the two is, that in the case of the chronometer error, the star may be at any zenith distance, but must have a well determined right ascension; whereas in the case of the collimation error, it must be close to the zenith, but need not be a Nautical Almanac star. Collimation stars have to be selected as close to

\* If the western end is higher these expressions are positive, and as a star transits too soon with the west end high, a positive correction is wanted.

the zenith as possible, in order to avoid the observed times of transit being affected by the deviation error of the instrument. It is necessary to observe every night *at least* two collimation stars: one of these should be taken E to W, the other W to E.

In one telescopic position wire I is the first wire crossed and wire VII the last: in the other telescopic position wire VII is the first wire crossed and wire I the last. Suppose in the position telescope east wire I was the first wire crossed by a zenith star, and that the time of transit (corrected for the inclination of the transit-axis level) was  $2^{\text{h}} 0^{\text{m}} 56^{\text{s}} \cdot 0$ : suppose now in the position telescopic west, after the second intersection of the star has been made, the corrected time of transit over wire I, which has become the last wire, is again noted and is  $2^{\text{h}} 2^{\text{m}} 2^{\text{s}} \cdot 0$ . The elapsed time between the two transits is  $66 \cdot 0$  seconds.

The equatorial interval of wire I is known: suppose it equal to  $28^{\text{s}} \cdot 55$ . The time, that the star would take to travel from wire I to wire IV, is equal to  $28 \cdot 55 \times \sec. \delta = 31 \cdot 0$  seconds suppose, where  $\delta =$  declination of star. Now, if there were no collimation error in the telescope, the centre wire would be transited by a star at the same time in both telescopic positions, and the interval that would elapse between its two times of transit over wire I would be  $2 \times 31 \cdot 0 = 62 \cdot 0$  seconds. The collimation *error* in seconds of arc is

$$\left( \frac{62 \cdot 0 - 66 \cdot 0}{2} \right) \times 15 \cos \delta = - 27'' \cdot 3.$$

In the above case the *actual* interval is *too long*, and therefore the system of wires is crossed *too soon* and is situated too far east with telescope east, and crossed too late and situated too far west with telescope west.

If the collimation star had been observed from W to E, the *first* time of transit would have been  $2^{\text{h}} 1^{\text{m}} 0^{\text{s}}$ , and the *second*  $2^{\text{h}} 1^{\text{m}} 58^{\text{s}}$ : the elapsed interval would then have been 58 seconds or 4 seconds *too short*. The collimation error in seconds of arc would then be

$$\left( \frac{62 \cdot 0 - 58 \cdot 0}{2} \right) \times 15 \cos \delta = + 27'' \cdot 3.$$

The two values of the collimation error are the same but their signs different: this is right enough, for if a star takes too long a time when observed E to W, it will take too short a time when observed W to E.

The following rule will prevent confusion of sign in correcting transits for collimation:—

Let  $c''$  represent the collimation constant for the correction of times of transits of stars, or in other words the quantity that has been called above the collimation error, but with a reverse sign: then  $c'' = \frac{x - y}{2} \times 15 \cos \delta$ , where  $x =$  observed interval,  $y =$  correct computed interval, both in seconds of time: if  $c''$  is positive, the wires are too far east for that position of the telescope, *in which the first of the two intersections are made*, and the correction to a transit taken in that position is positive. For example in the first instance given above,  $c'' = + 27'' \cdot 3$  when the star was taken E to W: the correction for collimation to the time of transit of a circumpolar, of declination  $85^\circ$ , which is being observed for deviation, would be  $+ \frac{27 \cdot 3}{15} \sec 85^\circ$ , when the transit occurred with telescope east, and  $- \frac{27 \cdot 3}{15} \sec 85^\circ$ , when it occurred with telescope west. Transits observed in that position, in which the *first* intersection was made, *when collimation was determined*, take the sign of their correction as given by the formula for  $c''$ : transits observed in the other position take the opposite sign from  $c''$ .

40. The deviation of the telescope from the meridian has to be determined nightly: it has to be taken account of in the recess in computing the distances from the meridian, at which the stars were intersected. It is necessary to compute out the deviation error every morning, and to re-adjust the instrument, if it is found large.

The deviation error is determined as follows:—The times of transit of a circumpolar star are noted at night over one or two wires in both telescopic positions. These times are “reduced to the centre” and the times of transit over wire IV in the two positions thus found. Corrections to the transits of wire IV have then to be applied, (i) for chronometer error, (ii) collimation, and (iii) inclination of the transit-axis. The corrections for chronometer error will have the same sign, when applied to



the transits of the circumpolar in the two positions: the corrections for collimation error will have opposite signs: the corrections for inclination may have the same or opposite signs, as the errors in the two positions are semi-independent of one another. Let  $T$  = the corrected time of transit,  $\delta$  = declination of the circumpolar, and  $\zeta$  = its zenith distance; then  $15 (T - R. A.) \cos \delta \operatorname{cosec} \zeta$  = deviation of the instrument.

If  $T$  is greater than  $R. A.$ , and the circumpolar has been observed at its  $\frac{\text{upper}}{\text{lower}}$  culmination, the deviation error is  $\frac{\text{west}}{\text{east}}$  of north.

Every night, after the time of transit of a circumpolar has been taken, the telescope should be brought down and the position of the collimator cross noted. The distance of the cross from the meridian can be computed, when the deviation of the centre wire is known. Then on any night when the pole is overclouded and no circumpolar observed, the distance between the centre wire and the collimator cross should be estimated accurately: if from the calculations of the preceding and succeeding nights, the collimator cross is found in the same position on both, it is but reasonable to assume that it has not moved in the interim. On a cloudy night therefore the deviation can be found from the observation of the collimator cross, whose distance from the meridian is known. If collimators are not being used, the distance of the centre wire from the meridian mark should be estimated every evening at sunset: if no circumpolar has been observed, this distance should be again estimated at sunrise: if no serious difference is found, the instrument may be considered to have remained unmoved in the interim, and the deviation error may be deduced from the position of the meridian mark. It should be borne in mind, however, that the collimators and the meridian mark are really intended solely for purposes of adjustment and not for determining deviation error, and that they should be used for the latter in cases of emergency only: there is always a chance that a collimator may have moved and returned to its original position between two observations made to it. The adjustment of the instrument in azimuth, and the determination of the deviation error are two distinct operations and must not be confused with one another.

CHAPTER III. DUTIES IN RECESS.

Under the head "duties in recess" are included the reduction of the observations and computations of the final results.

1. The computations should always be prefaced by a description of each latitude station. The dimensions of the pillar should be given, and its situation with reference to the trigonometrical station described. The distances and directions of the neighbouring villages should be noted, so that no difficulty may afterwards be experienced in finding the site: if the station be situated in a dense jungle or on a precipitous hill difficult of access, the best road leading to it should be mentioned.

The observer should at the time of observation estimate the direction, in which he would expect a deflection of the plumb-line, and put on record the reasons for his opinion. With the assistance of maps he can tell the distances and masses of the neighbouring mountain ranges and hills; from the direction of the rivers he can ascertain the general slope of the ground in the vicinity; and from the Geological Survey records he can find the character of the soil of the surrounding country.

The computations may be divided into three principal parts:—  
 (1) Zenith distance of each star. (2) North polar distance of each star.  
 (3) Resulting latitude.

(1). *Zenith distance of a star.*

Before proceeding to describe the computations it may be as well to note, that each star observed is supposed to have been intersected both in the position telescope east and in the position telescope west.

2. The first step in the computations is to take out the means of the four microscope readings, and to then correct these means for "run." A mean of the four readings has been designated  $c$ , and the "run" correction  $c'$ .

The amount of the "run" has first to be determined from the observations taken for the purpose, of which there ought to be from 70 to 90 per station. Suppose from all these we find that 299.736

micrometric divisions\* are equal to a  $5'$  division on the limb: then 299·736 divisions are equal to 300 seconds of arc, and the correction to the reading, when the micrometer has been run over a whole  $5'$  space, is  $0''\cdot264$ . If the micrometer has been only run over a  $1'$  space on the limb, the correction to its reading will be  $\frac{0''\cdot264}{5} = 0''\cdot053$ , and the correction to be applied to *one* division of the micrometer to obtain its value in seconds of arc will be  $0''\cdot00088$ .

The above shews how the *amount* of the "run" correction is found: its *sign* is determined by the three following rules, always supposing that the *run is too small*, (*i.e.*, that one micrometric division is larger than a second of arc):—

- (i) In the case of an observation, where a run has been taken, and both the "high" and "low" readings are recorded, the "high" reading only should be used in deducing the zenith distance: the sign of the run correction in that case is + ve.
- (ii) When a note has been made in the margin of the record thus "referred to  $n''$ ", meaning that the  $n''$  division of the limb has been intersected, the sign of the run correction is + ve, if the "reference minute" is less than the reading of A microscope. If the "reference minute" is greater than the reading of A microscope the sign is - ve.
- (iii) In all other cases, when the reading of A microscope falls between  $0' 0''$  and  $2' 30''$  or between  $5' 0''$  and  $7' 30''$ , the sign is + ve., and when between  $2' 30''$  and  $5' 0''$  or between  $7' 30''$  and  $10' 0''$ , the sign is - ve.

There is one troublesome, but happily very rare exception to the third rule: it is when the reading of A microscope falls so closely *below* a division of the limb, that the mean of the four microscopes is *above* that division: the sign is then obviously + and not -.

3. By the "instrumental" zenith is to be understood the intersection of the vertical axis of rotation with the celestial sphere. If the telescope is directed to this point, it will continue to point to it, when the instrument is

Instrumental zenith distance,  
and zero error.

\* A micrometric division here means the mean value of one division of each of the four microscopes.

turned about its axis, and the reading on the circle will remain the same. This reading has been designated the zero error ( $= \omega$ ), and is considered positive, when it makes positive arcs too great.

We will start with the assumption that latitude = declination + zenith distance *always*, so that north declinations and south zenith distances are positive, and south declinations and north zenith distances are negative. Facing the limb the circle reading of the zenith sector decreases, as the object-glass rises on the right hand or sinks on the left: consequently when the telescope is east of the pillar, circle readings are positive, and when west they are negative, as regards the sign of zenith distance. The graduation is continuous from  $0^\circ$  to  $360^\circ$ , and the zero may be placed at either the zenith or nadir.

One division of the telescope-micrometer corresponds very closely to  $\frac{3}{7}$ ths of a second of arc; it is in fact exactly equal to  $\frac{3}{7} \times 1'' \cdot 0007$ . The reading of the telescope-micrometer at any intersection reduced to seconds of arc is designated by the symbol M. Suppose  $N = c + c' + M$ ,  $N_E$  denoting that the readings  $c$  and  $M$  were taken for the intersection in the position "telescope east", and  $N_W$  for the intersection in the position "telescope west." Let  $Z =$  the distance of a star on the meridian from the "instrumental zenith."

Then  $N_E = \omega + 180^\circ + Z$ , and  $N_W = \omega + 180^\circ - Z$ , if the zero is at the nadir; and  $N_E = \omega + Z$ , and  $N_W = \omega + 360^\circ - Z$ , if the zero is at the zenith. In both cases  $Z = \frac{1}{2}(N_E - N_W)$ , and  $\omega = \frac{1}{2}(N_E + N_W) - 180^\circ$ .

The zero error does not enter in any way into the computations: it should be taken out by the recorder at the time of observation merely as a check against gross errors. It may also be *occasionally* employed as a criterion in rejecting observations, but great caution must be exercised in putting it to this use, as it is not even theoretically a constant quantity, but varies with the verticality of the vertical axis in the meridian.

4. The instrumental zenith distance has now to be corrected for the

Correction for inclination of the vertical axis.

inclination of the vertical axis. This inclination is measured by two independent levels, and

the results by both should agree closely, the mean of the two being adopted as the correction.

If the north end of the levels is too high, a south zenith distance is too small, and a north zenith distance is too large: having regard therefore to the signs of a zenith distance, the correction in both cases should be + ve. The proper sign will therefore be always attained in every case by deducting the south readings from the north. Suppose  $s$  = south reading of bubble, and  $n$  = north reading, and let  $S_E = n - s$ , in the position "telescope east," and  $S_W = n - s$  in the position "telescope west". Then the correction for inclination will be  $(S_W + S_E) \times \frac{x}{4}$ , where  $x''$  = one division of the level-scale. (In the printed forms for reduction  $S_W$  and  $S_E$  are put equal to  $s - n$ , and the correction made  $-(S_W + S_E) \times \frac{x}{4}$ : the result is the same).

The zero error of each level should be taken out for each star, for though it does not enter into the computations, it serves as a check against gross mistakes in reading the bubbles. In accordance with the system of signs adopted above the zero error is equal to  $\frac{S_W - S_E}{4}$ .

5. The instrumental zenith distance, when corrected for the inclination of the vertical axis, may be designated the *observed* zenith distance. Two corrections have to be applied to this observed zenith distance, *viz.*, (1) For refraction, (2) For intersection off the centre.

6. In computing the corrections for refraction the first step necessary is to find the readings of the barometer and thermometers at the time of observation of each star. A list of the stars observed on any one night is prepared, and their right ascensions to the nearest minute written against them. The corrected\* reading of the barometer and the mean corrected\* reading of the two thermometers are entered opposite those stars, which were observed immediately before the pressure and temperature were noted: barometric and thermometric readings are then interpolated between those actually taken during the night; the variation in both is considered proportional to the elapsed time, and this latter is derived from the right ascensions.

\* *i.e.*, corrected for index-error.

The actual correction for refraction is computed for each star by means of Bessel's three (Greenwich) tables. The argument with which to enter the first table is the zenith distance of the star, that for the second the barometric pressure, and that for the third the thermometer reading: a logarithmic number is obtained from each, the sum of the three being the logarithm of the "refraction."

The correction for refraction always *increases* a zenith distance, and is therefore—having regard to the signs of north and south zenith distance—*negative* for north stars and *positive* for south.

7. The observed zenith distance has to receive a correction on account of the extra-meridional position of the star at the time of intersection, due, in the first place to the observations being purposely and of necessity\* made off the centre, and in the second, to imperfect adjustment of the instrument. At no step in the reduction of the results is an observer so likely to fall into error, as when computing this correction, a portion of the work that must fall to his personal share.

There are four causes operating, which prevent a star being intersected *on* the meridian: they are (i) Intentional intersection off the centre. (ii) Collimation error. (iii) Inclination of the transit-axis. (iv) Deviation error.

The mistake of supposing that the effect of collimation error is cancelled, if an intersection be made in both telescopic positions, can only arise from oversight; the result of collimation error is to cause the zenith distance to be measured on a *small* circle parallel to the meridian instead of on the meridian; the small circle is in one telescopic position east, and in the other west, of the meridian, but at the same distance from it in both.

There is another and more serious mistake which is very likely to occur in computing the correction for "intersection off the centre": it is to assume that the difference between a meridional and an extra-meridional zenith distance of any star depends only on the distance from the meridian, at which the star is intersected. Having made this assumption, the observer *combines* the four causes, that lead to

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\* In order to give time for the instrument to be rotated and for each star to be intersected in both telescopic positions.

extra-meridional intersection and computes the total hour-angle of the star at the instant of its observation: he then substitutes the seconds of the hour-angle in some formula that is applicable to either deviation or collimation only, and determines a wholly erroneous correction to the zenith distance.

It should be borne in mind that the difference between a meridional and an extra-meridional zenith distance depends not only on the distance from the meridian, at which the intersection is made, but also on the displacement of the zenith, and the position of the particular circle of the sphere, on which the measurement is made.

In the case of intentional intersection off the centre, as also in that of collimation error, the zenith is displaced laterally in the prime vertical, and the measurement is made on a small circle parallel to the meridian: in the case of deviation error the measurement is made on a great circle cutting the meridian in the vertical line, and the zenith is not displaced at all: in the case of dislevelment of the transit-axis the zenith is displaced laterally in the prime vertical, and the measurement is made on a great circle cutting the meridian in the north-and-south diameter of the horizon. In no case will the correction to north and south zenith distances be the same, as the path of a north star is convex and that of a south concave to the zenith.

From what has been said above, it will be seen that intentional intersection off the centre and collimation error *must* be treated together: the correction to a zenith distance ( $\zeta$ ) depending on these two causes is expressed by the following formula:—

$$d\zeta_{(k+c)} = \frac{15^2}{2} (k+c)^2 \tan \delta \sin 1'',$$

in which  $(k+c)$  must be entered in seconds of time. This correction is the same in amount but opposite in sign for north and south zenith distances: it decreases the former as observed, and increases the latter. As north zenith distances have been considered negative and south zenith distances positive, the general formula\* for the correction involving both cases may be written

$$d\zeta_{(k+c)} = + \frac{15^2}{2} (k+c)^2 \tan \delta \sin 1''.$$

\* Chauvenet's Astronomy. Fourth Edition. Volume II, para. 229.  $\tau$  = hour-angle =  $(k+c) \sec \delta$ .

From this formula it will be seen, that it is not only advisable but imperative to treat  $k$  and  $c$  together: if treated separately the formula would be

$$d\zeta_k + d\zeta_c = + \frac{15^2}{2} (k^2 + c^2) \tan \delta \sin 1'',$$

and this is too small by

$$15^2 \times k \times c \times \tan \delta \sin 1''.$$

Suppose for instance that the first intersection was properly made at 21 seconds from the centre wire, and that the second was late and not made till 33 seconds from the centre: suppose also that owing to collimation error the centre wire is 30" too far east with "telescope west," which has the effect of making both intersections nearer the meridian. Then the mean value of  $k$  will be + 27<sup>s</sup>, and that of  $c$  will be - 2<sup>s</sup>, and the value of  $d\zeta_{(k+c)}$  will be

$$+ \frac{15^2}{2} \times 25^2 \times \tan \delta \sin 1''.$$

Now let the effect on zenith distance of an inclination in the transit axis be considered. If the transit-axis be inclined to the horizon at an angle  $b''$ , the zenith distance of a north star as observed will be *too great* by

$$\frac{b^2}{2} \cdot \frac{\cos (\Delta + \zeta) \cos \zeta \sin 1''}{\sin \Delta}, \text{ (Mr. Eccles's formula),}$$

where  $\zeta$  = north zenith distance, and  $\Delta$  = north polar distance of star.

As north zenith distances are negative, the correction will be

$$d\zeta_b = + \frac{b^2}{2} \cdot \frac{\cos (\Delta + \zeta) \cos \zeta \sin 1''}{\sin \Delta}.$$

When using this formula, the numerical value of  $\zeta$  must be entered regardless of sign. Thus suppose  $b = 40'' \cdot 2$ ,  $\Delta = 65^\circ$ , and  $\zeta =$  a north zenith distance of  $10^\circ$ , the correction will be

$$d\zeta_b = + \frac{(40 \cdot 2)^2}{2} \cdot \frac{\cos (65^\circ + 10^\circ) \cos 10^\circ \sin 1''}{\sin 65^\circ}.$$



It might perhaps have been more consistent, to have considered a north zenith distance =  $-\zeta$ , and to have so arranged the formula, that in the above example  $-10^\circ$  would have had to be substituted for  $\zeta$ : but this was tried, and was found to lead to much confusion; the formula has consequently been so written above, that in dealing with north stars the *numerical* value of the zenith distance has to be substituted for  $\zeta$  *without* changing any of the signs as given.

In the case of south stars the *true meridional* zenith distance is larger than the *observed* zenith distance: the latter has therefore to be increased, and as south zenith distances are positive, the correction will be positive: the correction is

$$d\zeta_b = + \frac{b^2}{2} \cdot \frac{\cos(\Delta - \zeta) \cos \zeta \sin 1''}{\sin \Delta}.$$

$b''$  is the *mean* inclination of the transit-axis in the two telescopic positions: it is a matter of indifference which end of the axis is high, and only *amount* must be considered. Thus suppose with telescope east the inclination is  $20''$  east, and with telescope west  $14''$  west, the mean inclination is  $\frac{20 + 14}{2} = 17''$ , and not  $\frac{20 - 14}{2}$ . If with telescope east the inclination is  $20''$  east and with telescope west  $14''$  east, the mean inclination is  $17''$  also.

Now let the effect on zenith distance of a deviation in azimuth be considered. Suppose  $a''$  to be the mean deviation of the telescope from the meridian in its two positions: the zenith distance of a north star as observed will be too great by

$$\frac{a^2}{2} \sin 1'' \times \frac{\sin(\Delta + \zeta) \sin \zeta}{\sin \Delta}, \text{ (Mr. Eccles's formula).}$$

As north zenith distances are negative the correction will be

$$d\zeta_a = + \frac{a^2}{2} \sin 1'' \times \frac{\sin(\Delta + \zeta) \sin \zeta}{\sin \Delta}.$$

When using this formula, the *numerical* value of  $\zeta$  must be entered regardless of sign, as explained before: thus if  $a'' = 112'' \cdot 1$ ,  $\Delta = 65^\circ$ , and  $\zeta =$  a north zenith distance of  $10^\circ$ , the correction will be

$$d\zeta_a = + \frac{(112 \cdot 1)^2}{2} \sin 1'' \times \frac{\sin(65^\circ + 10^\circ) \sin 10^\circ}{\sin 65^\circ}.$$

In the case of south stars the *observed* zenith distance will also be too great: the latter has therefore to be decreased, and as south zenith distances are positive, the correction will be negative: the correction is

$$d\zeta_a = - \frac{a^2}{2} \sin 1'' \times \frac{\sin (\Delta - \zeta) \sin \zeta}{\sin \Delta}.$$

In calculating  $a''$  only the *amount* of the deviation should be considered and no account should be taken of its sign. Thus suppose with telescope east the deviation is  $60''$  west, and with telescope west  $38''$  east, the value of  $a''$  will be  $\frac{60 + 38}{2} = 49''$  and not  $\frac{60 - 38}{2}$ .

Summing up the above conclusions, the three corrections to an observed zenith distance for intersection off the centre are:—

$$d\zeta_{(k+c)} = + \frac{15^2}{2} (k + c)^2 \cot \Delta \sin 1''$$

for both north and south stars,

$$d\zeta_b = + \frac{b^2}{2} \cdot \frac{\cos (\Delta \pm \zeta) \cos \zeta \sin 1''}{\sin \Delta}$$

for both  $\frac{\text{north}}{\text{south}}$  stars,

$$d\zeta_a = \pm \frac{a^2}{2} \cdot \frac{\sin (\Delta \pm \zeta) \sin \zeta \sin 1''}{\sin \Delta}$$

for  $\frac{\text{north}}{\text{south}}$  stars.

The first formula is independent of zenith distance: in substituting a numerical value for  $\zeta$  in the other two the sign should be retained as written whether north or south stars are being dealt with. The signs in front of the corrections in all three formulæ are given on the understanding, that north zenith distances are inherently negative.\*

The *instrumental* zenith distance was first found: this was corrected for "inclination of the vertical axis" and the result designated the *observed* zenith distance: this latter has now received a correction on

\* These formulæ are not applicable, if the zenith distance of a star is measured at its lower culmination.

account of refraction, and three corrections on account of the extra-meridional position of the star at the instant of intersection: the final result thus obtained is the *true meridional* zenith distance.

(2). *North polar distance of a star.*

Suppose the star No. 152 of the Greenwich Catalogue of 1872 to be observed in India for latitude on the 23rd December 1886: its apparent north polar distance at the moment of observation is required.

8. The mean north polar distance on the 1st of January 1872 is given in the Catalogue as  $75^{\circ} 18' 53'' \cdot 91$ . The first correction.

$$\left\{ p + \mu + \frac{sy}{200} \right\} y,$$

where  $y$  = number of years from 1872,  $p$  = annual precession in north polar distance =  $-18'' \cdot 703$ ,  $\mu$  = annual proper motion =  $0'' \cdot 00$ , and  $s$  = secular variation =  $+0'' \cdot 012$ . The amount of the correction is  $-4' 21'' \cdot 67$ , and this applied to the above north polar distance gives  $75^{\circ} 14' 32'' \cdot 24$  as the value of the mean north polar distance on January 1st, 1886.

9. The second correction is equal to  $Ee' + Ff' + Gg' + Hh' + L + l'$   $-300$ , where  $E, F, G, H, L$  are numbers: the logarithms of  $E, F, G, H$  and the value of  $L$  are given in the Nautical Almanac; they change with the day of the year but are the same for all stars;  $e', f', g', h', l'$  are functions of the star's place, the logarithms of  $e', f', g', h'$  and the value of  $l'$  are given in the Star Catalogue and are independent of the date of observation. The second correction amounts in the present example to  $-16'' \cdot 65$ .

10. The third correction consists simply of the product of the proper motion and the fraction of the year: in the present instance it is equal to  $\mu \times \frac{357}{365}$   $= 0'' \cdot 00$ .

By applying the second and third corrections to the *mean* N. P. D. on January 1st, 1886, the *apparent* N. P. D. at Greenwich mean midnight on the date of observation is obtained and is given at  $75^{\circ} 14' 15'' \cdot 59$ .

11. Having computed the apparent N. P. D. at Greenwich mean midnight on December 23rd, the same process is again gone through for another date, (say December 31st,) and the result found to be  $75^{\circ} 14' 16''.02$ . In eight days the N. P. D. has increased  $0''.43$ , and the daily variation is therefore  $+ 0''.054$ .

12. By means of the daily variation, a fourth correction has now to be applied to reduce the apparent N. P. D. from Greenwich mean midnight to time of observation. The local mean time of the star's upper transit is the mean time at *sidereal* noon + the star's right ascension.\* The local mean time of Greenwich mean midnight is  $12^h$  + longitude (in time). Therefore the interval from Greenwich mean midnight to time of transit is expressed by

$$(M. T. \text{ at Sidereal } 0^h + R.A.) - (12^h + \text{longitude}).$$

and this in the example quoted above is equal about to  $- .4$  of a day, the negative sign denoting that the transit occurred *before* Greenwich mean midnight. As the daily variation is  $+ 0''.054$ , the fourth correction is equal to  $+ .054 \times - .4 = - 0''.022$ . The apparent N. P. D. at time of observation is therefore on December 23rd,  $75^{\circ} 14' 15''.57$  and on December 31st,  $75^{\circ} 14' 16''.00$ . For any intermediate day it can be obtained by simple interpolation, thus for December 30th, it is  $75^{\circ} 14' 15''.57 + \frac{(16''.00 - 15''.57)7}{8} = 75^{\circ} 14' 15''.57 + 0''.38 = 75^{\circ} 14' 15''.95$ .

(3). *Resulting Latitude.*

As north polar distances and not declinations, are computed from the Catalogue it has been found advisable to take out the value of co-latitude by each observation instead of the latitude. Having regard to the adopted signs for zenith distance given above

$$\text{Co-latitude} = \text{north polar distance} - \text{zenith distance.}$$

The co-latitude should be taken out for each star: the mean co-latitude by north stars only and its probable error should then be computed:

\* The star's R.A. should theoretically be converted into its equivalent in mean time, but practically this is an unnecessary refinement, as also is the correction of mean time at sidereal noon for longitude.

and the mean co-latitude by south stars and its probable error: results by north and south stars must be retained quite distinct. The probable error of a co-latitude by north or south stars is

$$.6745 \sqrt{\frac{[e^2]}{n(n-1)}}$$

where  $n$  = number of north or south stars, and  $[e^2]$  = sum of squares of differences of all individual co-latitudes from the mean. The final value of co-latitude for a station is the mean of the two results by north and south stars respectively. If  $e_n$  = probable error of co-latitude by north stars only and  $e_s$  = probable error by south stars only, the probable error of the final mean value of co-latitude by all stars is

$$\sqrt{\left(\frac{e_n^2 + e_s^2}{4}\right)}.$$

Results by north and south stars will be often found to differ, occasionally considerably. If the difference is constant for stars of all zenith distances, it must be due to a personal peculiarity of the observer, which makes him intersect every star too high or too low: in this case the difference however large is immaterial, and does not affect the accuracy of the final mean value of co-latitude. If the difference *increases* with the zenith distance of the stars observed, it is due to some instrumental defect, probably flexure of the telescopic tube: in this case the difference is a very serious matter, the final resulting co-latitude cannot be accepted as trustworthy, and the instrument should be discarded from use.

In order to keep a watch on instrumental and personal peculiarities, the co-latitude of stations should be computed from observations taken only when the azimuthal stud was north, and compared with its value calculated from observations taken with the stud south.

If the two levels do not indicate the same inclination of the vertical axis, the co-latitude of stations should be taken out, 1stly, employing the inclination for every star as shewn by one level only, 2ndly, employing the inclination per star as shewn by the other level only; if the probable error of final co-latitude, calculated when the readings of but one level were utilised, is less than the probable error of co-latitude obtained by utilising both levels, it is evident that the second level should be discarded.

## PART VI.

## Tidal Observations.

## CHAPTER I. INTRODUCTORY REMARKS.

1. The immediate object of tidal observations is to determine the heights of the water, above some known datum mark at a station, for every instant of time during a more or less extended period. For this purpose, a self-registering tide-gauge is employed which exhibits the height of the tide in a graphical form by means of a pencil, driven by the rising and falling water with the help of suitable mechanical contrivances, marking a sheet of paper rolled round a drum driven by clock-work. The period during which the gauges are allowed to work is five years for minor stations, as this is considered sufficient to give a fair representation of the tidal oscillations at any place, and permanently at other stations or at least as long as the general tidal operations last, and certainly not less than nineteen years, 18·6 years being the period of a certain tide which is expected to give valuable information in regard to the rigidity of the earth. It is of the utmost importance that as few interruptions as possible may occur in the observations, and when they do occur that they may be of short duration; otherwise the method of interpolation employed in filling up the breaks, fails and a more complicated and less satisfactory one has to be adopted.

The results obtained are both of scientific and practical utility. With the scientific side of the subject we have little to do at present; but it may not be out of place to mention that a better knowledge of the laws of the tides is expected to lead to an evaluation of the mass of the moon, to more definite information regarding the rigidity of the earth, to an approximation to the depth of the sea from the observed velocities of the tide waves, and to information regarding the retardation of the earth's velocity due to tidal friction.

Practically, the correct determination of the tidal heights at any station, enables zeros of level to be fixed for the purposes of survey, and affords data for the calculation of the rise and fall of the tides at a future period and thus subserves the purposes of navigation.

2. To carry out the work properly the strength of the Tidal Party (excluding the Levelling Party) should be as follows:—

Strength of a tidal party.

- Two officers of the Senior Branch ;
- Two officers of the Junior Branch ;
- Two Native mechanics with a knowledge of clock-work ;
- Twelve sub-surveyors and computers ;
- One writer ;
- Twelve daftris, peons and khalasies.

## CHAPTER II. OBSERVATORY ARRANGEMENTS.

1. The choice of a site for the erection of a tide-gauge depends so much on local circumstances that a careful reconnaissance of the fore-shore is a necessary preliminary to the selection of the best of the generally limited number of suitable positions.

The gauge should be placed so as to obtain a fair representation of the tidal oscillations of the surrounding area, and to secure this it is necessary (1) that the sea should have direct communication with the gauge, and not approach it through tortuous channels ; (2) that the spot chosen should be sheltered from heavy weather, and (3) that there should be deep water at low-tide close to it. For example, a good position would be the end of a pier or jetty, or the wall of a dock. It must, however, be pointed out that a position in a cove or in a minor bay at the head of a large bay, though it may apparently answer the above requirements, is not a good one for a tidal observatory, as experience shows that, at stations where the range is small (as in the south of India) the tidal curves in such a spot often present a zig-zag appearance all along the rise and fall. The irregularities are certainly not caused by rough or lumpy water ; because it has frequently been noticed that they were being registered inside the float cylinder, at times when the surface of the water outside was perfectly smooth and no swell or ripple was apparent to the eye. There seems to be a slow throbbing or pulsatory action going on in such localities, during both rise and fall, which the eye does not readily detect ; for instance, during a rising tide the recording pencil will remain stationary, sometimes for nearly

five minutes, and then gradually fall for two or three minutes to an extent representing 2 or 3 inches in actual fall of tide, then again remain stationary for a few minutes, and afterwards move up on the rise. This will be repeated at intervals during the entire rising of the tide, and the same thing will recur in reversed order during the fall of the tide. In tidal rivers no such peculiarities have as yet been met with.

When a station has been selected near deep water, a vertical cylinder is fixed in the water in such a way as to admit it only through holes small enough to annul wave-motion and large enough to cause no sensible retardation of its rise and fall in the cylinder.

When an observatory has to be established at a place where such an adjunct as a pier, a jetty, or a dock-wall is not available, and cannot be specially constructed in deep water except at a prohibitory cost, it must be erected on the shore, the cylinder sunk in the ground, or in a masonry well and connected with the sea by piping, so as to maintain the level of the water to correspond at all times with that of the sea.

As an index to the method to be employed in determining the effect of a pipe connection, the subjoined example may be found useful.

Calculations to show the relative level of the water inside a cylinder of 22 inches diameter and that of the sea, the connection being made with a 2-inch pipe, 300 feet long, which has two bends of 90° each.

From Beardmore's Hydraulics, Table 8.

$$\text{The discharge for a 2-inch pipe} = \sqrt{\frac{26 \cdot 69}{\frac{l}{h}}} \text{ cubic feet per minute,}$$

where

$l$  = the length of the pipe,

$h$  = the head of water.

Now, assuming that the water in the sea has risen 1 inch higher than in the well, there is a head of 1 inch, and

$$\sqrt{\frac{l}{h}} = \sqrt{\frac{300 \times 12}{1}} = 60.$$

Therefore, the discharge = 768 cubic inches per minute, supposing that there are no bends in the pipe.



Now the quantity of water required to raise the level in the cylinder 1 inch is  $\pi r^2$  or  $11^2 \times \frac{22}{7}$  cubic inches nearly, *i.e.*, about 380 cubic inches.

Thus, a head of 1 inch would discharge the quantity required in something less than half a minute.

But when there are bends in the pipe, the head required to overcome them varies as the angles of the bends and the velocity of the water through the pipe.

The area of the connecting pipe in section =  $\pi r^2 = \frac{22}{7}$  square inches nearly.

Therefore the length of pipe which would contain a volume of water enough to raise the water in the cylinder 1 inch

$$= \frac{380}{\frac{22}{7}} \text{ inches} = 10 \text{ feet nearly,}$$

and if this amount is to be discharged into the cylinder in half a minute, the velocity of the water must be 20 feet per minute, diminishing as the head of water decreases.

Now by Beardmore's Hydraulics, Table 9, for a mean velocity of 20 feet per minute, and two bends of  $90^\circ$  each, we require a head of  $.0048 \times 2$  inches or  $\frac{1}{100}$  inch very nearly, so that the bends need hardly be taken into account.

Now suppose that there is an 18 feet rise and fall of tide at springs, *i.e.*, a mean of about 3 feet per hour, then the water will rise on an average  $\frac{3}{10}$  of an inch in  $\frac{1}{2}$  minute.

Combining the above arguments, it follows that if the sea-level was an inch higher than the water in the cylinder, the latter would be raised this height in  $\frac{1}{2}$  minute, by which time the sea would be  $\frac{3}{10}$  of an inch higher.

But the tide rising increases the head, and the water would tend on that account to flow quicker: also the sea-level would only be raised 1 inch in  $1\frac{2}{3}$  minutes; so that the differences of level under the conditions specified may be considered as inappreciable.

2. To facilitate the detailed description of the self-registering tide-gauge, it will be best to begin with a general account of it.

Description of instruments.

3. Mention has before been made of the 'cylinder' placed in communication with the sea, either directly or by a pipe connection, its use being to render the water still and undisturbed.

General outline.

Resting on the water in the cylinder is a 'float' which rises and falls with it, and to it is attached a copper 'band' which passes over a wheel called the 'stud-wheel'.

The rise and fall of the 'float' communicates motion to the 'stud-wheel' by means of the 'band,' and the 'stud-wheel' in turn, by means of a projecting axle on which is fastened a 'toothed-wheel,' communicates motion to another 'toothed-wheel'. On the same axis as the latter, and consequently moving with it, is another wheel round which a flexible 'chain' is passed, one end of which is attached to the wheel and the other to the 'pencil'. The 'chain' is kept taut by a 'counterpoise weight' to ensure the 'pencil' following the movements of the 'float'.

The 'pencil' moves longitudinally along a cylindrical 'drum' touching the surface with its point; the 'drum' revolves once in 24 hours by means of 'clock-work' at the opposite end to the float. The 'drum' is supported on a cast-iron 'bed-plate,' and the whole instrument on wooden trestles.

The instruments in use in India are of Newman's pattern, their distinguishing characteristic being very long drums whereby the curves are delineated on comparatively large scales, alterable at will. The instruments were originally made by the late Mr. P. Adie, but improvements were effected from time to time, and their construction is now entrusted to Messrs. A. L  g   & Co., London.

Having thus disposed of the apparatus as a whole, a detailed account of each part will now be given.

4. The size of the cylinder varies at different places, but it is generally 24 inches in internal diameter, and is usually made of thin iron plate, in sections of from 4 to 8 or even 10 feet in length, with angle-iron flanges at each end for bolting the lengths together: the bottom of the cylinder is ordinarily closed with an iron plate, while the top reaches to the floor of the observatory and sometimes to one or two feet above it.

In some places where durable wood can be obtained cheap, stout wooden boxes are substituted for the iron cylinder. They are 24 inches square in internal section: the bottom is closed with a wooden grating, which does not rest on the ground but is at a depth of some 4 or 5 feet below the lowest tides, the box being supported by the piles of the pier or wharf on which the observatory stands.

5. The easiest and best inlet for the water to the cylinder is through  
 a number of holes  $\frac{1}{4}$  to  $\frac{1}{2}$  an inch in diameter,  
 near the bottom of the cylinder, below the lowest  
 level of spring-tides, but at the same time well clear of the ground  
 on which the cylinder rests: if however the cylinder does not rest  
 on the ground, the bottom of the cylinder is the best place for the  
 holes.

Inlet for the water.

When the cylinder cannot be fixed in deep water, a pipe connection between the sea and the cylinder is necessary. The piping used is ordinary gas-pipe, in lengths of about 14 to 22 feet, with an internal diameter of 2 or 3 inches. In order to render the connections perfectly air-tight as well as to facilitate the joining together, the lengths of piping are fitted with cast-iron flanges made to screw on each end.

The piping is connected with the cylinder, at about 9 inches above the bottom, by a small bend and is then brought up vertically outside the cylinder to a height of one or two feet below the lowest high-water. At this point there is another bend fitted with a stop-cock, for a purpose explained hereafter, from which the pipe is taken straight down to the sea along the slope of the shore to the low-water line. Here lengths of flexible suction-pipe are joined on and taken out to deep water. This piping is a helix of copper wire coated with india-rubber and canvas, and 2 inches in diameter. It is made in lengths of 50 or 60 feet and fitted with couplings and means of connection. At the end of the outer length, a cylindrical copper rose of about 15 inches in length and 2 inches in diameter, and having some 150 holes of  $\frac{3}{10}$  of an inch bored in it, is screwed on. The rose is supported above the ground on a tripod fixed in the ground in deep water. Sometimes a small basket is made to answer the purpose of the copper rose.

The rigid and flexible pipes are connected as follows:—To the end of the rigid pipe a brass connecting piece is fitted. It is T-shaped and to either of its two outer extremities a length of the flexible pipe may be fixed, the other being closed with a brass disc with a good washer. When the flexible pipe has to be removed for cleaning, the brass disc is unscrewed, a length of spare flexible piping with a rose at its end attached and taken out to deep water temporarily: the original pipe is then taken off and cleaned, the disc being screwed on for the time in its place; then, when finished, the pipe and the disc are replaced in their original positions.

In connections of this sort, there is a decided tendency for air to collect in the piping and this causes a retardation of the flow of water into and out of the cylinder. It is to remedy this defect that the stop-cock, alluded to above, is placed at the upper bend of the rigid pipe. As this stop-cock is below high-water mark it is clear that, if it be opened at the time of any high-water, the air will be expelled and the levels of the water inside and outside the cylinder become identical. It is occasionally found necessary to keep the stop-cock open for 4 or 5 hours while the water is above the height of the cock.

6. The float is a cylindrical copper vessel 1 foot in diameter and 9 inches deep, and is of such a density that it will just sink if unsupported. The band is a copper ribbon, about 1 inch wide, perforated with holes about  $2\frac{1}{2}$  inches apart.

The float band and stud-wheel.

The stud-wheel is of brass, about  $9\frac{1}{2}$  inches in diameter, with a rim an inch wide: it has studs of the same diameter as the holes in the band, placed in the rim at intervals also of about  $2\frac{1}{2}$  inches, so that when the band is passed round the wheel the studs exactly fit into the holes, thus ensuring the revolution of the wheel as the float rises and falls. The band is attached at one end to the float, by means of two milled-headed screws passing through holes at the end of the band into a disc revolving underneath small rollers fixed on the top of the float: the arrangement forms a swivel and prevents the band being twisted. The band is cut to such a length that it passes over the stud-wheel and about 6 feet beyond when the float is in its lowest position in the cylinder. To the end of the band, as a counterpoise to the float, a weight is attached, and from its bottom a copper chain is suspended which theoretically should be equal in weight, length for length, to the copper band. The other end of the chain is attached to a hook below the float so as to form with the band a sort of endless chain passing over the stud-wheel and reaching to the bottom of the cylinder. This contrivance is introduced in order that the pull on the float shall be constant, otherwise a systematic error is introduced between rising and falling water. The counterpoise weight should be such as to give a decided preponderance, of say 3 or 4 lbs, on the float side; but when once adjusted it should not be altered, without noting the fact in the inspection book. When the whole system of float, band and counterpoise weight is hanging in position in the cylinder, there should be 3 inches space between the float

and cylinder on the one side and the counterpoise weight and cylinder on the other. The stud-wheel is supported on two uprights fixed to the bed-plate: its axle is about 8 inches long and carries at its other end a toothed-wheel which is in gearing with another, this latter being fixed on the axle of the wheel over which the chain connected to the pencil passes: this axle is fastened to one of the uprights which support the drum.

7. The wheel round which the chain winds is about 5 inches in diameter and is grooved for its reception, so that the chain lies on the wheel and does not overlap itself.

The chain wheel.

8. The toothed-wheels are constructed in couples so as to enable the working scale of the tidal diagram to be varied at pleasure, from the natural or full scale to that of  $\frac{1}{8}$ th, according to the range of the tide. Six couples are supplied with each gauge, giving scales of  $\frac{1}{1}$ ,  $\frac{1}{2}$ ,  $\frac{1}{3}$ ,  $\frac{1}{4}$ ,  $\frac{1}{6}$  and  $\frac{1}{8}$ ; and as the drum is 5 feet long any tide whose range does not exceed 30 feet can be safely registered. At Bhávnagar where the range is over 36 feet a  $\frac{1}{10}$  scale has been employed. In practice the couple is selected which enables the tidal curves to be exhibited on the largest scale possible.

The toothed-wheels.

9. The bed-plate is of cast-iron about 7 feet long, 1 foot broad and  $\frac{3}{4}$  of an inch thick, the upper surface being carefully planed. Underneath this plate and cast in the same piece with it, is a web 4 inches high which extends to within an inch of the edge of the upper plate both at the sides and at the ends, and has diagonals or stiffeners.

The bed-plate.

10. At about 10 inches from each end of the bed-plate is a brass upright, and on each of these a pair of friction-rollers is fixed, the pins on which the rollers turn being screwed into the uprights some 5 inches above the bed-plate.

The friction-rollers.

11. The drum which is 5 feet 3 inches long and exactly 24 inches in circumference revolves between the uprights: it is composed of sheet-brass and is made as nearly as possible a true circular cylinder. Axles project from each end and rest on the friction-rollers: one axle is elongated and passing through an oblique slot in the upright carries a toothed-wheel which

The drum.

gears with the driving clock. The position of the drum is horizontal and it carries the paper on which the tidal curve is registered.

Two grooves about  $\frac{1}{16}$  of an inch deep are cut round the drum near each end and exactly 5 feet apart and a third is cut midway between the two first. The groove at the clock-end is generally adopted as the *zero-line* of the gauge. When the paper is fixed on the drum, the zero, middle and end lines can be indicated on it by rubbing over the grooves with a hard pencil. The paper is nearly 5 feet 3 inches long and extends well beyond the extreme grooves.

12. Parallel to the drum and fixed above it to the brass uprights are two bars of solid brass drawn to angle shape and between them moves a slide carrying the pencil-holder in such a position that the pencil is exactly over the centre line of the drum. The pencil-holder slide which is a double T-shape moves along the bars and is pushed towards one of them by means of a spring so that it has no lateral motion. The bars are prevented from buckling or having lateral motion by two arched stiffeners which are screwed on to the outer sides of the bars and allow the pencil-holder slide to pass through them.

Arrangements for the pencil.

13. To each end of the upper plate of the pencil-slide, loops of wire are fixed: to the one of these nearest the float, a flexible chain is fastened and carried from thence round the chain-wheel, to which the end of it is made fast. The loop itself forms a swivel on a cylindrical capstan-headed screw which works into the upper plate of the slide and by means of which the pencil can be set exactly to the zero of the gauge for height. To the other loop, a silver wire or catgut-cord is tied which passes over a rimmed pulley at the top of the driving clock and has a weight of about 5 lbs attached to its other end. As the pencil-slide moves between the bars, the weight rises and falls and a sufficient space must be allowed for its drop.

The chain and counterpoise weight.

14. The pencil-holder is a small tube which screws into the slide and is adjusted so that it almost touches the paper on the drum. It is made to contain common leads which are pressed down on the paper from above by a weight of two or three ounces.

The pencil-holder.

15. The driving clock is a 'regulator clock' with English lever escapement (gold hair-spring). The movements are boxed in by movable brass slides and the oil-cups are protected by bushes. The drum is driven by a toothed-wheel of the clock gearing with another on the axle of the drum. The arrangement for connecting and disconnecting the drum and the clock is as follows:—A clamping screw with a milled head is connected with an interior arbor so that when the screw is clamped the driving toothed-wheel of the clock is not movable on the arbor, but when the screw is released the wheel can turn freely and thus permit the drum to be placed in any position required. The drum can thus be set so that the pencil shows the correct time on the diagram and the clamping of the screw then brings the two toothed-wheels into connection with the rest of the wheel work of the clock.

The clock.

16. In order to prevent any back-lash which may exist between the gearing of the clock and the drum, a cord carrying a weight of about 5 lbs, is attached to and encircles a barrel on the axle of the drum and passes over a pulley on the bed-plate. This barrel carries a pawl which drops into a ratchet-wheel on the drum: it also carries a crown-wheel which gears with a bevel-pinion. This pinion turns freely in a socket fastened to the upright which supports the clock end of the drum. The outer end of the pinion is square so that the key which winds the clock fits it. In winding the pinion the barrel is also turned, thus winding up the weight which prevents the back-lash, while any backward motion is stopped by the pawl and ratchet on the drum.

The back-lash weight.

17. As the drum for the diagram may not be quite circular the position of the pencil, as the clock shows the exact hour at four different times of the day, is marked on the diagram which is afterwards re-divided in accordance with these marks.

Ellipticity of the drum.

18. The tide-gauge is set up on a wooden trestle the top of which is 5 feet long, 1 foot broad and 2 inches thick: the legs are splayed and firmly braced. The trestle is placed longitudinally in the observatory and touches the top of the float cylinder at one end.

Trestle.

19. No system of tidal observations can be considered complete which does not contain a continuous record of the atmospheric condition at the station. Consequently every tidal station (with the exception of Bombay where the information is supplied from Colaba) is furnished with a self-registering aneroid barometer to measure the momentary variations of pressure, and a standard mercurial barometer to check the aneroid from time to time and enable its index error to be determined; also with a self-registering anemometer indicating the velocity and direction of the wind at every moment, a maximum and a minimum thermometer and a rain-gauge.

20. In the self-registering aneroid barometer there are seven vacuum-chambers or boxes coupled together; the top one is attached to a screw used for setting the metallic registering pointer and to the lowest is fixed a fork with hardened steel knife-bearings. On these bearings rests, by means of knife-edges, a lever which connects the balancing-spring with the vacuum-boxes, being pivoted on other knife-edges midway between the attaching points. The vacuum-boxes and balancing-spring are placed on a brass frame.

The balancing-spring is a spiral one hooked at the bottom to the lower edge of the lever and at the top to a screw working in the upper part of the frame, by which its pull in connection with the lever is so adjusted that the reading of the instrument corresponds to that of the mercurial barometer under the particular atmospheric conditions existing at the time of its first adjustment. This adjustment, made by the maker, ought not to be altered unless the instrument has to be taken to pieces.

The amount of motion produced by the variation of the atmospheric pressure on the boxes is multiplied by the lever above mentioned and then again by a second lever which is supported on two uprights and counterpoised. These two levers are connected together by a steel rod pointed at both ends and pivoted in conical holes out of which it is prevented from slipping by means of forks.

Projecting from the clock is a third lever of the same length as the second and attached to it by a joint whose length is half the height of the recording barrel. The joint is movable and at its centre a



metallic pointer is fixed for the purpose of marking on the prepared paper on the barrel; the pointer can be adjusted to press more or less heavily as required. The mode of marking is as follows:—Attached to the back of the third lever there is a sliding-piece which is pulled by the clock movement three times per hour and by this motion the joint is twisted and the pointer pressed against the paper.

To the brass frame on which the vacuum-boxes and balancing-spring are placed there is fixed a steel tube and on this the revolving drum pivots, being maintained in position by a nut screwing in the top of the tube. The tube is hollow to admit of a turnscrew being inserted to set the recording pointer to agree with the mercurial barometer. At the bottom of the revolving drum there is a toothed crown-wheel which gears with a pinion driven by the clock. The drum revolves in  $8\frac{1}{2}$  days. The recording barrel on which the specially prepared paper is fixed rests by its own weight on the revolving drum and has a knob on the top with a hole through it of the same size as the hollow in the steel tube. The several parts are fitted on a substantial brass plate, about 21 inches long and 6 inches wide, screwed on to a board an inch thick and the whole instrument is fixed in a neat case with a glass front, the top and front being made to open on hinges.

21. The self-registering anemometer is fitted with Robinson's cups and steering-vanes and is about half the size of Beckley's standard anemometer. A special long protecting tube is fixed on the cross-bars carrying the cups to prevent dust or rain blowing into the bearings. The steering-vanes are made as large as possible with a sharper angle than usual and the screw working the wheel of the indicating shaft has a quicker thread so as to make the motion of the vane still more rapid.

The recording instrument has larger crown-wheels than usual so as to produce an easier gearing. The driving-barrel of the clock movement is drilled through its centre and a shaft passed through the hole having at one end a pinion gearing with the reading barrel and at the other, in front of the clock, a milled-headed screw. This arrangement allows the registering drum to be easily set, for by loosening the screw the shaft becomes free and after the drum is set to correct time, the clock connection is made by clamping the screw.

To prevent the back-lash of the recording barrel, the pinion on the centre spindle of the driving-barrel of the clock in gearing with the recording barrel is double; one part of this pinion is fixed on to the spindle and the other is loose but attached to the fixed part by means of a circular wire spring and this being pressed back when gearing the double pinion causes the tooth of the recording drum, which is in gear, to be clamped on both sides and thus prevents lost motion.

The clock has a good lever escapement with gold hair-spring and is driven by a weight; a dial showing minutes is fitted on to it.

The recording is done by the usual spiral metallic pencil marking on especially prepared paper.

Sometimes it is necessary to place the anemometer at a distance from the tidal observatory in order to obtain a site freely exposed to the wind from every direction.

22. A tidal observatory is constructed of wood and is usually so made as to be readily taken to pieces and put together again in order to be removed and re-erected.

The observatory.

It is about 12 feet by 9 feet in plan and 12 feet high in the middle up to the ridge, from which the roof slopes down to the sides which are 8 feet high.

23. Outside the observatory and attached to a pile of the pier or to the wharf wall, a graduated staff is fixed vertically, in such a position as to be easily read, so that a comparison of the level of the water outside and inside the cylinder may be readily made by simultaneous readings of the pencil on the diagram and of the water on the graduated staff.

The graduated staff.

24. Two or three bench-marks are laid down in the vicinity of every tidal observatory and connected with the bed-plate by first-class levelling described afterwards in Part VII. They are either cut in the dock-wall, or on the steps of some neighbouring building, or are cubical blocks of masonry, about  $3\frac{1}{2}$  feet each way, containing a large stone imbedded on the upper face. The stone is inscribed to show that it is a bench-mark of the Great

Bench-marks.

Trigonometrical Survey, and the year on which it was laid down is also given. In the centre of this stone, a square depression of 5-inch side and  $\frac{1}{4}$  inch deep is cut and nicely smoothed, its size being just sufficient to allow the levelling-staff to turn freely in it.

25. The trestle is first put in position, longitudinally in the observatory and touching the float cylinder at one end. The tide-gauge is next set up on the trestle, in such a position that the centre of the float, the band and the counterpoise weight shall all be in a diametral plane of the float cylinder, and also so that the float and counterpoise weight shall each be about 3 inches from the sides of the cylinder. The trestle is then wedged up so that its top is nearly level, and its legs are firmly screwed to the floor. After this the bed-plate is levelled, longitudinally and transversely, by driving wedges in between the web and the top of the trestle; and the various parts of the instrument are tested, to see that they work freely and that there is enough drop, at extreme tides, for the counterpoise weight attached to the pencil traveller.

To set up and start the self-registering tide-gauge.

The wheels to govern the scale of the diagram are now placed in gear with the float-wheel, a trial diagram put on, and the instrument approximately adjusted so that at half-tide the pencil will be at the centre of the drum, the extreme rise and fall being roughly known beforehand. [Sometimes the zero of the gauge, hereafter defined, is made to correspond with some particular level which has been taken as the datum for local surveys and the instrument is adjusted accordingly. For example, when soundings are being taken in the vicinity and the times are noted, the gauge readings may be made to indicate the amounts to be subtracted from the soundings to find the distance of the surface of the water above the datum.]

Whatever be the approximate adjustments, careful measurements must next be taken to determine the distance of the water below the surface of the bed-plate when the pencil is on the zero of the gauge. In making these measurements, called 'zero measurements', a special apparatus is employed.

26. A flat strip of brass with a right-angled band is fixed by two or three countersunk screws to the top of a box-wood scale, divided into tenths, hundredths and

Zero measurements.

five-hundredths of a foot, so that when the flat piece of brass lies on the top of the bed-plate, the scale hangs vertically down; care is taken to have the under surface of the flat piece, which rests on the top of the bed-plate, corresponding exactly to the zero mark of the box-wood scale.

A small circular wooden disc, 3 inches in diameter and  $\frac{1}{2}$  an inch thick, bevelled at its edge from the bottom towards the top, is attached by means of a brass slip fixed on the top of the disc, to the end of a Chesterman's tape. The slip is made of two brass plates, about  $2\frac{1}{2}$  inches long and 1 inch broad, one fixed vertically nearly at the centre of the disc and the other attached to the first by four screws at its corners, so as to be removed at pleasure. The tape is held between the plates, and, when the screws are clamped, the disc is suspended by the tape; there is a line drawn across the tape, so as to show exactly how it is to be placed relatively to the top of the brass clip. The top of the disc is slightly loaded with lead, so that, when hanging from the tape, its under surface is quite horizontal. The distance from the bottom surface of the disc to the top of the clip is exactly 3 inches. The tape itself is now marked at every foot from 5 feet to 30 feet, by applying it to a standard levelling staff, the disc arrangement being held suspended all the time, so as to have the tape in tension under the same condition as it would be when in use.

The measurements are taken as follows:—The box-wood scale is suspended from the bed-plate above the float-cylinder, and the disc is lowered into the cylinder, care being taken to keep the tape close to the side of the scale; this is done by running off more tape than is required, passing it across the bed-plate and holding the tape on the top so that it can be paid out easily and in very minute increments of length. When the disc is seen to be close to the surface of the water, warning is given to the clerk who is standing by, ready to mark the exact position of the pencil on the barrel. The lowering is continued very carefully, and actual contact with the water is noticed by the disc causing a tremor on the surface; at the moment of contact a signal is made to the clerk who marks on the diagram the position of the centre of the lead pencil, and the distance in feet is read off the tape while the tenths, hundredths and thousandths are read off the scale. This distance is entered in the inspection book in one column: the measurement of the position of the pencil above the zero-cut on the

drum is then carefully made, by using another scale, and entered in another column. The latter entry is multiplied by the denominator of the fraction indicating the working scale of the gauge and entered in a third column. The sum of the entries in the first and third columns gives a value of the distance of the water from the bed-plate with the pencil on the zero-line, or, in other words, the distance of the zero of the gauge from the bed-plate.

As a rule, twenty such measurements are made and the mean taken. The measurements have to be taken both during a rising and a falling tide (when it is well on the rise or fall) and the mean of the two sets is the adopted value of the zero below bed-plate: this eliminates the influence of the lost motion or back-lash between the two toothed-wheels connecting the stud-wheel and chain-wheel. If the value of the distance of the zero below bed-plate agrees with the true zero, previously fixed upon in regard to the datum, there is nothing more to be done; but if not, the chain must be lengthened or shortened by means of the milled-headed screw until the agreement is complete; the gauge may then be started and left to itself.

27. For each observatory there is a clerk provided, who is generally one of the writers in the Port Office and receives a small increase of pay for the observatory duties; but in some cases a special man has to be engaged. Printed instructions are given to him concerning his work which should be carried out as follows:—

28. The observatory should be visited each day at 7 and 10 A.M., and at 4 and 6 P.M., except on Sundays when two visits are considered sufficient; and also twice a month at some other hour to change the diagram.

29. The tide-gauge clock must be wound up twice a week and the back-lash weight every evening. The positions of the pencil on the barrel should be marked by a circle of ink round the pencil, on each visit to the observatory, at the exact hour and the date of the month written alongside, the preceding day's curve should also be inked in with one of the coloured inks supplied. Simultaneous readings of the position of the pencil on the diagram and of certain white lines marked on the float-band should be taken once daily

and entered in the report, so as to make sure that the band has not been displaced. The heights of the tidal curve for each hour of the preceding 24 hours must be carefully measured off the diagram and entered in the report.

30. The clock of the tide-gauge must be compared daily with the gun or time-ball, or with a watch previously taken to the telegraph office, where Madras time can be correctly obtained at 4 P.M., and local time deduced by applying a correction for longitude: the error of the clock should be noted in the report with a mark as to whether it has been corrected or allowed to stand. In setting the clock, supposing it is  $1^m 40^s$  fast, the button on its face should be pulled out when the second hand is exactly at the 60 and released at the end of  $1^m 40^s$ ; but if it is slow, the minute hand should be turned forward  $2^m$  and the clock stopped as before for  $20^s$ . Any stoppage of the clock must be noted on the diagram and also in the report. The reading of the pencil on the diagram and the height of the water at the same moment on the graduated staff (if there is one) should be taken once a day and entered in the report. If there is an unmistakable difference of two-tenths of a foot when the water is calm, the cylinder should be flushed out and the communication holes examined. The diagram must have the zero, mid and end lines marked by rubbing a hard pencil over the grooves cut in the barrel at these points. A line about half an inch long should be rubbed at the hours of 10 A.M., 2 P.M. and midnight in a part of the paper not marked by any curve, if possible. The marking should be done on the day after the diagram is put on and again on the day it is taken off, and the date entered against each set of marks.

31. The diagram must be changed once a fortnight, generally every second Monday, when the tide has well turned so as to make sure of getting the highest and lowest tides. The change is made as follows:—the new diagram is numbered, dated, and has a narrow slip one inch long cut out in readiness for setting the zero-line: it is damped all over with clean water and paste applied to the overlap. The hour and date that the work was stopped is noted in ink on the old diagram and the pencil-holder is taken off. The diagram on the barrel is next cut carefully along the 12 o'clock line, the back-lash weight removed, the clips unscrewed and the diagram taken off, carefully rolled up and put

aside. The barrel and the clock are then disconnected and the new diagram put on the barrel, by first making the 12 o'clock line of the paper agree with that marked on the barrel and the zero of the diagram with that of the barrel; this can be done by means of the small slits that have been cut out of the diagram. The clips are now screwed down and the barrel turned round by hand till the outer edge of the diagram comes in contact with the 12 o'clock line and the height lines meet. The pencil-holder must then be fixed and the clock and barrel clamped, care being taken that the pencil is over that part of the diagram which corresponds, as nearly as possible in time, with the time of the clock. The back-lash weight is carefully and slowly put on and the hour of commencing work noted on the diagram. It only remains to regulate the position exactly for time. This is done by selecting any convenient hour, say noon, and at one minute before unclamping the clock-barrel connecting-screw and, when the second hand shows the complete hour exactly, bringing the centre of the pencil exactly over the hour-line of the diagram and clamping very firmly, otherwise the clock may fail to drive the barrel.

32. Any remarks regarding the stoppage of the clock, or in fact anything unusual, must be noted on the diagram and in the daily report. If the clock should stop, the weight must be removed from the pencil-cup and the pencil slightly raised; the clock-barrel connecting-screw must then be unclamped, the barrel, being held so that the back-lash weight does not run down, revolved by hand so as to bring the pencil over that part of the diagram which corresponds as nearly as possible to about 5 minutes in advance of the aneroid clock or reference watch time, and the pencil weight replaced: the clock must then be started and stopped again; when it shows the first exact hour after starting the pencil should be brought exactly over the hour-line, the clock and the barrel firmly clamped and the clock re-started when the exact hour is shown by the watch.

33. If by any chance the band should come off the stud-wheel, it should be replaced carefully by turning the wheel until the pencil is on the zero-line of the diagram and fitting the hole of the band marked with paint on the stud similarly marked; as the marks are made when the zero-line marked on the float-band is brought to agree precisely with the bed-plate, on the wheel being released, the pencil will assume its proper position.

Stoppage of clock.

Displacement of band.

34. Should the chain between the pencil-traveller and the float end break, the pencil-holder must be removed from the traveller, the counterpoise weight detached and the two pieces of the chain taken out and re-riveted. The ends must then be attached to the traveller and the sheave as before, and the stud-wheel turned by hand till the 2.5 line painted on the band is on a level with the bed-plate. If the deviation of the pencil from the 2.5 line on the barrel is small, it can be set right by the adjusting screw attached to the pencil-holder; but if the deviation is more than the screw admits of, one of the change wheels will have to be taken off, turned one or two cogs and refixed, the final adjustment being made by the screw attached to the holder.

Breakage of chain or wire.

If a silver wire is in use instead of a chain, 6 feet 6 inches of new wire should be taken, one end inserted through the hole in the rim of the sheave and tied firmly with a stop-knot. The wire should then be wound round the groove of the sheave and the zero of the band held in position agreeing with the plane of the bed-plate. The pencil-holder should now be brought exactly over the zero-line of barrel and the other end of the wire securely fixed through the stud-loop on the traveller. The traveller should then be released, the float-wheel being still held so that the zero on the band is in position, in order to ascertain if the counterpoise weight will allow the pencil to be over the zero of the barrel; if not, it must be adjusted until it is so. In carrying out this, great care must be taken that the band does not kink.

35. The stop-cock should be opened at some time every day, at high water or as nearly so as convenient; but on no account must it be opened if the level of the water is nearly the same as the height of the stop-cock.

Stop-cock.

36. The aneroid and mercurial barometers and the thermometers attached thereto should be read daily at 7 and 10 A.M., and at 4 and 6 P.M.

Aneroid barometer.

Should the clock of the aneroid stop, the hand must be gently turned round till it points to the proper time as shown by the tide-gauge clock. The barrel of the diagram must then be turned until the pencil points to the proper time, but in doing this great care must



be taken, otherwise the gold thread in Adie's pattern or the marker in Légé's may be broken. In the former, it should not be attempted if the clock hands are between 5 minutes to an hour and 10 minutes past the hour as the pencil marker suspended to the gold thread is, at these times, either pressing or close to the barrel; and in the latter, it may be best done at 5 minutes past a full hour.

The aneroid clock must be wound up every Monday morning and may be regulated by stopping for a few minutes if fast or pushing the minute hand forward if slow; but for one or two minutes' error it need not be altered.

The aneroid diagrams should be carefully numbered and changed every Monday morning, and the sheets should be carefully inked as they are taken off and put away.

37. The anemometer clock must be wound up every morning by pulling the cord with the small weight and thus raising the heavy weight close to the bed-plate.

Anemometer.

The anemometer diagram must be changed daily at 7 A.M. and the diagrams dated and numbered, the hour being recorded as put on at such a time and taken off at such a time. The number of miles of wind for the last 24 hours must be entered in the report, and is obtained by counting the number of velocity lines and multiplying by 10. The diagrams must be carefully inked in daily.

The instruments should be all oiled occasionally; and in the case of the anemometer, if the direction of the wind has been steadily from one point for many days without altering, as in the S.W. monsoon, the fans of the direction gear should be turned with the hand until the vane has made one or two complete revolutions.

38. The daily reports must be made up in duplicate and one copy sent by post to the head office. Anything unusual must be marked on the diagram and noted on the back of the report, and if anything emergent is required to be done, the port officer must telegraph to the officer in charge of the tidal party.

Reports.

39. As a rule an inspection is made once a year, but sometimes oftener. Of course if any interruption has taken place, such as the removal of instruments by the port officer for safety on account of a cyclone, as has occurred more than once, or for the settlement of the observatory, thus necessitating a temporary suspension of the observations pending repairs, then an inspection should be made as soon as possible after the information has been received.

Inspection of a tidal observatory. General remarks.

When it is intended to make an inspection, the first thing to be done is to test the accuracy of the one-foot graduations of the Chesterman's tape with which measurements for determination of working zero will be made, and which have been marked on the tape in the way described at page 195. The tape should not be used continuously for more than a couple of months without being retested and, if necessary, corrected.

The inspecting officer ought to carry in his inspection box, a copy of this handbook and of the current tide tables for Indian ports, and the necessary scales, measuring tape, and other instruments required at an inspection.

The inspecting officer, accompanied by a mechanic to dismantle, clean, repair if necessary, and refit the instruments, attends to the following points when making an inspection; and at the time of inspection writes in the observatory inspection book, under appropriate heads, a report, a copy of which is forwarded to the head office. The usual heads of the report are:—

General remarks.

Bench-marks.

Details of levelling.

Self-registering tide-gauge.

Details of determination of working zero.

Auxiliary instruments.

The general remarks should contain an account of the working of all the instruments since the last inspection, and should draw attention to the manner in which the observatory clerk performs his duties, and to anything else requiring special notice.

40. On arrival at the tidal observatory, it is necessary in the first place to ascertain if any settlement of the tide-gauge has taken place, by connecting the float end of its bed-plate by spirit levelling of precision with the bench-mark of reference, which in its turn should be similarly connected with the other bench-marks, and with the graduated staff, in order to test the accuracy of the zero of the latter. The report should give both the results and the details of the levelling, and should mention the condition in which each bench-mark and the staff is found; it should also mention whether the bed-plate is level both longitudinally and transversely.

41. Then, a set of measurements for the determination of working zero at a rising tide and another at a falling tide, each set to consist generally of not less than 20 measurements, should be taken; their mean will give a value of the distance of the working zero below the bed-plate. This eliminates the influence of the lost motion, or back-lash, between the toothed-wheels connecting the stud-wheel with the sheave for the wire of the pencil-slide (page 196); it also cancels the error arising from looseness of the pencil in the pencil-holder. If it should be necessary to continue these measurements, another pair of sets should be taken before proceeding with the inspection. These measurements for determination of the working zero should be made when the tide is well on the rise and fall, and not when it is almost high or low water. The method of taking them is explained at page 195 and, in entering them in the inspection report, it should be stated that they were made *before cleaning* the gauge.

Concurrently with the measurements for the determination of the working zero, mentioned in the preceding paragraph, a comparison of the reading of the pencil on the drum with the reading of the bed-plate on the float-band, and with the reading of the graduated staff, should be made both at a rising and falling tide, and entered in the report.

Before cleaning the gauge, its clock should be compared with the telegraph or gun time by the inspecting officer, as a check on the previous recent comparisons entered by the observatory clerk in his daily reports.

42. The balance of the gauge ought also to be tested before beginning to clean the instrument. This may be done with sufficient accuracy by raising the float completely out of the water by gently turning the stud-wheel, and then taking the reading of a spring-balance hooked, for the purpose, to one of the holes of the band on the counterpoise side. The reading of the spring-balance will give the preponderance of the float. There ought to be a decided preponderance of, say, 3 or 4 lbs. on the float side, but the weight when once adjusted should require no alteration, as it would affect the value of the zero-line. Should the preponderance be found to have increased, it points to a probable flaw in the float sufficient to have admitted an influx of water.

After testing the balance of the gauge, and before cleaning it, the 2.5 painted line of reference on the band should be brought to the level of the bed-plate, when the pencil should be exactly on the 2.5 or mid-line of the drum (not diagram). If the pencil be found out of this position, the discrepancy should be measured and noted.

43. Then, the float and band should be raised into the observatory for examination and measurement, and the time noted. This is the first step in the dismantling of the gauge, preparatory to cleaning it. The total length of the band should be measured, also the distances from the painted 2.5 line and from the painted 0 line upon it to its junction with the float. During the inspection of the Bhávnagar tidal observatory in December 1887, it was found that the readings of the band and pencil agreed, but the measurements for determination of the working zero differed largely from what they ought to have been. The measurements and examination of the band disclosed that it had broken close to the float, and it was found that the observatory clerk had tried to conceal the breakage by attaching the float to the band at the place where the breakage had occurred. The float should be closely examined and, if any water is found in it, it should be repaired or renewed. In such a case it is interesting if the quantity of water which found its way into the float can be ascertained. The influx may have been sufficient to alter the balance of the instrument and raise the working zero. As an instance of this it was found that water had entered the float of the Dublat tide-gauge in November 1882, in sufficient quantity to raise the working zero 0.14 of a foot.

The dismantling of the gauge should now be completed, and it should be thoroughly cleaned and oiled where necessary; but the reference lines painted on the band and stud-wheel at the last inspection should not as yet be obliterated. It should be mentioned specially whether or not the driving clock required cleaning.

44. After the several parts of the gauge are cleaned, they should be refitted carefully, the band being replaced so that when its painted 2.5 line of reference is level with the bed-plate, the pencil shall be at, or close to, the 2.5 line on the drum. The working of the band on the stud-wheel, while the pencil is being moved along the drum from its zero to the highest line of the diagram, should be tested; and it is best if this can be done at low water so as to reduce as much as possible the chance of the band kinking. The working should be smooth, and each hole of the band should fit over the studs freely. Any hole found too tight may be enlarged slightly with a file.

If the gauge is fitted, between the pencil and the float, with a silver wire, (which it is hoped will soon be superseded in every gauge by the new chain), and if the wire has been in use for a whole year, it should be renewed in the manner described at page 199, the best time for renewing it being about low water as there is then least danger of causing a kink in the band. The method described on page 199, indicates sufficiently well how a chain should be fitted, but a renewal of the latter is not likely to be often required.

The bed-plate should be made level both longitudinally and transversely if necessary, and if this operation be found, by spirit-levelling, to have altered the level of the bed-plate relatively to that of the bench-mark of reference, the alteration of level should be recorded.

45. The refitted gauge being now clean, level, and connected by levelling with the bench-mark of reference, is still unadjusted. To ascertain the amount of adjustment required, measurements for determination of the working zero at rising and falling tides should now be taken and recorded as having been made *after cleaning* the gauge. If a combination of the results of these measurements with the final level of the bed-plate make the distance below the bench-mark of reference of the

working zero, thus obtained, to differ from the distance of the true zero below the same bench-mark by a quantity appreciable on the scale of the diagram, the position of the pencil must be adjusted until the working zero coincides with the true zero. It is usual to take one more pair of sets of measurements for determination of the working zero (which should be registered as having been made *after cleaning* the gauge) as a final test of the perfect adjustment of the instrument.

The gauge being in adjustment, the reference lines painted on the band at last inspection should be compared with the pencil readings on the drum, and if the former are found out of position, they may now be obliterated and new lines substituted for them, special care being taken in the painting of the streaks, the *upper edges* of which mark where the readings of the bed-plate on the band correspond with the readings of the pencil on the engraved lines of the drum.

46. The dismantling, cleaning, and refitting of the auxiliary instruments proceed hand in hand with the similar duties in connection with the self-registering tide-gauge. The auxiliary instruments are:—

- Standard mercurial barometer.
- Self-registering aneroid barometer.
- Maximum and minimum thermometer.
- Anemometer.
- Rain-gauge.

Of these only the self-registering aneroid barometer and the anemometer require dismantling, cleaning, and refitting, and these operations are carried out as described in the following paragraphs.

47. The aneroid should be compared with the mercurial barometer, and its clock should be rated.

The position of the pencil-marker on the diagram should be made to agree with the reading shown on the dial, and adjusted, if necessary, by the screw; the diagrams should be examined to see if the marker is working freely; if they show a straight line, *i.e.*, no rises at 10 o'clock nor depressions at 4 o'clock, then the marker will be moving stiffly and requires cleaning.

Thermometer comparisons should be made between that on the aneroid and that attached to the mercurial barometer.

The clerk should be made to read the aneroid and mercurial barometers and the thermometer, and to set the maximum and minimum thermometers.

The diagrams should be examined, and the clerk told if the inking in has been properly done or not: the supply of blank diagrams should be noted, to see that there are plenty for future work.

48. The direction of the vane with the wind, and the marking of the direction on the barrel should be tested.  
*Anemometer.* The upper part of the instrument should be oiled and the cups so marked that they cannot be put wrong.

The diagrams should be examined and the clock looked at, to see if new catgut or anything else is required. If the diagrams are faintly marked the bearing of the helices should be looked at—they should be quite free.

49. Before ending his inspection, the inspecting officer should see the observatory clerk make an accurate comparison of the tide-gauge clock (which, like all the other clocks in the observatory, must keep *local time*) with the telegraphic or gun time, and enter the comparison in the report. The clerk must also show that, in addition to being able to rate the clock, he knows how to bring it to correct time when it is fast or slow, according to his printed instructions. A comparison of the reading of the pencil on the drum with that of the band at the bed-plate, and with the level of the water on the graduated staff, should be made and recorded after the tide-gauge has been put into adjustment. The inspecting officer should see that a conspicuous note is contained in the observatory report book, for the information not only of the observatory clerk but of the local official appointed to supervise him and superintend the working of the observatory, to the effect that whenever any interruption in the working of the tide-gauge takes place, owing *e.g.* to the stoppage of the driving clock, hourly readings should be taken on the diagram by day and night during the interruption. If for any reason these readings cannot be taken, then hourly readings of the graduated staff (the zero of which should agree with that of the

*Miscellaneous duties before closing inspection.*

gauge) should be taken by day and night and entered in the daily reports. Should this amount of frequency be unattainable, then it is indispensable that readings at high and low water should be taken day and night and registered in the daily reports. If the cause of the interruption be of so serious a nature as to render necessary the removal of the instruments from the observatory, the promptest information should be sent to the officer in charge of the tidal operations to enable him to arrange for an inspection at the earliest possible date. The inspecting officer ends his inspection by taking a note of whatever diagrams, ink, books, pencils or other necessaries are required to be sent to the observatory.

### CHAPTER III. COMPUTATIONS.

1. The tidal diagrams are examined and prepared for reduction in the head-quarter office in the following manner:—Vertical lines in red ink are drawn through each set of the points which have been marked by the clerk of the observatory, showing the position of the pencil at the exact hours of 7 A.M., 10 A.M., 4 P.M., and 6 P.M., and these lines are the bases for drawing vertical lines at the intermediate hours of the day. The daily reports are next examined to see if there are any clock errors amounting to 3 minutes or more, as compared with telegraphic time or gun signal; if there are, then ( $\times$ 's) crosses in red ink are made *on* the tidal curves to show the exact position of each hour. The limit of 3 minutes error has been adopted because  $\frac{1}{20}$  of an inch is the smallest distance which can be conveniently and accurately laid down in measuring along the curve, and  $\frac{1}{20}$  inch = 3 minutes.

If the clock is fast, the cross is placed in *advance* of the hour-line; if slow, then *behind* the vertical time-line. Thus, suppose the clock 4 minutes *fast* the cross ( $\times$ ) is placed between the 2 and 3 P.M. lines at  $\frac{1}{15}$  of an inch from the 2 P.M. line; if, however, the clock was *slow* by 4 minutes, then the cross ( $\times$ ) is put between the 1 and 2 P.M. lines at  $\frac{1}{15}$  of an inch from the 2 P.M. line. As a rule, however, there is rarely any correction of this kind required, for when the clocks are properly attended to, errors of over 30 seconds are at once corrected by the clerks.

Interrupted curves or non-recorded curves caused by the stoppage of the clock, or other suspension of the tidal registration, are carefully



filled in by drawing a curve in dotted lines exactly between the two contiguous instrumental curves.

The zero-lines to which all the measurements for height are referred are now laid down as indicated in the rules below, in which the terms 'true zero,' 'working zero,' 'accepted value of true zero,' and 'adopted level of bed-plate' have the following meanings:—

2. The true zero is that which has been adopted in determining the datum-line for heights in the tide-tables.

True zero.

Its relative level with regard to the bench-mark of reference is fixed. As a rule, the zero corresponds to that originally adopted when the gauge was started, and its distance below the bed-plate was determined when the level of the bed-plate with regard to the bench-mark was fixed.

3. The working zero is the level of the water with reference to the bench-mark, corresponding to the pencil

Working zero.

being on the zero-groove cut on the drum. In starting the instrument the working zero of course corresponds to the true zero, but from various causes the instrument may get out of adjustment, and its working zero may be altered. The position of the working zero on the diagram is always marked by the clerk rubbing over the groove cut in the drum with a hard pencil. In general at an inspection, the working zero is made to agree with the true zero by adjusting the instrument.

4. The accepted value of the true zero is the distance of the true zero from the bed-plate, which was determined when

Accepted value of true zero.

the bed-plate was fixed as regards its relative level with the bench-mark of reference.

5. The adopted level of the bed-plate means the level of the bed-plate with reference to the bench-mark, which

Adopted level of bed-plate,

has been adopted in determining the true zero; as a rule this will correspond to the level obtained when the observations commenced.

6. The inspection book must first of all be examined to see if the bed-plate has altered in level relatively to the

Rules for fixing true zero on diagrams.

bench-mark. If there is any difference from

the adopted level exceeding  $\cdot 02$  of a foot, a correction will have to be applied on this account. The measurements for the determination of the *working zero* at the various inspections are next examined. If no alteration has been made in the adjustment of the gauge during the inspection, then the whole of the sets of measurements should be grouped, and the mean value would represent the distance of the working zero from the bed-plate on the day of the inspection.

If an adjustment has been made during an inspection, then those measurements for determination of zero before and after adjustment must be grouped separately, and the means respectively applied to the preceding and the following diagrams.

In treating the diagrams for any period between two inspections, the distance of the working zero from the bed-plate must be taken as the mean of the values obtained at the inspections.

The following are the cases which may occur and the ways of adjusting for them.

I. *Bed-plate settled below adopted level.*—The true zero will have to be placed above the working zero at a distance proportioned to the amount of the settlement in accordance with the scale of the diagram; hence the measurements from the true zero will be *less* than from the working zero.

II. *Bed-plate raised above adopted level.*—In this case the true zero will be placed below the working zero.

III. *Bed-plate unaltered and working zero at greater distance from bed-plate than accepted value for true zero.*—The true zero in this case will be placed above the working zero.

IV. *Bed-plate unaltered and working zero at less distance from bed-plate than accepted value for true zero.*—In this case the true zero will be placed below the working zero.

V. *Bed-plate settled and working zero at greater distance from bed-plate than true zero.*—In this case the true zero would be placed above the working zero at a distance equal to the sum of the corrections on account of each event.

VI. *Bed-plate settled and working zero at less distance from bed-plate than true zero.*—If the correction for settlement is the *greater of the two* the true zero will be placed *above* the working zero, and if the correction on account of the difference of zero-measurements was the greater then the true zero should be placed below the working zero. Obviously the amount in each case would be the difference of the two corrections.

VII. *Bed-plate raised and distance of working zero from bed-plate less than that accepted for true zero.*—In this case the true zero would be placed *below* the working zero at a distance equal to the sum of the two corrections.

VIII. *Bed-plate raised and distance of working zero from bed-plate greater, &c.*—The true zero should be placed *below* the working zero if the correction on account of the raising is the greater, and *above* if the latter correction is the greater. The distance between the two zeros is difference of the corrections.

N.B. If the determination of the true shows that the working zero comes within 0.005 of the true zero *on the diagram*, then no correction is considered necessary, and the working zero is used as the line of reference in measuring the heights from the diagram. What is meant by being within .005 *on the diagram* is the actual difference between the true and working zeros reduced to scale.

*Water getting into the float* would have the effect of making the working zero *nearer* the bed-plate than the value formerly obtained, and this would have to be treated under IV, VI, or VII, according as the bed-plate had remained unaltered, had settled, or had been raised.

*A kink in the band.*—If this occurred, and zero-measurements were taken, it would have the effect of showing the zero so determined as being nearer the bed-plate than it would be if the kink were removed, and if the band righted itself in the course of working, the determination for zero at next inspection would be at a greater distance from the bed-plate than formerly.

Information to be recorded in the book entitled 'Determination of the True Zero on the Diagram' should be somewhat as follows:—

(1) Level of bed-plate with reference to B. M. unaltered, or settled  
by . . . . . or raised by . . . . .



two successive equal and similarly placed ordinates. The constant  $a$  is called the 'amplitude' of the curve because its value is that of the greatest displacement. The angle  $nx + b$  or  $\frac{2\pi}{\lambda}x + b$  is called the 'phase' of the curve: the constant  $b$  is therefore known if the phase is given for any value of  $x$ .

Any two curves of equal wave-length may be combined into another of the same wave-length. For the equation

$$\begin{aligned} y &= a_1 \cos \left( \frac{2\pi}{\lambda} x + b_1 \right) + a_2 \cos \left( \frac{2\pi}{\lambda} x + b_2 \right) \\ &= (a_1 \cos b_1 + a_2 \cos b_2) \cos \frac{2\pi}{\lambda} x - (a_1 \sin b_1 + a_2 \sin b_2) \sin \frac{2\pi}{\lambda} x \\ &= A \cos \left( \frac{2\pi}{\lambda} x + B \right), \end{aligned}$$

where  $A^2 = a_1^2 + a_2^2 + 2 a_1 a_2 \cos (b_1 - b_2),$

and  $\tan B = \frac{a_1 \sin b_1 + a_2 \sin b_2}{a_1 \cos b_1 + a_2 \cos b_2},$

represents a harmonic curve of the same wave-length as the two components. Similarly for any number of curves of the same wave-length.

Two or more curves of different wave-lengths can however not be combined into a single harmonic curve; but if the wave-lengths are commensurable the resultant curve is periodic. For let

$$y = a_1 \cos \left( \frac{2\pi}{\lambda_1} x + b_1 \right) + a_2 \cos \left( \frac{2\pi}{\lambda_2} x + b_2 \right) + a_3 \cos \left( \frac{2\pi}{\lambda_3} x + b_3 \right) + \&c.,$$

and let  $\lambda$  be the least common multiple of  $\lambda_1, \lambda_2, \lambda_3, \&c.$ , so that their actual values are  $\frac{\lambda}{m_1}, \frac{\lambda}{m_2}, \frac{\lambda}{m_3}, \&c.$ , where  $m_1, m_2, m_3, \&c.$ , are integers;

then  $y = a_1 \cos \left( \frac{2\pi}{\lambda} m_1 x + b_1 \right) + a_2 \cos \left( \frac{2\pi}{\lambda} m_2 x + b_2 \right) + \&c.,$

and if  $x + \lambda$  is put for  $x$  the value of  $y$  is unaltered, so that the resulting curve is a periodic curve of wave-length  $\lambda$ .

9. Such are a few of the properties of harmonic curves, and the next thing to be done is to point out the connection between harmonic curves and tidal observations.

In a *Report of a Committee for the Harmonic Analysis of Tidal Observations* for the British Association, 1883, Professor G. H. Darwin has deduced an expression for the height of the tide at any time: each term, which is of the harmonic form  $R \cos(nt - \zeta)$ , arises from some specific cause in the elaboration of the equilibrium theory of tides and is regarded as a separate tide due to this cause. Thus there are as many tides as there are terms in the series, and the height of each simple tide is equal to a constant,  $R$ , multiplied by the cosine of a certain angle  $nt - \zeta$  called the 'argument', which is partly made up of a simple function of the time and partly dependent on the position of the sun or moon or both.

The maximum value of the cosine being unity, the constant,  $R$ , gives the greatest height above the mean of the particular tide, that is, the 'semi-range' or 'amplitude'.

The part of the argument which is a function of the time is of the form  $nt$ , so that  $n$  represents the rate at which the argument increases: it is called the 'speed' of the tide and is reckoned in mean solar hours. Also since the tide's maximum occurs when the remainder of the argument, *viz.*,  $\zeta$  is equal to  $nt$ , it follows that  $\frac{\zeta}{n}$  gives the time which must elapse from the beginning of the observations till the time of the first high water of the tide:  $\zeta$  is therefore called the 'epoch'.

10. For the purpose of arithmetical calculation the form  $R \cos(nt - \zeta)$ , in which the tide is presented, is not convenient and it is therefore expanded into

$$A \cos nt + B \sin nt,$$

so that  $R^2 = A^2 + B^2$  and  $\tan \zeta = \frac{B}{A}$ ;

and the immediate object of the numerical reductions is to find the  $A$ 's and  $B$ 's, from which the  $R$ 's and  $\zeta$ 's are at once obtained by means of

the above equations. It now remains to explain how the A's and B's are determined from the observations.

The expression deduced by Darwin gives for the height of the tide at any time,

$$h = A_0 + \Sigma (A \cos nt) + \Sigma (B \sin nt).$$

Now among these  $n$ 's will be found  $n_1, 2n_1, 3n_1$  &c.;  $n_2, 2n_2, 3n_2$ ; and so on, and it would be practically impossible to determine the corresponding A's and B's in a direct manner. It has, however, been found possible, by a method of manipulation of the observed quantities which will be explained below, to separate the terms containing  $n_1$  from all the others and then the problem presents no difficulty. As will be explained, it is reduced to the question of determining the constants from a series of equations of the form

$$h = A_0 + A_1 \cos nt + B_1 \sin nt + A_2 \cos 2nt + B_2 \sin 2nt + \&c.,$$

where  $t$  has any integral value from 0 to  $\frac{2\pi}{n}$ , so that  $nt$  goes through its variations from 0 to  $2\pi$ .

Now it is clear, if  $r$  and  $s$  be any two integers, and the summation extends from  $t = 0$  to  $t = \frac{2\pi}{n}$ , that

$$\Sigma \cos rnt = 0, \text{ and } \Sigma \sin rnt = 0,$$

since to each positive value, there is a corresponding negative value of the cosine or sine.

$$\text{Also } \Sigma \cos^2 rnt = \frac{1}{2} \Sigma (1 + \cos 2rnt) = \frac{\pi}{n},$$

$$\text{and } \Sigma \sin^2 rnt = \frac{1}{2} \Sigma (1 - \cos 2rnt) = \frac{\pi}{n};$$

$$\Sigma \cos rnt \cos snt = \frac{1}{2} \Sigma \cos(r + s)nt + \frac{1}{2} \Sigma \cos(r - s)nt = 0,$$

and similarly

$$\Sigma \cos rnt \sin snt \quad \text{and} \quad \Sigma \sin rnt \sin snt \quad \text{are each equal to zero.}$$

Consequently for determining the constants there are the following equations:—

$$\Sigma h = \frac{2\pi}{n} A_0, \quad \text{or} \quad A_0 = \frac{n}{2\pi} \Sigma h;$$

$$\Sigma h \cos rnt = \frac{\pi}{n} A_r, \quad \text{or} \quad A_r = \frac{n}{\pi} \Sigma h \cos rnt;$$

$$\Sigma h \sin rnt = \frac{\pi}{n} B_r, \quad \text{or} \quad B_r = \frac{n}{\pi} \Sigma h \sin rnt.$$

The A's and B's being now determined, the R's and  $\zeta$ 's are calculated from the two formulæ given above.

11. But in order that a comparison of the records of different years may be made, it is necessary to exhibit the height of the tide in yet a different form; for when it is represented by  $R \cos (nt - \zeta)$ , it is clear that  $\zeta$  may have any value from 0 to  $360^\circ$  and that the results of the analysis of successive years of observations will not be comparable with each other.

Such being the case, let it be supposed that the results of the analysis are presented in a number of terms of the form

$$f H \cos (V + u - \kappa).$$

Here  $V$  is a linear function of the moon's and the sun's mean longitudes, the mean longitudes of the moon's and the sun's perigees, and the local mean solar time at the place of observation reduced to angle at  $15^\circ$  per hour.  $V$  therefore increases uniformly with the time and its rate of increase per mean solar hour is the  $n$  of the first method, or the 'speed' of the tide.

It is supposed that  $u$  stands for a certain function of the longitude of the node of the lunar orbit, at an epoch half a year later than 0<sup>h</sup> of the first day. Strictly speaking,  $u$  should be taken as this same function of the longitude of the moon's node, varying as the node moves; but as the variation is but small in the course of a year,  $u$  may be treated as a constant and put equal to an average value for the year, which average value is taken as the true value of  $u$  at exactly mid-year.

Together  $V + u$  constitute the whole 'argument'\* according to the equilibrium theory of tides, with the sea covering the whole earth; and it therefore follows that  $\frac{\kappa}{n}$  is the lagging of the tide which arises

\* See Schedules [B, i], [B, ii], [B, iii] and C of Professor Darwin's Report.



from kinetic action, friction of the water, imperfect elasticity of the earth, and the distribution of the land.

It is also supposed that  $H$  is the mean value in British feet of the semi-range of the particular tide in question; and  $f$  is a numerical factor of augmentation or diminution due to the variability of the obliquity of the lunar orbit: the method of determining it is fully explained in, and its values tabulated at the end of, Professor Darwin's Report and also in the Auxiliary Tables appended.

It is obvious, then, that, if the tidal observations are consistent from year to year,  $H$  and  $\kappa$  should come out the same from each year's reductions: and it is only when the results are presented in such a form as this, that it will be possible to judge whether the harmonic analysis is giving satisfactory results.

The determination of  $H$  and  $\kappa$  from  $R$  and  $\zeta$  is made as follows:—  
Clearly  $H = \frac{R}{f}$  and is at once found; also  $nt - \zeta$  is identical with  $V + u - \kappa$ , so that if  $V_0$  be the value of  $V$  at  $0^h$  of the first day, that is when  $t = 0$ , then,

$$-\zeta = V_0 + u - \kappa;$$

so that

$$\kappa = \zeta + V_0 + u.$$

Thus the rule for the determination of  $\kappa$  is:—add to the value of  $\zeta$  the value of the 'argument'\* at  $0^h$  of the first day.

12. Hitherto the tides have been spoken of as if they each were treated in the same way, but this is not the case. For the purposes of calculation they are divided into 'short-period' and 'long-period' tides. The short-period tides are still further sub-divided into semi-diurnal and diurnal tides. The former have periods equal or nearly equal to 12 mean solar hours and the latter have periods equal or nearly equal to 24 mean solar hours. Besides these there are also some 'over-tides' and 'compound-tides'; their origin is explained in Professor Darwin's Report, and their periods approximate to some submultiple of 12 mean solar hours. The long-period tides have periods of about a fortnight, a month, half a year or a year as the case may be.

\* See Schedules [B, i], [B, ii], [B, iii] and C of Professor Darwin's Report.

The method of determining the A's and B's described above is only applicable to the short-period tides: the means of determining them for the long-period tides will be described hereafter.

13. The tide-gauge gives a graphical record of the height of the water above some known datum for every instant of time. The first operation performed on the tidal record is the measurement, in feet and decimals, of the height of the water above the true zero of the gauge (the height of which relatively to the datum is known) at every mean solar hour. The period chosen for analysis is about one year, and the first measurement corresponds to noon, but it has been found inconvenient hitherto to have the same initial noon at the several ports.

Description of numerical harmonic analysis for short-period tides.

It would seem, at first sight, preferable to take the measurements at each mean lunar hour; but the whole of the actual process in use is based on measurements taken at the mean solar hours, and a change to lunar time would involve a great deal of fresh labour and expense.

If  $T$  be the period of any one of the diurnal tides, or the double period of any one of the semi-diurnal tides, it approximates more or less nearly to 24 mean solar hours, and if it be divided into 24 equal parts, each part may be spoken of as a  $T$ -hour, while for brevity mean solar time will be referred to as  $S$ -time.

Suppose, now, that there are two clocks, each marked with  $360^\circ$  or 24 hours, and that the hand of the first or  $S$ -clock goes round once in 24  $S$ -hours and that of the second or  $T$ -clock goes round once in 24  $T$ -hours; and suppose that the two clocks are started at  $0^\circ$  or  $0^h$  at noon of the initial day. For the sake of distinctness, imagine that a  $T$ -hour is longer than an  $S$ -hour so that the  $T$ -clock goes slower than the  $S$ -clock.

The measurements of the tide-curve give the height of the water exactly at each  $S$ -hour; and it is required from these data to determine the height of the water at each  $T$ -hour. For this end, it is necessary to count  $T$ -time; but this must be done with reference to  $S$ -time and, moreover, the time must always be specified as an integral number of hours.

Beginning, then, with  $0^h$  of the first day, it is necessary to count 0, 1, 2, &c., as the  $T$ -hand comes up to its hour marks. But as the

S-hand gains on the T-hand, there will come a time when the T-hand, being exactly at the  $p$  hour-mark, the S-hand is nearly as far as  $p + \frac{1}{2}$ . When, however, the T-hand has advanced to the  $p + 1$  hour-mark, the S-hand will be a little beyond  $p + 1 + \frac{1}{2}$ ; that is to say, a little less than half-an-hour before  $p + 2$ . Counting, then, the T-hours in S-time, it is necessary to jump from  $p$  to  $p + 2$ . The counting will go on continuously for a number of hours nearly equal to  $2p$ , and then another number will be dropped, and so on throughout the whole year. If the T-hand went faster than the S-hand, it is obvious that one number would be repeated at two successive hours instead of one being dropped. Each such process may be described as a 'change'.

Now if there is a sheet marked for entries of heights of water according to T-hours from results measured at S-hours, the S-measurements must be entered continuously up to  $p$ : then comes a 'change' and the dropping of one of the S-series, after which the entry goes on continuously until another 'change' when another is dropped, and so on.

Since a 'change' occurs at the time when a T-hour falls almost exactly between two S-hours, it will be more accurate to insert the two S-entries which fall on each side of the truth. If this be done the whole of the S-series of measurements is entered on the T-sheet. Similarly if it is the T-hand which goes faster than the S-hand, a gap may be left in the T-series instead of duplicating an entry. For the analysis of the T-tide there is, therefore, prepared a sheet arranged in rows and columns: each row corresponds to one T-day and the columns are marked  $0^h, 1^h, \dots, 23^h$ ; the  $0^h$ 's may be called T-noons. A dot is put in each space for entry, and where there is a 'change' two dots are put if there is to be a double entry, and a bar if there is to be no entry; black vertical lines mark the end of each S-day. These black lines will, of course, fall into slightly irregular diagonal lines across the page, being steeper and steeper the more nearly T-time approaches to S-time. They slope downwards from right to left if the T-hour is longer than the S-hour, and the other way in the opposite case. The 'changes' also run diagonally with a slope in the opposite direction to that of the black lines when the T-hour is longer than the S-hour, and in the same direction in the opposite case.

A sample is annexed of parts of pages drawn up for the entries of the M-series and J-series of tides, in the former of which T-time is mean lunar time.

CHAP. III.]

COMPUTATIONS.

| M-SERIES |                |                |                |                |                |                |                |                |                |                |                 |                 | J-SERIES        |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |        |                                 |  |
|----------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|--------|---------------------------------|--|
| No.      | 0 <sup>h</sup> | 1 <sup>h</sup> | 2 <sup>h</sup> | 3 <sup>h</sup> | 4 <sup>h</sup> | 5 <sup>h</sup> | 6 <sup>h</sup> | 7 <sup>h</sup> | 8 <sup>h</sup> | 9 <sup>h</sup> | 10 <sup>h</sup> | 11 <sup>h</sup> | 12 <sup>h</sup> | 13 <sup>h</sup> | 14 <sup>h</sup> | 15 <sup>h</sup> | 16 <sup>h</sup> | 17 <sup>h</sup> | 18 <sup>h</sup> | 19 <sup>h</sup> | 20 <sup>h</sup> | 21 <sup>h</sup> | 22 <sup>h</sup> | 23 <sup>h</sup> | S-hour | No. of days                     |  |
| 1        | .              | .              | .              | .              | .              | .              | .              | .              | .              | .              | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .      | 2 <sup>d</sup> 0 <sup>h</sup>   |  |
| 2        | .              | .              | .              | .              | .              | .              | .              | .              | .              | .              | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .      | 3 1                             |  |
| 3        | .              | .              | .              | .              | .              | .              | .              | .              | .              | .              | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .      | 4 2                             |  |
| 4        | .              | .              | .              | .              | .              | .              | .              | .              | .              | .              | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .      | 5 2                             |  |
| 5        | .              | .              | .              | .              | .              | .              | .              | .              | .              | .              | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .      | 6 3                             |  |
|          |                |                |                |                |                |                |                |                |                |                |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |        |                                 |  |
| 67       | .              | .              | .              | .              | .              | .              | .              | .              | .              | .              | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .      | 70 7                            |  |
| 68       | .              | .              | .              | .              | .              | .              | .              | .              | .              | .              | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .      | 71 8                            |  |
| 69       | .              | .              | .              | .              | .              | .              | .              | .              | .              | .              | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .      | 72 9                            |  |
| 70       | .              | .              | .              | .              | .              | .              | .              | .              | .              | .              | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .      | 73 10                           |  |
| 71       | .              | .              | .              | .              | .              | .              | .              | .              | .              | .              | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .      | 74 11                           |  |
| Sum      | .              | .              | .              | .              | .              | .              | .              | .              | .              | .              | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .      | No. of days                     |  |
| No.      | 73             | 73             | 74             | 74             | 73             | 73             | 73             | 73             | 75             | 73             | 73              | 73              | 74              | 74              | 74              | 73              | 73              | 75              | 74              | 73              | 73              | 74              | 73              | 74              | 74     | 73 <sup>d</sup> 12 <sup>h</sup> |  |
|          |                |                |                |                |                |                |                |                |                |                |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |        |                                 |  |
| &c.      |                |                |                |                |                |                |                |                |                |                |                 |                 | &c.             |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |        |                                 |  |
| 1        | .              | .              | .              | .              | .              | .              | .              | .              | .              | .              | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .      | 1 <sup>d</sup> 22 <sup>h</sup>  |  |
| 2        | .              | .              | .              | .              | .              | .              | .              | .              | .              | .              | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .      | 2 21                            |  |
| 3        | .              | .              | .              | .              | .              | .              | .              | .              | .              | .              | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .      | 3 20                            |  |
| 4        | .              | .              | .              | .              | .              | .              | .              | .              | .              | .              | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .      | 4 19                            |  |
| 5        | .              | .              | .              | .              | .              | .              | .              | .              | .              | .              | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .      | 5 19                            |  |
|          |                |                |                |                |                |                |                |                |                |                |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |                 |        |                                 |  |
| 72       | .              | .              | .              | .              | .              | .              | .              | .              | .              | .              | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .      | 70 6                            |  |
| 73       | .              | .              | .              | .              | .              | .              | .              | .              | .              | .              | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .      | 71 5                            |  |
| 74       | .              | .              | .              | .              | .              | .              | .              | .              | .              | .              | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .      | 72 4                            |  |
| 75       | .              | .              | .              | .              | .              | .              | .              | .              | .              | .              | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .      | 73 3                            |  |
| 76       | .              | .              | .              | .              | .              | .              | .              | .              | .              | .              | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .      | 74 3                            |  |
| Sum      | .              | .              | .              | .              | .              | .              | .              | .              | .              | .              | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .               | .      | No. of days                     |  |
| No.      | 73             | 74             | 73             | 73             | 74             | 73             | 73             | 74             | 73             | 73             | 73              | 74              | 72              | 74              | 74              | 71              | 74              | 73              | 73              | 73              | 73              | 72              | 74              | 73              | 73     | 73 <sup>d</sup> 4 <sup>h</sup>  |  |

Since the first day is numbered 1 and the first hour 0<sup>h</sup>, it follows that to find the number of hour values entered in the form from 0<sup>h</sup> of the first day, it is necessary to subtract 1 from the number of the day and to add 1 to the number of the hour.

For each class of tide there are five pages similar to the annexed examples, giving in all about 370 values for the height of the water at each of the 24 special hours: the number of values for each hour varies slightly according as more or less 'changes' fall into each column.

The numbers entered in each column are summed on each of the five pages; the five sets of results are now summed and the results divided each by the proper divisor for its column, thus giving a mean value for that column. In this way 24 numbers are found which give the mean height of the water at each of the 24 special hours.

14. It is obvious that if this process were continued over a very long time, in the end, the tide under analysis would be extracted from among all the others; but as the process only extends over about a year, the elimination of the others is not quite complete. The elimination of the effects of the other tides may be improved by choosing the period for analysis not exactly equal to one year. For, suppose that the expression for the height of the water is

$$A_1 \cos n_1 t + B_1 \sin n_1 t + A_2 \cos n_2 t + B_2 \sin n_2 t,$$

where  $n_2$  is nearly equal to  $n_1$ , and that it is required to eliminate the  $n_2$ -tide so as to be left only with the  $n_1$ -tide.

The expression may then be put equal to

$$\begin{aligned} & \{ A_1 + A_2 \cos (n_1 - n_2) t - B_2 \sin (n_1 - n_2) t \} \cos n_1 t \\ & + \{ B_1 + A_2 \sin (n_1 - n_2) t + B_2 \cos (n_1 - n_2) t \} \sin n_1 t; \end{aligned}$$

which shows that the tide may be regarded as oscillating with a speed  $n_1$ , but with slowly varying range. Now, in the column appertaining to any hour in the form,  $n_1 t$  is a multiple of  $15^\circ$  if  $n_1$  be a diurnal, and of  $30^\circ$ , if  $n_1$  be a semi-diurnal tide. Consider the column headed 'p-hours'; then  $n_1 t = 15^\circ p$  for diurnals and  $30^\circ p$  for semi-diurnals.

Hence the sum of all the entries, of which suppose there are  $q$ , in the column numbered  $p$ -hours, is for diurnal tides,

$$\begin{aligned} & \cos 15^\circ p \left\{ A_1 q + A_2 \left[ \cos (n_1 - n_2) \frac{15p}{n_1} + \cos (n_1 - n_2) \left( \frac{2\pi}{n_1} + \frac{15p}{n_1} \right) \right. \right. \\ & \left. \left. + \cos (n_1 - n_2) \left( 2 \frac{2\pi}{n_1} + \frac{15p}{n_1} \right) \right] + B_2 [\&c.] \right\} + \sin 15^\circ p \left\{ \&c. \right\} \dots (a) \end{aligned}$$

and for semi-diurnal tides, the arguments of all the circular functions in the expression (a) are to be doubled.

Now such a number of terms is to be chosen, that the series by which  $A_2$  and  $B_2$  are multiplied may vanish. This is exactly the case, if the series is exactly re-entrant and is nearly the case, if nearly re-entrant.

The condition is exactly satisfied, if

$$(n_1 - n_2) q \frac{2\pi}{n_1} = 2\pi r \quad \text{for diurnal tides,}$$

or 
$$(n_1 - n_2) q \frac{4\pi}{n_1} = 2\pi r \quad \text{for semi-diurnal tides,}$$

where  $r$  is either a positive or negative integer.

That is to say, if

$$(n_1 - n_2) q = n_1 r \quad \text{for diurnal tides,}$$

or 
$$(n_1 - n_2) q = \frac{1}{2} n_1 r \quad \text{for semi-diurnal tides.}$$

It is not worth while attempting to eliminate the effects of the semi-diurnal tides on the diurnal tides and *vice versa*, because the periods could only differ by the fraction of a day and owing to the incommensurability of the speeds, it is impossible to avoid being wrong to that amount.

It is of course impossible to choose for each tide  $n_1$ , a period which shall minimise the effects of more than one of the tides of short-period  $n_2$ , in vitiating the values of the mean semi-ranges of the tide  $n_1$ ; and accordingly the periods have been chosen so as to minimise the effect of the principal solar semi-diurnal tide  $S_2$  on the principal lunar semi-diurnal tide  $M_2$ , and of the  $M_2$  tide upon the other semi-diurnal tides: in the case of the diurnal tides, the periods are chosen to minimise the effects of either  $O$  or  $K_1$ .

Returning now to the general notation and considering the 24 mean values, each pertaining to the 24 T-hours, it may be supposed that all the tides, excepting the T-tide, are adequately eliminated, and, in fact, a computation of the necessary corrections for the absence of complete elimination, which is given in the Tidal Report of 1872, of the Tidal Committee of the B. A. under the presidency of Sir William Thomson, shows that this is the case.

15. Now it is obvious that any one of the 24 values does not give the true height of the T-tide at that T-hour, but gives the average height of the water, as due to the T-tide, estimated over half a T-hour before and half a T-hour after that hour. A correction must therefore be determined on this account.

The required expression for the height of the tide at any T-hour is

$$h = A_1 \cos \theta + B_1 \sin \theta + \&c., \&c. + A_r \cos r \theta + B_r \sin r \theta + \&c.$$

But the results of analysis give instead of this the mean of all the  $h$ 's between the limits  $\theta + \frac{a}{2}$  and  $\theta - \frac{a}{2}$ .

That is  $h a = \sum h$  between these limits

$$= \&c. + \sum A_r \cos r \theta + \sum B_r \sin r \theta + \&c. \text{ between these limits,}$$

or 
$$h a = \&c. + A_r \frac{2}{r} \sin \frac{r a}{2} \cos r \theta + B_r \frac{2}{r} \sin \frac{r a}{2} \sin r \theta + \&c.$$

whence 
$$h = \&c. + \frac{\sin \frac{r a}{2}}{\frac{r a}{2}} A_r \cos r \theta + \frac{\sin \frac{r a}{2}}{\frac{r a}{2}} B_r \sin r \theta + \&c.$$

Consequently the coefficients that express the oscillation which goes through its period  $r$  times in 24 T-hours, must be augmented by the

factor  $\frac{2}{\sin \frac{r a}{2}}$  to give the true  $A_r$  and  $B_r$ .

Remembering that  $a$  is  $15^\circ$  and putting for  $r$ , 1, 2, 3, &c., in succession, the augmenting factors for the diurnal, semi-diurnal, ter-diurnal oscillations, &c., become

$$\frac{7.5 \pi}{180 \sin 7^\circ 30'}; \quad \frac{15 \pi}{180 \sin 15^\circ}; \quad \frac{22.5 \pi}{180 \sin 22^\circ 30'}; \quad \&c.$$

Thus, the augmenting factors are:—

|     |            |           |         |
|-----|------------|-----------|---------|
| for | $A_1, B_1$ | . . . . . | 1.00286 |
|     | $A_2, B_2$ | . . . . . | 1.01152 |
|     | $A_3, B_3$ | . . . . . | 1.02617 |
|     | $A_4, B_4$ | . . . . . | 1.04720 |
|     | $A_6, B_6$ | . . . . . | 1.11072 |
|     | $A_8, B_8$ | . . . . . | 1.20920 |

In the reduction of the S-series of tides, the numbers treated are the actual heights of the water exactly at the S-hours, and therefore no augmenting factor is requisite.

16. If now  $t$  denotes T-time expressed in hours and  $n$  is  $15^\circ$ , the height  $h$ , as expressed by the averaging process explained above, is given by the formula

Determination of A's and B's.

$$h = A_0 + A_1 \cos nt + B_1 \sin nt + A_2 \cos 2nt + B_2 \sin 2nt + \&c.,$$

where  $t$  is 0, 1, 2 . . . . 23.

Then, if  $\Sigma$  is the sum of the series of 24 terms found by giving  $t$  its 24 values, as before shown,

$$A_0 = \frac{1}{24} \Sigma h; \quad A_1 = \frac{1}{12} \Sigma h \cos nt; \quad B_1 = \frac{1}{12} \Sigma h \sin nt;$$

$$A_2 = \frac{1}{12} \Sigma h \cos 2nt; \quad B_2 = \frac{1}{12} \Sigma h \sin 2nt; \quad \&c., \&c.$$

Also, since  $n = 15^\circ$  and  $t$  is an integer, all the cosines and sines involved are equal to one of the following:— 0;  $\pm \sin 15^\circ$ ;  $\pm \sin 30^\circ$ ;  $\pm \sin 45^\circ$ ;  $\pm \sin 60^\circ$ ;  $\pm \sin 75^\circ$ ;  $\pm 1$ . These are denoted in the computation forms by 0,  $\pm S_1$ ,  $\pm S_2$ ,  $\pm S_3$ ,  $\pm S_4$ ,  $\pm S_5$ ,  $\pm 1$ .

This enables the forms to be arranged in the neat tabular form on pages 1, 2 and 3 of the Analysis of Short-Period Tides, a specimen of which is given in Form No. VI, where the 24 hourly values to be submitted to analysis are written continuously down columns I and II. The subsequent operations are sufficiently indicated by the headings to the columns, and it will be found on examination that the results are in reality the sums of the several series given above.



LIST OF

| Initial          | Speed in symbols and in m. s. hours                                     | Name of tide                        |
|------------------|-------------------------------------------------------------------------|-------------------------------------|
| $S_1$            | $\gamma - \eta$ 15°                                                     | Principal Solar Diurnal             |
| $S_2$            | $2(\gamma - \eta)$ 30                                                   | " " Semi-Diurnal                    |
| $S_4$            | $4(\gamma - \eta)$ 60                                                   | } Principal Solar Series Over-Tides |
| $S_6$            | $6(\gamma - \eta)$ 90                                                   |                                     |
| $S_8$            | $8(\gamma - \eta)$ 120                                                  |                                     |
| $P_1$            | $\gamma - 2\eta$ 14 '9589314                                            |                                     |
| $T_2$            | $2\gamma - 3\eta$ 29 '9589314                                           | Larger Solar Elliptic               |
| $M_1$            | $\gamma - \sigma - \omega$ and $\gamma - \sigma + \omega$ 14 '4920521   | Principal Lunar Diurnal             |
| $M_2$            | $2(\gamma - \sigma)$ 28 '9841042                                        | " " Semi-Diurnal                    |
| $M_3$            | $3(\gamma - \sigma)$ 43 '4761563                                        | } Principal Lunar Series Over-Tides |
| $M_4$            | $4(\gamma - \sigma)$ 57 '9682082                                        |                                     |
| $M_6$            | $6(\gamma - \sigma)$ 86 '9523126                                        |                                     |
| $M_8$            | $8(\gamma - \sigma)$ 115 '9364164                                       |                                     |
| $K_1$            | $\gamma$ 15 '0410686                                                    | Luni-Solar Diurnal (Declinational)  |
| $K_2$            | $2\gamma$ 30 '0821372                                                   | " " Semi-Diurnal "                  |
| $N_2$            | $2\gamma - 3\sigma + \omega$ 28 '4397296                                | Larger Lunar Elliptic               |
| $2N_2$           | $2\gamma - 4\sigma + 2\omega$ 27 '8953548                               | Lunar Elliptic, 2nd order           |
| $L_2$            | $2\gamma - \sigma - \omega$ and $2\gamma - \sigma + \omega$ 29 '5284788 | Smaller Lunar Elliptic              |
| $\nu_2$          | $2\gamma - 3\sigma - \omega + 2\eta$ 28 '5125830                        | Larger Lunar Evectional             |
| $O_1$            | $\gamma - 2\sigma$ 13 '9430356                                          | Lunar (Declinational) Diurnal       |
| $Q_1$            | $\gamma - 3\sigma + \omega$ 13 '3986609                                 | Larger Lunar Elliptic Diurnal       |
| $J_1$            | $\gamma + \sigma - \omega$ 15 '5854433                                  | Lunar Elliptic Diurnal              |
| $(MS)_4$         | $4\gamma - 2\sigma - 2\eta$ 58 '9841042                                 | } Compound Tides                    |
| $\mu_2$ or $2MS$ | $2\gamma - 4\sigma + 2\eta$ 27 '9682084                                 |                                     |
| $(2SM)_2$        | $2\gamma + 2\sigma - 4\eta$ 31 '0158958                                 |                                     |
| $(M_2K_1)_3$     | $3\gamma - 2\sigma$ 44 '0251728                                         |                                     |
| $(2M_2K_1)_3$    | $3\gamma - 4\sigma$ 42 '9271398                                         |                                     |
| $(M_2N)_4$       | $4\gamma - 5\sigma + \omega$ 57 '4238338                                |                                     |
| $MSf$            | $2\sigma - 2\eta$ 1 '0158958                                            | Luni-Solar Synodic Fortnightly      |
| $Mm$             | $\sigma - \omega$ 0 '5443747                                            | Lunar Monthly                       |
| $Mf$             | $2\sigma$ 1 '0980330                                                    | " Fortnightly                       |
| $Sa$             | $\eta$ 0 '0410686                                                       | Solar Annual                        |
| $Ssa$            | $2\eta$ 0 '0821372                                                      | " Semi-Annual                       |

CHAP. III.]

COMPUTATIONS.

TIDES.

| Initial Argument = $V_0 + u$                                                                                     | Factor for reduction = $\frac{1}{f}$                                                                                          | Initial          |
|------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------|------------------|
| Zero                                                                                                             | Unity                                                                                                                         | $S_1$            |
| "                                                                                                                | "                                                                                                                             | $S_2$            |
| "                                                                                                                | "                                                                                                                             | $S_4$            |
| "                                                                                                                | "                                                                                                                             | $S_6$            |
| "                                                                                                                | "                                                                                                                             | $S_8$            |
| $-\hbar_0 + \frac{1}{2}\pi$                                                                                      | "                                                                                                                             | $P_1$            |
| $-(\hbar_0 - p_1)$                                                                                               | "                                                                                                                             | $T_2$            |
| $(\hbar_0 - \nu) - (s_0 - \xi) + Q - \frac{1}{2}\pi$ where<br>$\tan Q = \frac{1}{2} \tan P$                      | Fac. O $\div \sqrt{\frac{3}{2} + \frac{3}{2} \cos 2P}$                                                                        | $M_1$            |
| $2(\hbar_0 - \nu) - 2(s_0 - \xi)$                                                                                | $\left(\frac{\cos \frac{1}{2}\omega \cos \frac{1}{2}i}{\cos \frac{1}{2}I}\right)^4$                                           | $M_2$            |
| $\frac{3}{2}$ Arg. $M_2$                                                                                         | (Fac. $M_2$ ) $^{\frac{3}{2}}$                                                                                                | $M_3$            |
| $2$ Arg. $M_2$                                                                                                   | (Fac. $M_2$ ) $^2$                                                                                                            | $M_4$            |
| $3$ Arg. $M_2$                                                                                                   | (Fac. $M_2$ ) $^3$                                                                                                            | $M_6$            |
| $4$ Arg. $M_2$                                                                                                   | (Fac. $M_2$ ) $^4$                                                                                                            | $M_8$            |
| $\hbar_0 - \nu' - \frac{1}{2}\pi$ where<br>$\tan \nu' = \frac{\sin \nu}{\cos \nu + .464k}$                       | $\frac{1.46407k}{\sqrt{1 + (.464k)^2 + .928k \cos \nu}}$ where $k = \frac{\sin 2\omega(1 - \frac{3}{2}\sin^2 i)}{\sin 2I}$    | $K_1$            |
| $2\hbar_0 - 2\nu''$ where<br>$\tan 2\nu'' = \frac{\sin 2\nu}{\cos 2\nu + .464k}$                                 | $\frac{1.46407k}{\sqrt{1 + (.464k)^2 + .928k \cos 2\nu}}$ where $k = \frac{\sin^2 \omega(1 - \frac{3}{2}\sin^2 i)}{\sin^2 I}$ | $K_2$            |
| Arg. $M_2 - (s_0 - p_0)$                                                                                         | Fac. $M_2$                                                                                                                    | $N_2$            |
| Arg. $N_2 - (s_0 - p_0)$                                                                                         | Fac. $M_2$                                                                                                                    | $2N_2$           |
| Arg. $M_2 + (s_0 - p_0) - R + \pi$ where<br>$\tan R = \frac{\sin 2P}{\frac{1}{2} \cot^2 \frac{1}{2}I - \cos 2P}$ | Fac. $M_2 \div \sqrt{1 - 12 \tan^2 \frac{1}{2}I \cos 2P}$                                                                     | $L_2$            |
| Arg. $M_2 + (s_0 - p_0) + 2\hbar_0 - 2s_0$<br>$(\hbar_0 - \nu) - 2(s_0 - \xi) + \frac{1}{2}\pi$                  | $\frac{\text{Fac. } M_2 \sin \omega \cos^2 \frac{1}{2}\omega \cos^4 \frac{1}{2}i}{\sin I \cos^2 \frac{1}{2}I}$                | $\nu_2$<br>$O_1$ |
| Arg. $O_1 - (s_0 - p_0)$<br>$(\hbar_0 - \nu) + (s_0 - p_0) - \frac{1}{2}\pi$                                     | $\frac{\text{Fac. } O_1 \sin 2\omega(1 - \frac{3}{2}\sin^2 i)}{\sin 2I}$                                                      | $Q_1$<br>$J_1$   |
| Arg. $M_2$                                                                                                       | Fac. $M_2$                                                                                                                    | $(MS)_4$         |
| Arg. $M_4$                                                                                                       | Fac. $M_4$                                                                                                                    | $\mu_2$ or $2MS$ |
| $2\pi - \text{Arg. } M_2$                                                                                        | Fac. $M_2$                                                                                                                    | $(2SM)_2$        |
| Arg. $M_2 + \text{Arg. } K_1$                                                                                    | Fac. $M_2 \times \text{Fac. } K_1$                                                                                            | $(M_2K_1)_3$     |
| Arg. $M_4 - \text{Arg. } K_1$                                                                                    | Fac. $M_4 \times \text{Fac. } K_1$                                                                                            | $(2M_2K_1)_3$    |
| Arg. $M_2 + \text{Arg. } N_2$                                                                                    | Fac. $M_2 \times \text{Fac. } N_2$                                                                                            | $(M_2N)_4$       |
| $2\pi - \text{Arg. } M_2$                                                                                        | Fac. $M_2$                                                                                                                    | MSf              |
| $s_0 - p_0$                                                                                                      | $\frac{(1 - \frac{3}{2}\sin^2 \omega)(1 - \frac{3}{2}\sin^2 i)}{(1 - \frac{3}{2}\sin^2 I)}$                                   | Mm               |
| $2(s_0 - \xi)$                                                                                                   | $\frac{\sin^2 \omega \cos^4 \frac{1}{2}i}{\sin^2 I}$                                                                          | Mf               |
| $\hbar_0$                                                                                                        | Unity                                                                                                                         | Sa               |
| $2\hbar_0$                                                                                                       | "                                                                                                                             | Ssa              |

17. The preceding table contains a list of the tides, at present subjected to analysis, together with their speeds, arguments and factors for reduction. The symbols in the table have the following meanings:—

List of tides.

$\gamma$  = earth's angular velocity of rotation.

$\sigma$  = mean motion of the moon.

$\eta$  = " " " sun.

$\omega$  = " " " lunar perigee.

$h$  = sun's mean longitude.

$p$  = mean longitude of the moon's perigee.

$\nu$  = right ascension of the 'intersection' or descending node of the equator on the lunar orbit.

$s$  = moon's mean longitude.

$\xi$  = longitude 'in the moon's orbit' of the 'intersection'.

$I$  = obliquity of the lunar orbit to the equator.

$\omega$  = " " " ecliptic.

$i$  = inclination of the moon's orbit to the ecliptic.

$P$  = longitude of the moon's perigee at mid-year measured from the 'intersection'.

The letters with the zero subscript represent the values of the corresponding functions at 0<sup>h</sup> of the 1st day of the year of observation.

18. For the purpose of determining the tides of long period, it is necessary to eliminate the oscillations of water-level arising from the tides of short period. As the quickest of the tides of long period has a period of many days, the height of the water at one instant for each day gives sufficient data. Thus there will, in a year's observations, be 365 heights to be submitted to harmonic analysis. In leap-years, the last day's observation must be dropped, because the treatment is adapted for analysing 365 values.

In finding the value of the height of the water for each day, the algebraical mean of 24 consecutive hourly values, beginning with the height at noon, is taken: the result will then apply to the middle instant of the period 0<sup>h</sup> to 23<sup>h</sup>, that is to say to 11<sup>h</sup> 30<sup>m</sup> at night.

19. The formation of a daily mean does not obliterate the tidal oscillations of short period, because none of the tides, excepting those of the principal solar series, have commensurable periods in mean solar time.

Clearance for short-period tides.

A correction, or 'clearance of the daily mean', should therefore be applied for all the important tides of short period, excepting for the solar tides.

Let  $R \cos(nt - \zeta)$  be the expression for one of the tides of short period, as evaluated by the harmonic analysis for the same year: and let  $a$  be the value of  $nt - \zeta$  at any noon. Then the 24 consecutive hourly heights of the water due to this tide, beginning with that noon, are:—

$$R \cos a; \quad R \cos(n + a); \quad R \cos(2n + a) \dots R \cos(23n + a);$$

the sum of these is  $R \frac{\sin 12n}{\sin \frac{1}{2}n} \cos(a + 11\frac{1}{2}n)$ , so that the 'clearance of the daily mean' is  $-\frac{1}{24}R \frac{\sin 12n}{\sin \frac{1}{2}n} \cos(a + 11\frac{1}{2}n)$ , and is additive.

It has been found, practically, that only three tides of short period *viz.*,  $M_2$ ,  $N$ ,  $O$ , exercise any appreciable effect, so that clearances for them have to be applied. It was formerly the custom to compute the clearances for these three tides, for every day in the year, as above and to correct the daily means accordingly: but the procedure now is different, and a single correction, for each short-period tide, is applied to each of the final equations, instead of to each daily mean. The process will be explained more fully below.

20. The mean of the 365 quantities is now taken to give the mean height of the water for the year; and it is evident that, even if the daily means are uncleared from the effects of the short-period tides, as is the case in practice, this yearly mean cannot be sensibly vitiated.

The yearly mean height is next subtracted from each of the 365 daily means, and 365 quantities,  $\delta h$ , are found giving the mean daily height of the water above the mean yearly height.

These quantities are to be the subject of harmonic analysis: and the tides chosen for evaluation are those which have been denoted above as  $Mm$ ,  $Mf$ ,  $MSf$ ,  $Sa$  and  $Ssa$ .

Let

$$\begin{aligned} \delta h = & A \cos(\sigma - \omega)t + B \sin(\sigma - \omega)t \\ & + C \cos 2\sigma t + D \sin 2\sigma t \\ & + C' \cos 2(\sigma - \eta)t + D' \sin 2(\sigma - \eta)t \\ & + E \cos \eta t + F \sin \eta t \\ & + G \cos 2\eta t + H \sin 2\eta t, \end{aligned}$$

where  $t$  is time measured from the first 11<sup>h</sup> 30<sup>m</sup>.

Then a little manipulation, for which the reader is referred to Professor Darwin's Report of 1883, gives the following equations:—

|                                                   | Coefficient of A | Coefficient of B | Coefficient of C | Coefficient of D | Coefficient of C' | Coefficient of D' | Coefficient of E | Coefficient of F | Coefficient of G | Coefficient of H |
|---------------------------------------------------|------------------|------------------|------------------|------------------|-------------------|-------------------|------------------|------------------|------------------|------------------|
| $\Sigma \delta h \times \cos(\sigma - \omega)t =$ | +183.05          | + 2.14           | + 0.73           | + 4.29           | + 0.77            | + 5.04            | + 4.88           | - 0.34           | + 4.96           | - 0.69           |
| „ $\times \sin(\sigma - \omega)t =$               | + 2.14           | +181.95          | - 4.15           | + 1.02           | - 4.90            | + 1.07            | + 3.80           | + 0.34           | + 3.88           | + 0.69           |
| „ $\times \cos 2\sigma t =$                       | + 0.73           | - 4.15           | +183.18          | + 0.88           | + 0.61            | + 0.92            | - 1.50           | - 0.10           | - 1.51           | - 0.19           |
| „ $\times \sin 2\sigma t =$                       | + 4.29           | + 1.02           | + 0.88           | +181.82          | + 0.92            | - 0.75            | + 3.05           | - 0.08           | + 3.06           | - 0.17           |
| „ $\times \cos 2(\sigma - \eta)t =$               | + 0.77           | - 4.90           | + 0.61           | + 0.92           | +183.19           | + 0.97            | - 1.68           | - 0.11           | - 1.70           | - 0.23           |
| „ $\times \sin 2(\sigma - \eta)t =$               | + 5.04           | + 1.07           | + 0.92           | - 0.75           | + 0.97            | +181.81           | + 3.25           | - 0.10           | + 3.27           | - 0.23           |
| „ $\times \cos \eta t =$                          | + 4.88           | + 3.80           | - 1.50           | + 3.05           | - 1.68            | + 3.25            | +182.43          | + 0.00           | - 0.14           | + 0.00           |
| „ $\times \sin \eta t =$                          | - 0.34           | + 0.34           | - 0.10           | - 0.08           | - 0.11            | - 0.10            | + 0.00           | +182.57          | + 0.00           | + 0.00           |
| „ $\times \cos 2\eta t =$                         | + 4.96           | + 3.88           | - 1.51           | + 3.06           | - 1.70            | + 3.27            | - 0.14           | + 0.00           | +182.43          | + 0.00           |
| „ $\times \sin 2\eta t =$                         | - 0.69           | + 0.69           | - 0.19           | - 0.17           | - 0.23            | - 0.23            | + 0.00           | + 0.00           | + 0.00           | +182.57          |

21. The left-hand sides of these equations must now be cleared from the effects of the three tides of short period. This is done in the following manner:—

It has been shown before that the 'clearance' is

$$- \frac{1}{24} R \frac{\sin 12n}{\sin \frac{1}{2}n} \cos(nt - \zeta + 11\frac{1}{2}n).$$

The proper clearances therefore to be applied to the left-hand sides of the first and second equations will be

$$- \frac{1}{24} \Sigma R \frac{\sin 12n}{\sin \frac{1}{2}n} \cos(nt - \zeta + 11\frac{1}{2}n) \cos(\sigma - \omega)t,$$

and 
$$- \frac{1}{24} \Sigma R \frac{\sin 12n}{\sin \frac{1}{2}n} \cos(nt - \zeta + 11\frac{1}{2}n) \sin(\sigma - \omega)t,$$

the summation extending over 365 days.

Writing  $R\cos\zeta=A$  and  $R\sin\zeta=B$ , Professor Darwin in his report of 1883 has deduced these clearances in the forms

$$X_1 A + X_2 B \quad \text{and} \quad Y_1 A + Y_2 B,$$

where the X's apply to the left-hand sides containing a cosine and the Y's to those containing a sine, and the A's and B's are taken from the analysis of the corresponding short-period tides. Simple formulæ are also given for computing the X's and Y's; their values are shown in the annexed table, and are to be applied with the signs there given:—

|                                         |       | $\sigma - \omega$ | $2\sigma$ | $2(\sigma - \eta)$ | $\eta$    | $2\eta$   |
|-----------------------------------------|-------|-------------------|-----------|--------------------|-----------|-----------|
| $M_2$<br>$n = 2(\gamma - \sigma)$       | $X_1$ | - 0.05557         | + 0.00302 | + 5.7393           | - 0.10410 | - 0.10465 |
|                                         | $X_2$ | - 0.17036         | - 0.03773 | - 2.9228           | - 0.07525 | - 0.07546 |
|                                         | $Y_1$ | - 0.17075         | + 0.04170 | - 2.8400           | - 0.00176 | - 0.00353 |
|                                         | $Y_2$ | + 0.04410         | + 0.01052 | - 5.7271           | + 0.00476 | + 0.00958 |
| $N$<br>$n = 2\gamma - 3\sigma + \omega$ | $X_1$ | - 0.05884         | + 0.03680 | + 0.02938          | - 0.01760 | - 0.01760 |
|                                         | $X_2$ | - 0.07758         | - 0.22337 | - 0.19384          | + 0.00254 | + 0.00254 |
|                                         | $Y_1$ | - 0.02059         | - 0.15245 | - 0.12210          | + 0.00020 | + 0.00041 |
|                                         | $Y_2$ | + 0.11381         | - 0.08544 | - 0.08081          | + 0.00007 | + 0.00015 |
| $O$<br>$n = \gamma - 2\sigma$           | $X_1$ | - 0.06485         | + 0.01673 | + 0.01582          | - 0.19240 | - 0.19340 |
|                                         | $X_2$ | - 0.34765         | - 0.07788 | - 0.08158          | - 0.18260 | - 0.18311 |
|                                         | $Y_1$ | - 0.34523         | + 0.08418 | + 0.08748          | - 0.00460 | - 0.00926 |
|                                         | $Y_2$ | + 0.04052         | + 0.03379 | + 0.03295          | + 0.00897 | + 0.01802 |

22. Having now obtained the 'cleared' values of the left-hand sides of the equations, the left-hand side of the

Solution of equations.

first equation is divided by the coefficient of A in that equation, the left-hand side of the second equation by the coefficient of B in that equation and so on, the results being approximate values of A, B, C, &c. These are now substituted in the ten equations and the final values of A, B, C, &c. deduced. But the initial instant of time is the first 11<sup>h</sup> 30<sup>m</sup> in the year, instead of the first noon. Hence, if as before,

$$R^2 = A^2 + B^2 \quad \text{and} \quad \tan \zeta_1 = \frac{B}{A},$$

then, in order to reduce the results to the normal form in which noon of the first day is the initial instant of time, the increment of the corresponding argument for 11<sup>h</sup> 30<sup>m</sup> must be added to  $\zeta_1$  to get  $\zeta$ . Having thus determined  $R$  and  $\zeta$ ,  $H$  and  $\kappa$  will be found as before by multiplying  $R$  by its proper factor of reduction and by adding to  $\zeta$  the initial argument.

23. It may happen from time to time that the tide-gauge breaks down for a few days, from the stoppage of the clock, the choking of the tube, or some other accident and that other readings are not taken during the interruption. In this case there will be a hiatus in the values of  $\delta h$ . Now the whole process employed depends on the existence of 365 continuous values of  $\delta h$ . Unless, therefore, the year's observations are to be sacrificed, this hiatus must be filled. If not more than three or four days observations are wanting, it is best to plot out the values of  $\delta h$  graphically on each side of the hiatus and, filling in the gap with a curve drawn by hand, use the values of  $\delta h$  given by the conjectural curve. If the gap is somewhat longer several plans might be adopted, for example, if there is another station in the neighbourhood the values of  $\delta h$  for that station might be inserted; or, the values of  $\delta h$  for another part of the year, in which the moon's and sun's declinations are as nearly as possible the same as they were during the gap might be used and, as a matter of fact, these methods have been used. When the hiatus is of considerable length the preceding methods are inapplicable, and the plan employed is as follows:—The actual  $\delta h$ 's are entered in their proper places; then in the ten final equations all the terms with small coefficients are neglected, and in the terms whose coefficients are approximately 182.5, a coefficient equal to 182.5 diminished by half the number of days of hiatus, is substituted; the computations are then carried out, as if there was no gap, until the values of  $R$  and  $\zeta$  are obtained for each long-period tide. From these approximate values of  $R$  and  $\zeta$ , the height of each tide for each day of the gap is computed from the formula  $R \cos(nt - \zeta)$ , where  $t$  is the number of days since the commencement of the year of observation and  $n$  the speed of the particular long-period tide per mean solar day. Thus five heights, above or below mean-water level, are obtained for each day of the gap. These five heights are added together and the sum is the missing  $\delta h$  for the particular day. The gap having been thus

Interruption in record.

filled in with computed  $\delta h$ 's, the whole computation is repeated with the completed series.

Where a break extends over two or three months in the first half of a working year, the observations antecedent to the break in that half-year are rejected, and the date of the working year put forward to the date following the end of the break: but if the break occurs in the second half of the working year, that year is considered to end at the commencement of the break, and to begin 365 days before it. And the year following the break will begin at the end of the break.

24. As the determination of each of the ten quantities  $\Sigma \delta h \cos(\sigma - \varpi)t$ ,  $\Sigma \delta h \sin(\sigma - \varpi)t$ , &c., by multiplying each of the 365  $\delta h$ 's by its proper cosine or sine and adding the results together, would be extremely laborious, the method of equivalent multipliers has been devised by Professor Adams. The values of the respective cosines and sines are divided into eleven groups, according as they fall nearest to 1.0, .9, .8, .7 . . . .2, .1, 0. Then, as all the values of  $\delta h$  are to be multiplied by some value of the cosine or sine, and that value must fall into one of these groups, all the values of  $\delta h$  which belong to one of these groups are collected together, summed and the sum multiplied by the corresponding multiplier. Since there are as many positive as negative values of the cosine or sine, the signs of half of the  $\delta h$ 's must be changed: this is effected mechanically as follows:—In the spaces in the forms for the entry of the  $\delta h$ 's, those  $\delta h$ 's whose signs are to be unchanged are to be entered on the left side of the space if positive, and to the right if negative. Thus, in the column corresponding to each multiplier, there are two sub-columns: these are separately summed and the difference of these sums gives the total of the column for the  $\delta h$ 's whose signs are to be unchanged. This process is carried out in the upper half of the form and the result is called  $a$ . Exactly the same course is adopted in the lower half of the form with the  $\delta h$ 's whose signs have to be changed, and the result is denoted by  $b$ . The complete sum of the  $\delta h$ 's is thus  $a - b$ , and the value of  $a - b$  in each column is multiplied by the multiplier corresponding to that column, when the sum of the products will give the result required. A pair of forms, one for the cosine and the other for the sine series, is of course required for each long-period tide.



25. From the analysis of the observations at any port, one value of  $H$  (the mean semi-range) and one value of  $\kappa$  (which may be called the mean epoch) are obtained for each constituent for each year of observation analysed. For the larger tides  $M_2$ ,  $S_2$ ,  $K$ , &c., the values obtained are very accordant, but in the smaller tides there are considerable discrepancies from year to year. The means of the values of  $H$  and  $\kappa$  for each tide are accepted as the best results.

Suppose, now, it is required to predict the tides at Karáchi for 1891; the values of the  $R$  and  $\zeta$  are computed for 0<sup>h</sup> January 1st, 1891 for Karáchi for each of the constituent tides represented on the tide-predicting machine. The value of  $R$  gives the proper throw of the corresponding crank and  $\zeta$  the angle at which it has to be set in starting the tide-predicting machine belonging to the Indian Government at present located in the India Store Depôt at Lambeth.

The computations of the  $R$ 's and  $\zeta$ 's are carried out in the way described in the account of the reductions of the observations, the only difference being that to find  $R$  from  $H$  and  $\zeta$  from  $\kappa$ , the formulæ  $R=fH$  and  $\zeta=\kappa-(V_0+u)$  are used.

When all the  $R$ 's and  $\zeta$ 's have been computed, the results are sent to Mr. E. Roberts of the Nautical Almanac Office for employment in the tide-predicter.

The depth, also, below mean sea-level of the datum-line of reference from which the heights are estimated is required, in order to enable the pen which draws the datum-line on the tide-diagram to be set. Until lately there was no scientific definition of the usual term "Low-water Ordinary Spring-Tides", but, after consultation between Professor Darwin, Major Baird and Captain Wharton, the British Hydrographer, it was determined to use for Indian charts a datum termed the "Indian Low-water Spring-Tides". This is the depth below mean sea-level of the sum of the mean lunar semi-diurnal, the mean solar semi-diurnal, the mean luni-solar diurnal and the mean lunar diurnal tide, or in the notation used in this handbook,

$$A_0 - (H \text{ of } M_2 + H \text{ of } S_2 + H \text{ of } K_1 + H \text{ of } O_1).$$

The machine is started with the proper setting of the cranks and the tide-curve for a whole year is automatically traced on a roll of

paper in about four hours. The time and height of each high and low water are measured, and the tabulated results form the tide-tables for the port.

This is the procedure followed in regard to open coast stations: the riverain stations are treated in a different way, the observations being reduced separately for each month in the year, and the semi-monthly curves obtained from the means of the corresponding months throughout the total period of the observations.

26. In entering the heights of the water, read off the diagrams, in the computation forms, various precautions are taken to guard against error. The procedure adopted in regard to these forms will now be explained, in so far as is not self-evident from the forms themselves. The forms given below have for convenience been numbered consecutively from I to XI, but this is not the case in the actual forms.

27. After the diagrams have been prepared, the heights are successively measured and entered in the respective hour columns of this form, the first entry being that of 0<sup>h</sup> of the 1st day. The first day is called 1 in the forms for short-period tides and 0 in those for long-period tides. The date for each day is generally written in pencil on the left-hand margin.

The measurements are made to hundredths of a foot, by means of a paper scale divided into tenths and hundredths in accordance with working scale of the instrument which registered the diagram under treatment. When the reader has called out the entry for each 23rd hour, the recorder checks the record by calling out 'end of the day' and then gives the date of the next day.

The reading and entry of the heights in the S-Series is done in duplicate, the original set being generally measured by a surveyor and the duplicate by a native computer. While the measurement for the original set, the clerk watches the entries in the original set, and checks the duplicate set to within 0.05 of a foot ac-

remeasured. The original set is measured by a surveyor and the duplicate by a native computer. While the measurement for the original set, the clerk watches the entries in the original set, and checks the duplicate set to within 0.05 of a foot accuracy. If there are any discrepancies larger than those above mentioned, the original set is re-measured.

paper in about four hours. The time and height of each high and low water are measured, and the tabulated results form the tide-tables for the port.

This is the procedure followed in regard to open coast stations: the riverain stations are treated in a different way, the observations being reduced separately for each month in the year, and the semi-monthly curves obtained from the means of the corresponding months throughout the total period of the observations.

26. In entering the heights of the water, read off the diagrams, in the computation forms, various precautions are taken to guard against error. The procedure adopted in regard to these forms will now be explained, in so far as is not self-evident from the forms themselves. The forms given below have for convenience been numbered consecutively from 1 to XI, but this is not the case in the actual forms.

27. After the diagrams have been prepared, the heights are successively measured and entered in the respective hour columns of this form, the first entry being that of 0<sup>h</sup> of the 1st day. The first day is called 1 in the forms for short-period tides and 0 in those for long-period tides. The date for each day is generally written in pencil on the left-hand margin.

The measurements are made to hundredths of a foot, by means of a paper scale divided into tenths and hundredths in accordance with working scale of the instrument which registered the diagram under treatment. When the reader has called out the entry for each 23rd hour, the recorder checks the record by calling out 'end of the day' and then gives the date of the next day.

The reading and entry of the heights in the S-Series is done in duplicate, the original set being generally measured by a surveyor and the duplicate by a native computer. While the measurement for the duplicate is going on, a second clerk watches the entries in the original and if the reading differs by more than 0.01, 0.03 or 0.05 of a foot according as it is the natural, half or smaller scale, that height is at once remeasured and a correct value entered both in the original and the duplicate. The original and duplicate are now compared, and if there are any discrepancies larger than those above mentioned, they are

noted, remeasured at the completion of the comparison, and correct values entered in both copies.

28. The heights are next copied from the original of the S-Series into the M-Series, from the M-Series into the O-Series, from that into the K-Series and so on, the last being now the 2MK-Series.

Where a double dot occurs in the forms, it indicates that two successive hourly values of the S-Series are to be entered, the first above the second: when a horizontal line takes the place of one of the usual dots, it means that no entry is to be made there, but that the next entry is to be in the next column to the right.

A black vertical line means that the solar day divides at the line and that the height immediately preceding corresponds to a 23rd solar hour. A double dot with a short black vertical line opposite the upper one, means that the entry made at the upper dot is the height corresponding to the 23rd hour of one solar day, and that at the lower dot to the 0 hour of the next solar day. By these marks, the copyist knows that he is at the end of a mean solar day.

At the right-hand side of all the forms, except the S-form, is a column headed S-hour giving the day and hour of the S-Series corresponding to the 23rd hour of the particular day of the series in question. This is a further check but is rarely used.

29. When the 2MK-Series has been copied from the MK-Series, it is compared simultaneously with the original and duplicate of the S-Series, and if it agrees with the original the copying is perfect. The comparison with the duplicate guards against gross errors which may have escaped notice in the comparison of the original and duplicate.

Errors found in the 2MK-Series are searched for in the other series in the reverse order until one is reached in which the error does not occur, and corrections are made accordingly.

30. The heights in each column are then added together, the units, tens and hundreds being separately summed, and the sums entered at the bottom of the page; some weeks afterwards they are verified by fresh computers.

Besides this, for the S-Series, the sum of the 24 hourly heights is taken for each day and entered in the column 'daily sum', which, divided by 24, gives the quantity in the column 'daily mean'.

31. For the S-Series the total of the 'daily sums' should be equal to the total of the horizontal line at the bottom of the page in each of the five pages of the form.

Checks.

For the other series, for example the M-Series, the total of the heights on page 1 of M should be the total of the heights on page 1 of S, less the sum of the last 12 hours on page 1 of S, since the last entry on page 1 of M corresponds to 7<sup>d</sup> 11<sup>h</sup> of S. In comparing the totals after the first page, account must be taken of the number of entries in excess or defect at the beginning as well as at the end of each page, as compared with the entries on the corresponding page of S.

32. This requires no explanation; but care must be taken that the number of observations is correct: it should be the sum of the five quantities, one on each page, at the bottom of the page under the corresponding hour.

Form V. Summations and means.

The remainder of the forms for short-period tides are self explanatory.

33. The mean height of the water for each day is taken from the column 'daily means' in the S-Series, and the mean height of the water for the whole year, or  $A_0$ , is determined in the form for the S-Series which corresponds to Form VI. The latter mean is subtracted from each of the former quantities, giving a number of small positive and negative quantities  $\delta h$ , one for each day. These are entered in Form IX, of which there are two for each long-period tide, in the manner described in the footnote.

Form IX. Long-period tides.

In solving the first equation, the second line is obtained by introducing for B, C, D, &c., the first approximate values obtained in the preceding form. In solving the second equation, the second approximate value of A is introduced and the first of the other quantities, C, D, &c., and so on.

Form XI.

The rest of the computations is sufficiently evident from an inspection of the forms given below.

COMPUTATIONS.

[PART VI.]

KARACHI, 1883-84.

SHORT-PERIOD TIDES.

Commencing 0 hours.

Astronomical time, 1st May, 1883.

Motion per mean Solar hour = 15°.

FORM I.—SERIES S.

Argument ( $\gamma - \eta$ ).

|     | h 0    | h 1    | h 2    | h 3    | h 4    | h 5    | h 6    | h 7    | h 8    | h 9    | h 10   | h 11   | h 12   | h 13   | h 14   | h 15   | h 16   | h 17   | h 18   | h 19   | h 20   | h 21   | h 22   | h 23   | Daily Sum | Daily Mean |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|-----------|------------|
| 1   | 5'22   | 6'35   | 7'60   | 8'62   | 9'42   | 9'86   | 9'76   | 9'00   | 8'16   | 7'30   | 6'30   | 5'67   | 5'44   | 5'76   | 6'46   | 7'18   | 7'92   | 8'56   | 8'80   | 8'36   | 7'41   | 6'52   | 5'62   | 4'91   | 176'20    | 7'342      |
| 2   | 4'81   | 5'51   | 6'56   | 7'81   | 8'98   | 9'82   | 10'36  | 10'24  | 9'34   | 8'10   | 6'90   | 5'77   | 4'75   | 4'36   | 4'97   | 5'86   | 6'95   | 8'15   | 9'07   | 9'42   | 9'09   | 8'14   | 6'90   | 5'76   | 177'62    | 7'401      |
| 3   | 5'00   | 4'83   | 5'50   | 6'72   | 8'10   | 9'28   | 10'18  | 10'74  | 10'48  | 9'26   | 7'81   | 6'20   | 4'59   | 3'36   | 3'32   | 4'95   | 5'50   | 6'91   | 8'50   | 9'64   | 10'14  | 9'76   | 8'48   | 7'08   | 175'43    | 7'310      |
| 4   | 5'79   | 4'96   | 4'71   | 5'37   | 6'77   | 8'24   | 9'51   | 10'44  | 10'92  | 10'27  | 8'76   | 6'96   | 5'05   | 3'23   | 3'07   | 2'34   | 3'45   | 5'28   | 7'06   | 8'84   | 10'24  | 10'84  | 10'20  | 8'72   | 170'03    | 7'084      |
| 5   | 7'24   | 5'96   | 4'90   | 4'48   | 5'23   | 6'84   | 8'51   | 9'84   | 10'74  | 11'04  | 10'20  | 8'36   | 6'13   | 4'04   | 3'21   | 1'32   | 1'75   | 3'29   | 5'48   | 7'48   | 9'40   | 10'82  | 11'40  | 10'59  | 167'25    | 6'969      |
| 70  | 10'02  | 10'67  | 10'74  | 9'90   | 8'52   | 7'06   | 5'83   | 5'03   | 5'05   | 5'72   | 6'77   | 7'87   | 8'80   | 9'20   | 8'78   | 7'80   | 6'77   | 5'74   | 5'02   | 4'80   | 5'01   | 5'66   | 6'77   | 8'07   | 175'60    | 7'317      |
| 71  | 9'23   | 10'04  | 10'50  | 10'14  | 9'00   | 7'60   | 6'43   | 5'30   | 4'72   | 5'00   | 5'84   | 6'87   | 7'68   | 8'18   | 8'50   | 8'27   | 7'32   | 6'35   | 5'83   | 5'64   | 5'48   | 5'40   | 6'24   | 6'48   | 172'04    | 7'168      |
| 72  | 6'83   | 7'27   | 7'66   | 8'04   | 8'24   | 8'31   | 6'90   | 5'86   | 5'06   | 4'77   | 5'68   | 5'77   | 6'28   | 6'63   | 6'96   | 7'26   | 7'41   | 7'14   | 6'71   | 6'47   | 6'20   | 5'87   | 6'10   | 6'66   | 159'48    | 6'645      |
| 73  | 7'27   | 7'60   | 7'87   | 8'13   | 8'30   | 8'41   | 7'50   | 6'41   | 5'53   | 4'94   | 4'81   | 4'98   | 5'52   | 6'33   | 6'93   | 7'33   | 7'61   | 7'58   | 7'44   | 7'10   | 6'71   | 6'38   | 6'34   | 6'54   | 163'56    | 6'815      |
| 74  | 6'96   | 7'68   | 8'45   | 8'92   | 8'96   | 8'46   | 7'88   | 6'87   | 5'92   | 5'00   | 4'44   | 4'32   | 4'64   | 5'11   | 5'69   | 6'52   | 7'19   | 7'74   | 7'87   | 7'76   | 7'38   | 7'02   | 6'87   | 6'77   | 164'42    | 6'851      |
| Sum | 646'01 | 647'27 | 641'04 | 644'89 | 604'65 | 591'02 | 584'87 | 587'09 | 591'62 | 592'54 | 587'31 | 570'00 | 535'33 | 489'49 | 442'73 | 408'68 | 393'70 | 398'91 | 430'37 | 472'53 | 519'28 | 564'88 | 604'18 | 634'41 | 13162'80  |            |

and

Compared by

Measured from Diagram by

Summed by

Checked by

CHAP. III.]

COMPUTATIONS.

[KARACHI, 1883-84.

SHORT-PERIOD TIDES.

FORM II.—SERIES M.

Motion per mean Solar hour =  $14^{\circ}.4920521$ .

Commencing 0 hours.

Astronomical time, 1st May, 1883.

Argument ( $\gamma - \sigma$ ).

|     | 0 <sup>h</sup> | 1 <sup>h</sup> | 2 <sup>h</sup> | 3 <sup>h</sup> | 4 <sup>h</sup> | 5 <sup>h</sup> | 6 <sup>h</sup> | 7 <sup>h</sup> | 8 <sup>h</sup> | 9 <sup>h</sup> | 10 <sup>h</sup> | 11 <sup>h</sup> | 12 <sup>h</sup> | 13 <sup>h</sup> | 14 <sup>h</sup> | 15 <sup>h</sup> | 16 <sup>h</sup> | 17 <sup>h</sup> | 18 <sup>h</sup> | 19 <sup>h</sup> | 20 <sup>h</sup> | 21 <sup>h</sup> | 22 <sup>h</sup> | 23 <sup>h</sup> | S. hour      |                                   |
|-----|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|--------------|-----------------------------------|
| 1   | 5.22           | 6.35           | 7.60           | 8.62           | 9.42           | 9.86           | 9.76           | 9.00           | 8.16           | 7.30           | 6.30            | 5.67            | 5.44            | 5.76            | 6.46<br>7.18    | 7.92            | 8.56            | 8.80            | 8.36            | 7.41            | 6.52            | 5.62            | 4.91            | 4.81            | 4.81         | 2 <sup>d</sup> 0 <sup>h</sup>     |
| 2   | 5.51           | 6.56           | 7.81           | 8.98           | 9.82           | 10.36          | 10.24          | 9.34           | 8.10           | 6.90           | 5.77            | 4.75            | 4.36            | 4.97            | 5.86            | 6.95            | 8.15            | 9.07            | 9.42<br>9.09    | 8.14            | 6.90            | 5.76            | 5.00            | 4.83            | 4.83         | 3 1                               |
| 3   | 5.50           | 6.72           | 8.10           | 9.28           | 10.18          | 10.74          | 10.48          | 9.26           | 7.81           | 6.20           | 4.59            | 3.36            | 3.32            | 4.05            | 5.50            | 6.91            | 8.50            | 9.64            | 10.14           | 9.76            | 8.48            | 7.08            | 5.79            | 4.96            | 4.71         | 4 2                               |
| 4   | 5.37           | 6.77           | 8.24           | 9.51           | 10.44          | 10.92          | 10.27          | 8.76           | 6.96           | 5.05           | 3.23            | 2.07            | 2.34            | 3.45            | 5.28            | 7.06            | 8.84            | 10.24           | 10.84           | 10.20           | 8.72            | 7.24            | 5.96            | 4.90            | 5 2          |                                   |
| 5   | 4.48           | 5.23           | 6.84           | 8.51<br>9.84   | 10.74          | 11.04          | 11.20          | 8.36           | 6.13           | 4.04           | 2.21            | 1.32            | 1.75            | 3.29            | 5.48            | 7.48            | 9.40            | 10.82           | 11.40           | 10.59           | 8.86            | 7.36            | 5.99            | 4.90            | 6 3          |                                   |
| 67  | 5.22           | 6.16           | 7.14           | 7.97           | 8.68           | 9.10           | 8.50           | 7.32           | 5.84           | 4.60           | 3.96            | 3.96            | 4.64            | 5.70            | 7.33            | 8.88            | 10.02           | 10.67           | 10.74           | 9.90            | 8.52            | 7.06            | 5.83            | 5.03            | 70 7         |                                   |
| 68  | 5.05           | 5.72           | 6.77           | 7.87           | 9.20           | 8.78           | 7.80           | 6.77           | 5.74           | 5.02           | 4.80            | 5.01            | 5.66            | 6.77            | 8.07            | 9.23            | 10.04           | 10.50           | 10.14           | 9.00            | 7.60            | 6.43            | 5.30            | 4.72            | 71 8         |                                   |
| 69  | 5.00           | 5.84           | 6.87           | 7.68           | 8.18           | 8.50           | 8.27           | 7.32           | 6.35<br>5.83   | 5.64           | 5.48            | 5.40            | 6.24            | 6.48            | 6.83            | 7.27            | 7.66            | 8.04            | 8.24            | 8.31            | 6.90            | 5.00            | 5.06            | 4.77            | 72 9         |                                   |
| 70  | 5.00           | 5.77           | 6.28           | 6.63           | 6.96           | 7.26           | 7.41           | 7.14           | 6.71           | 6.47           | 6.20            | 5.87            | 6.10            | 6.66<br>7.27    | 7.60            | 7.87            | 8.13            | 8.30            | 8.41            | 7.50            | 6.41            | 5.53            | 4.94            | 4.81            | 73 10        |                                   |
| 71  | 4.98           | 5.52           | 6.33           | 6.93           | 7.33           | 7.61           | 7.58           | 7.44           | 7.10           | 6.71           | 6.38            | 6.34            | 6.54            | 6.96            | 7.68            | 8.45            | 8.92            | 8.96<br>8.46    | 7.88            | 6.87            | 5.92            | 5.00            | 4.44            | 4.32            | 74 11        |                                   |
| Sum | 361.34         | 447.11         | 555.96         | 641.43         | 700.00         | 735.57         | 723.91         | 647.44         | 565.07         | 456.26         | 375.65          | 330.69          | 317.89          | 425.92          | 512.91          | 596.94          | 673.81          | 739.51          | 728.88          | 662.97          | 575.35          | 489.01          | 403.65          | 366.47          | No. of Days. |                                   |
| No. | 73             | 73             | 74             | 74             | 73             | 73             | 73             | 73             | 75             | 73             | 73              | 73              | 74              | 74              | 74              | 73              | 73              | 75              | 74              | 73              | 73              | 74              | 73              | 74              | 74           | 73 <sup>d</sup> 12 <sup>h</sup> . |

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13\*02-24

Commencing 0 hours.

Astronomical time, 1st May, 1883.

Argument ( $\gamma = 2\sigma$ ).

SHORT-PERIOD TIDES.

[KARACHI, 1883-84.]

FORM III.—SERIES O.

Motion per mean Solar hour =  $13^{\circ}.9430356$ .

|     | 0 <sup>h</sup>                     | 1 <sup>h</sup>    | 2 <sup>h</sup>                       | 3 <sup>h</sup>                       | 4 <sup>h</sup>    | 5 <sup>h</sup>                     | 6 <sup>h</sup>                     | 7 <sup>h</sup>                     | 8 <sup>h</sup>                     | 9 <sup>h</sup>    | 10 <sup>h</sup>                    | 11 <sup>h</sup>                    | 12 <sup>h</sup>   | 13 <sup>h</sup>                     | 14 <sup>h</sup>                    | 15 <sup>h</sup>   | 16 <sup>h</sup>                    | 17 <sup>h</sup>   | 18 <sup>h</sup>                    | 19 <sup>h</sup>                    | 20 <sup>h</sup>   | 21 <sup>h</sup>                    | 22 <sup>h</sup>                    | 23 <sup>h</sup>                    | S. hour                          |
|-----|------------------------------------|-------------------|--------------------------------------|--------------------------------------|-------------------|------------------------------------|------------------------------------|------------------------------------|------------------------------------|-------------------|------------------------------------|------------------------------------|-------------------|-------------------------------------|------------------------------------|-------------------|------------------------------------|-------------------|------------------------------------|------------------------------------|-------------------|------------------------------------|------------------------------------|------------------------------------|----------------------------------|
| 1   | 5 <sup>22</sup>                    | 6 <sup>35</sup>   | 7 <sup>60</sup>                      | 8 <sup>62</sup>                      | 9 <sup>42</sup>   | 9 <sup>86</sup>                    | 9 <sup>76</sup><br>9 <sup>00</sup> | 8 <sup>16</sup>                    | 7 <sup>30</sup>                    | 6 <sup>30</sup>   | 5 <sup>67</sup>                    | 5 <sup>44</sup>                    | 5 <sup>76</sup>   | 6 <sup>46</sup>                     | 7 <sup>18</sup>                    | 7 <sup>92</sup>   | 8 <sup>56</sup>                    | 8 <sup>80</sup>   | 8 <sup>36</sup>                    | 7 <sup>41</sup><br>6 <sup>52</sup> | 5 <sup>62</sup>   | 4 <sup>91</sup>                    | 4 <sup>81</sup>                    | 5 <sup>51</sup>                    | 2 <sup>d</sup> 1 <sup>h</sup>    |
| 2   | 6 <sup>56</sup>                    | 7 <sup>81</sup>   | 8 <sup>98</sup>                      | 9 <sup>82</sup>                      | 10 <sup>36</sup>  | 10 <sup>24</sup>                   | 9 <sup>34</sup>                    | 8 <sup>10</sup>                    | 6 <sup>90</sup><br>5 <sup>77</sup> | 4 <sup>75</sup>   | 4 <sup>36</sup>                    | 4 <sup>97</sup>                    | 5 <sup>86</sup>   | 6 <sup>95</sup>                     | 8 <sup>15</sup>                    | 9 <sup>07</sup>   | 9 <sup>42</sup>                    | 9 <sup>09</sup>   | 8 <sup>14</sup>                    | 6 <sup>90</sup>                    | 5 <sup>76</sup>   | 5 <sup>00</sup>                    | 4 <sup>83</sup><br>5 <sup>50</sup> | 6 <sup>72</sup>                    | 3 3                              |
| 3   | 8 <sup>10</sup>                    | 9 <sup>28</sup>   | 10 <sup>18</sup>                     | 10 <sup>74</sup>                     | 10 <sup>48</sup>  | 9 <sup>26</sup>                    | 7 <sup>81</sup>                    | 6 <sup>20</sup>                    | 4 <sup>59</sup>                    | 3 <sup>36</sup>   | 3 <sup>32</sup>                    | 4 <sup>05</sup><br>5 <sup>50</sup> | 6 <sup>91</sup>   | 8 <sup>50</sup>                     | 9 <sup>64</sup>                    | 10 <sup>14</sup>  | 9 <sup>76</sup>                    | 8 <sup>48</sup>   | 7 <sup>08</sup>                    | 5 <sup>79</sup>                    | 4 <sup>96</sup>   | 4 <sup>71</sup>                    | 5 <sup>37</sup>                    | 6 <sup>77</sup>                    | 4 4                              |
| 4   | 8 <sup>24</sup><br>9 <sup>51</sup> | 10 <sup>44</sup>  | 10 <sup>92</sup>                     | 10 <sup>27</sup>                     | 8 <sup>76</sup>   | 6 <sup>96</sup>                    | 5 <sup>05</sup>                    | 3 <sup>23</sup>                    | 2 <sup>07</sup>                    | 2 <sup>34</sup>   | 3 <sup>45</sup>                    | 5 <sup>28</sup>                    | 7 <sup>06</sup>   | 8 <sup>84</sup><br>10 <sup>24</sup> | 10 <sup>84</sup>                   | 10 <sup>20</sup>  | 8 <sup>72</sup>                    | 7 <sup>24</sup>   | 5 <sup>96</sup>                    | 4 <sup>90</sup>                    | 4 <sup>48</sup>   | 5 <sup>23</sup>                    | 6 <sup>84</sup>                    | 8 <sup>51</sup>                    | 5 6                              |
| 5   | 9 <sup>84</sup>                    | 10 <sup>74</sup>  | 11 <sup>04</sup><br>10 <sup>20</sup> | 8 <sup>36</sup>                      | 6 <sup>13</sup>   | 4 <sup>04</sup>                    | 2 <sup>21</sup>                    | 1 <sup>32</sup>                    | 1 <sup>75</sup>                    | 3 <sup>29</sup>   | 5 <sup>48</sup>                    | 7 <sup>48</sup>                    | 9 <sup>40</sup>   | 10 <sup>82</sup>                    | 11 <sup>40</sup>                   | 10 <sup>59</sup>  | 8 <sup>86</sup><br>7 <sup>36</sup> | 5 <sup>99</sup>   | 4 <sup>90</sup>                    | 4 <sup>56</sup>                    | 5 <sup>48</sup>   | 7 <sup>26</sup>                    | 8 <sup>96</sup>                    | 10 <sup>28</sup>                   | 6 8                              |
| 65  | 4 <sup>64</sup><br>5 <sup>70</sup> | 7 <sup>33</sup>   | 8 <sup>88</sup>                      | 10 <sup>02</sup>                     | 10 <sup>67</sup>  | 10 <sup>74</sup>                   | 9 <sup>90</sup>                    | 8 <sup>52</sup>                    | 7 <sup>06</sup>                    | 5 <sup>83</sup>   | 5 <sup>03</sup>                    | 5 <sup>05</sup>                    | 5 <sup>72</sup>   | 6 <sup>77</sup>                     | 7 <sup>87</sup><br>8 <sup>80</sup> | 9 <sup>20</sup>   | 8 <sup>78</sup>                    | 7 <sup>80</sup>   | 6 <sup>77</sup>                    | 5 <sup>74</sup>                    | 5 <sup>02</sup>   | 4 <sup>80</sup>                    | 5 <sup>01</sup>                    | 5 <sup>66</sup>                    | 70 21                            |
| 66  | 6 <sup>77</sup>                    | 8 <sup>07</sup>   | 9 <sup>23</sup>                      | 10 <sup>04</sup><br>10 <sup>50</sup> | 10 <sup>14</sup>  | 9 <sup>00</sup>                    | 7 <sup>60</sup>                    | 6 <sup>43</sup>                    | 5 <sup>30</sup>                    | 4 <sup>72</sup>   | 5 <sup>00</sup>                    | 5 <sup>84</sup>                    | 6 <sup>87</sup>   | 7 <sup>68</sup>                     | 8 <sup>18</sup>                    | 8 <sup>50</sup>   | 8 <sup>27</sup><br>7 <sup>32</sup> | 6 <sup>35</sup>   | 5 <sup>83</sup>                    | 5 <sup>64</sup>                    | 5 <sup>48</sup>   | 5 <sup>40</sup>                    | 6 <sup>24</sup>                    | 6 <sup>48</sup>                    | 71 23                            |
| 67  | 6 <sup>83</sup>                    | 7 <sup>27</sup>   | 7 <sup>66</sup>                      | 8 <sup>04</sup>                      | 8 <sup>24</sup>   | 8 <sup>31</sup><br>6 <sup>90</sup> | 5 <sup>86</sup>                    | 5 <sup>06</sup>                    | 4 <sup>77</sup>                    | 5 <sup>08</sup>   | 5 <sup>77</sup>                    | 6 <sup>28</sup>                    | 6 <sup>63</sup>   | 6 <sup>96</sup>                     | 7 <sup>26</sup>                    | 7 <sup>41</sup>   | 7 <sup>14</sup>                    | 6 <sup>71</sup>   | 6 <sup>47</sup><br>6 <sup>20</sup> | 5 <sup>87</sup>                    | 6 <sup>10</sup>   | 6 <sup>66</sup>                    | 7 <sup>27</sup>                    | 7 <sup>60</sup>                    | 73 1                             |
| 68  | 7 <sup>87</sup>                    | 8 <sup>13</sup>   | 8 <sup>30</sup>                      | 8 <sup>41</sup>                      | 7 <sup>50</sup>   | 6 <sup>41</sup>                    | 5 <sup>53</sup>                    | 4 <sup>94</sup><br>4 <sup>81</sup> | 4 <sup>98</sup>                    | 5 <sup>52</sup>   | 6 <sup>33</sup>                    | 6 <sup>93</sup>                    | 7 <sup>33</sup>   | 7 <sup>61</sup>                     | 7 <sup>58</sup>                    | 7 <sup>44</sup>   | 7 <sup>10</sup>                    | 6 <sup>71</sup>   | 6 <sup>38</sup>                    | 6 <sup>34</sup>                    | 6 <sup>54</sup>   | 6 <sup>96</sup><br>7 <sup>68</sup> | 8 <sup>45</sup>                    | 8 <sup>92</sup>                    | 74 3                             |
| 69  | 8 <sup>96</sup>                    | 8 <sup>46</sup>   | 7 <sup>88</sup>                      | 6 <sup>87</sup>                      | 5 <sup>92</sup>   | 5 <sup>00</sup>                    | 4 <sup>44</sup>                    | 4 <sup>32</sup>                    | 4 <sup>64</sup>                    | 5 <sup>11</sup>   | 5 <sup>69</sup><br>6 <sup>52</sup> | 7 <sup>19</sup>                    | 7 <sup>74</sup>   | 7 <sup>87</sup>                     | 7 <sup>76</sup>                    | 7 <sup>38</sup>   | 7 <sup>02</sup>                    | 6 <sup>87</sup>   | 6 <sup>77</sup>                    | 6 <sup>73</sup>                    | 7 <sup>20</sup>   | 7 <sup>88</sup>                    | 8 <sup>37</sup>                    | 8 <sup>73</sup><br>8 <sup>74</sup> | 75 5                             |
| Sum | 536 <sup>93</sup>                  | 520 <sup>49</sup> | 523 <sup>78</sup>                    | 509 <sup>52</sup>                    | 509 <sup>01</sup> | 493 <sup>46</sup>                  | 508 <sup>06</sup>                  | 509 <sup>57</sup>                  | 535 <sup>11</sup>                  | 532 <sup>66</sup> | 566 <sup>22</sup>                  | 569 <sup>10</sup>                  | 587 <sup>27</sup> | 588 <sup>44</sup>                   | 606 <sup>51</sup>                  | 594 <sup>24</sup> | 583 <sup>08</sup>                  | 572 <sup>95</sup> | 571 <sup>37</sup>                  | 565 <sup>98</sup>                  | 562 <sup>28</sup> | 560 <sup>03</sup>                  | 551 <sup>53</sup>                  | 552 <sup>86</sup>                  | No. of<br>Days.                  |
| No. | 75                                 | 73                | 75                                   | 74                                   | 74                | 74                                 | 74                                 | 75                                 | 75                                 | 74                | 74                                 | 75                                 | 73                | 74                                  | 75                                 | 74                | 74                                 | 73                | 75                                 | 74                                 | 75                | 74                                 | 75                                 | 74                                 | 74 <sup>d</sup> 6 <sup>h</sup> . |

COMPUTATIONS.

[PART VI.]



Commencing o hours.

SHORT-PERIOD TIDES.

[KARACHI, 1883-84.]

Astronomical time, 1st May, 1883.

FORM IV.—SERIES J.

Motion per mean Solar hour =  $15^{\circ} 58' 54.433''$ .

Argument ( $\gamma + \sigma - \omega$ ).

CHAP. III.]

|     | 0 <sup>h</sup> | 1 <sup>h</sup> | 2 <sup>h</sup> | 3 <sup>h</sup> | 4 <sup>h</sup> | 5 <sup>h</sup> | 6 <sup>h</sup> | 7 <sup>h</sup> | 8 <sup>h</sup> | 9 <sup>h</sup> | 10 <sup>h</sup> | 11 <sup>h</sup> | 12 <sup>h</sup> | 13 <sup>h</sup> | 14 <sup>h</sup> | 15 <sup>h</sup> | 16 <sup>h</sup> | 17 <sup>h</sup> | 18 <sup>h</sup> | 19 <sup>h</sup> | 20 <sup>h</sup> | 21 <sup>h</sup> | 22 <sup>h</sup> | 23 <sup>h</sup> | S. hour. |                                  |
|-----|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|----------|----------------------------------|
| 1   | 5'22           | 6'35           | 7'60           | 8'62           | 9'42           | 9'86           | 9'76           | 9'00           | 8'16           | 7'30           | 6'30            | 5'67            | 5'44            | —               | 5'76            | 6'46            | 7'18            | 7'92            | 8'56            | 8'80            | 8'36            | 7'41            | 6'52            | 5'62            | 5'62     | 1 <sup>d</sup> 22 <sup>h</sup>   |
| 2   | 4'91           | 4'81           | 5'51           | 6'56           | 7'81           | 8'98           | 9'82           | 10'36          | 10'24          | 9'34           | 8'10            | 6'90            | 5'77            | 4'75            | 4'36            | 4'97            | —               | 5'86            | 6'95            | 8'15            | 9'07            | 9'42            | 9'09            | 8'14            | 8'14     | 2 21                             |
| 3   | 6'90           | 5'76           | 5'00           | 4'83           | 5'50           | 6'72           | 8'10           | 9'28           | 10'18          | 10'74          | 10'48           | 9'26            | 7'81            | 6'20            | 4'59            | 3'36            | 3'32            | 4'05            | 5'50            | —               | 6'91            | 8'50            | 9'64            | 10'14           | 10'14    | 3 20                             |
| 4   | 9'76           | 8'48           | 7'08           | 5'79           | 4'96           | 4'71           | 5'37           | 6'77           | 8'24           | 9'51           | 10'44           | 10'92           | 10'27           | 8'76            | 6'96            | 5'05            | 3'23            | 2'07            | 2'34            | 3'45            | 5'28            | —               | 7'06            | 8'84            | 8'84     | 4 19                             |
| 5   | 10'24          | 10'84          | 10'20          | 8'72           | 7'24           | 5'96           | 4'90           | 4'48           | 5'23           | 6'84           | 8'51            | 9'84            | 10'74           | 11'04           | 10'20           | 8'36            | 6'13            | 4'04            | 2'21            | 1'32            | 1'75            | 3'29            | 5'48            | 7'48            | 7'48     | 5 19                             |
| 72  | 5'22           | 6'16           | 7'14           | 7'97           | 8'68           | 9'10           | 8'50           | 7'32           | 5'84           | 4'60           | 3'96            | 3'96            | 4'64            | —               | 5'70            | 7'33            | 8'88            | 10'02           | 10'67           | 10'74           | 9'90            | 8'52            | 7'06            | 5'83            | 5'83     | 70 6                             |
| 73  | 5'03           | 5'05           | 5'72           | 6'77           | 7'87           | 8'80           | 9'20           | 8'78           | 7'80           | 6'77           | 5'74            | 5'02            | 4'80            | 5'01            | 5'66            | 6'77            | —               | 8'07            | 9'23            | 10'04           | 10'50           | 10'14           | 9'00            | 7'60            | 7'60     | 71 5                             |
| 74  | 6'43           | 5'30           | 4'72           | 5'00           | 5'84           | 6'87           | 7'68           | 8'18           | 8'50           | 8'27           | 7'32            | 6'35            | 5'83            | 5'64            | 5'48            | 5'40            | 6'24            | 6'48            | —               | 6'83            | 7'27            | 7'66            | 8'04            | 8'24            | 8'24     | 72 4                             |
| 75  | 8'31           | 6'90           | 5'86           | 5'06           | 4'77           | 5'08           | 5'77           | 6'28           | 6'63           | 6'96           | 7'26            | 7'41            | 7'14            | 6'71            | 6'47            | 6'20            | 5'87            | 6'10            | 6'66            | 7'27            | 7'60            | —               | 7'87            | 8'13            | 8'13     | 73 3                             |
| 76  | 8'30           | 8'41           | 7'50           | 6'41           | 5'53           | 4'94           | 4'81           | 4'98           | 5'52           | 6'33           | 6'93            | 7'33            | 7'61            | 7'58            | 7'44            | 7'10            | 6'71            | 6'38            | 6'34            | 6'54            | 6'96            | 7'68            | 8'45            | 8'92            | 8'92     | 74 3                             |
| Sum | 530'35         | 540'31         | 525'07         | 529'69         | 543'21         | 541'22         | 538'65         | 552'76         | 540'83         | 548'94         | 551'26          | 556'58          | 551'56          | 542'43          | 559'33          | 557'96          | 535'36          | 553'85          | 542'24          | 544'27          | 537'84          | 529'88          | 541'24          | 535'56          | 535'56   | No. of Days.                     |
| No. | 73             | 74             | 73             | 73             | 74             | 73             | 73             | 74             | 73             | 73             | 73              | 74              | 73              | 72              | 74              | 74              | 71              | 74              | 73              | 73              | 73              | 72              | 74              | 73              | 73       | 73 <sup>d</sup> 4 <sup>h</sup> . |

COMPUTATIONS.

TIDAL OBSERVATIONS.

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SHORT-PERIOD TIDES.

[KARACHI, 1883-84.]

FORM V.—SUMMATIONS AND MEANS OF SERIES M.

|                 | 0 <sup>h</sup>  | 1 <sup>h</sup>  | 2 <sup>h</sup>  | 3 <sup>h</sup>  | 4 <sup>h</sup>  | 5 <sup>h</sup>  | 6 <sup>h</sup>  | 7 <sup>h</sup>  | 8 <sup>h</sup>  | 9 <sup>h</sup>  | 10 <sup>h</sup> | 11 <sup>h</sup> |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Sum (p. 1)      | 381.34          | 447.11          | 555.96          | 641.43          | 700.00          | 735.57          | 722.91          | 647.44          | 565.07          | 456.26          | 375.65          | 330.69          |
| " (p. 2)        | 343.74          | 428.05          | 526.68          | 615.67          | 695.14          | 721.19          | 699.40          | 641.14          | 544.08          | 442.65          | 368.45          | 332.16          |
| " (p. 3)        | 345.81          | 419.59          | 505.63          | 596.08          | 681.28          | 718.74          | 709.65          | 634.70          | 545.90          | 459.52          | 385.92          | 345.39          |
| " (p. 4)        | 364.14          | 435.23          | 536.65          | 632.80          | 691.73          | 730.42          | 711.59          | 651.35          | 553.59          | 451.15          | 366.11          | 324.73          |
| " (p. 5)        | 353.49          | 423.23          | 527.79          | 625.39          | 690.07          | 724.24          | 714.22          | 631.25          | 539.98          | 433.03          | 358.82          | 318.28          |
| Sum(pp.1-5)     | 1788.52         | 2153.21         | 2652.71         | 3111.37         | 3458.22         | 3630.16         | 3557.77         | 3205.88         | 2748.62         | 2242.61         | 1854.95         | 1651.25         |
| No. of Obs.     | 369             | 369             | 371             | 370             | 369             | 368             | 369             | 369             | 372             | 368             | 369             | 369             |
| Means ...       | 4.847           | 5.835           | 7.150           | 8.409           | 9.372           | 9.865           | 9.642           | 8.688           | 7.389           | 6.094           | 5.027           | 4.475           |
| 12 <sup>h</sup> | 13 <sup>h</sup> | 14 <sup>h</sup> | 15 <sup>h</sup> | 16 <sup>h</sup> | 17 <sup>h</sup> | 18 <sup>h</sup> | 19 <sup>h</sup> | 20 <sup>h</sup> | 21 <sup>h</sup> | 22 <sup>h</sup> | 23 <sup>h</sup> |                 |
| 347.89          | 425.92          | 512.91          | 596.94          | 673.81          | 739.51          | 728.38          | 662.97          | 575.35          | 489.01          | 403.65          | 366.47          | 13082.24        |
| 345.45          | 412.26          | 519.02          | 599.31          | 680.90          | 707.83          | 697.21          | 630.58          | 532.87          | 434.23          | 363.64          | 330.45          | 12612.10        |
| 360.08          | 422.98          | 511.07          | 602.91          | 657.22          | 693.69          | 676.94          | 624.31          | 528.24          | 424.97          | 354.08          | 320.60          | 12525.30        |
| 351.46          | 417.95          | 512.74          | 605.23          | 695.54          | 736.54          | 733.18          | 660.95          | 570.73          | 482.95          | 394.27          | 345.06          | 12956.09        |
| 330.04          | 400.14          | 503.91          | 591.91          | 660.43          | 706.30          | 702.20          | 627.34          | 541.38          | 443.44          | 362.66          | 328.93          | 12538.47        |
| 1734.92         | 2079.25         | 2559.65         | 2996.30         | 3367.90         | 3583.87         | 3537.91         | 3206.15         | 2748.57         | 2274.60         | 1878.30         | 1691.51         | 63714.20        |
| 370             | 369             | 371             | 368             | 368             | 369             | 369             | 368             | 369             | 369             | 368             | 369             |                 |
| 4.689           | 5.635           | 6.899           | 8.142           | 9.152           | 9.712           | 9.588           | 8.712           | 7.449           | 6.164           | 5.104           | 4.584           |                 |

COMPUTATIONS.

[PART VI.]

[KARACHI, 1883-84.]

SHORT-PERIOD TIDES.

FORM VI.—ANALYSIS OF SERIES M.

| I              | II    | III                   | IV     | V                         | VI   | VII                     | VIII                    | IX   | X              | XI                      | XII                     | XIII               | XIV      | XV      |
|----------------|-------|-----------------------|--------|---------------------------|------|-------------------------|-------------------------|------|----------------|-------------------------|-------------------------|--------------------|----------|---------|
|                |       | I+II                  | I-II   | Lower half of IV reversed | IV+V | M                       | M                       | IV-V | M              | M                       | First half of III       | Second half of III | XII+XIII | XIV     |
| 0 <sup>h</sup> | 4'847 | 12 <sup>h</sup> 4'689 | 9'536  | +158                      | +158 | 0                       | 0                       | +158 | +158           | 1                       | 9'536                   | 19'230             | 28'766   | -9'694  |
| 1              | 5'835 | 13 5'635              | 11'470 | +200                      | +091 | S <sub>1</sub>          | S <sub>3</sub>          | +309 | S <sub>5</sub> | S <sub>3</sub>          | 11'470                  | 17'400             | 28'870   | -5'930  |
| 2              | 7'150 | 14 6'899              | 14'049 | +251                      | +174 | S <sub>2</sub>          | I                       | +328 | S <sub>4</sub> | 0                       | 14'049                  | 14'838             | 28'887   | -0'789  |
| 3              | 8'409 | 15 8'142              | 16'551 | +267                      | +197 | S <sub>3</sub>          | S <sub>3</sub>          | +337 | S <sub>2</sub> | -S <sub>3</sub>         | 16'551                  | 12'258             | 28'809   | +4'293  |
| 4              | 9'372 | 16 9'152              | 18'524 | +220                      | +160 | S <sub>4</sub>          | 0                       | +280 | S <sub>2</sub> | -I                      | 18'524                  | 10'131             | 28'655   | +8'393  |
| 5              | 9'865 | 17 9'712              | 19'577 | +153                      | +129 | S <sub>5</sub>          | -S <sub>3</sub>         | +177 | S <sub>1</sub> | 0                       | 19'577                  | 9'059              | 28'636   | +10'518 |
| 6              | 9'642 | 18 9'588              | 19'230 | +054                      | +054 | I                       | -I                      | +054 | 0              | 0                       | 19'577                  | 9'059              | 28'636   | +10'518 |
| 7              | 8'688 | 19 8'712              | 17'400 | +024                      | +054 |                         |                         |      |                |                         |                         |                    |          |         |
| 8              | 7'389 | 20 7'449              | 14'838 | +060                      |      | 12                      | 12                      |      |                |                         |                         |                    |          |         |
| 9              | 6'094 | 21 6'164              | 12'258 | +070                      |      | 12                      | 12                      |      |                |                         |                         |                    |          |         |
| 10             | 5'027 | 22 5'104              | 10'131 | +077                      |      |                         |                         |      |                |                         |                         |                    |          |         |
| 11             | 4'475 | 23 4'584              | 9'059  | +109                      |      |                         |                         |      |                |                         |                         |                    |          |         |
|                |       |                       |        |                           |      | B <sub>1</sub> = +0'473 | B <sub>3</sub> = +0'193 |      |                | A <sub>1</sub> = +0'970 | A <sub>3</sub> = -0'223 |                    |          |         |

| M                        | XVI                      | XVII                     | XVIII                    | XIX        | XX                                     | XXI                                    | XXII                     | XXIII                    | XXIV                    | XXV                     | XXVI            | XXVII                    |                          |
|--------------------------|--------------------------|--------------------------|--------------------------|------------|----------------------------------------|----------------------------------------|--------------------------|--------------------------|-------------------------|-------------------------|-----------------|--------------------------|--------------------------|
|                          | M                        | M                        | M                        | M          | First half of XIV                      | Second half of XIV                     | XX-XXI                   | M                        | M                       | XX+XXI                  | M               | M                        |                          |
| 0                        | 0'000                    | 1 - 9'694                | 0'000                    | 1 - 9'694  | 28'766                                 | 28'809                                 | -0'043                   | 0                        | 0                       | 57'575                  | 0               | 1 + 57'575               |                          |
| S <sub>2</sub>           | - 2'065                  | S <sub>4</sub> - 5'136   | 1 - 5'930                | 0          | 28'870                                 | 28'655                                 | +0'215                   | S <sub>4</sub>           | 1 - 0'043               | S <sub>1</sub> + 49'818 | S <sub>2</sub>  | - 28'763                 |                          |
| S <sub>1</sub>           | - 0'683                  | S <sub>2</sub> - 0'395   | 0                        | +0'789     | 28'887                                 | 28'636                                 | +0'251                   | S <sub>4</sub>           | S <sub>2</sub> + 0'108  | -S <sub>1</sub>         | -S <sub>2</sub> | - 28'763                 |                          |
| I                        | + 4'293                  | 0                        | - 4'293                  | 0          | 0                                      | 0                                      | 0                        | 0                        | -S <sub>2</sub> - 0'126 | 57'523                  | -S <sub>1</sub> | - 28'761                 |                          |
| S <sub>4</sub>           | + 7'269                  | -S <sub>2</sub> - 4'196  | 0                        | +8'393     | 0 = nat. sin 0°                        | 0 = nat. sin 0°                        | 0                        | 0                        | 12 - 0'061              |                         | 12              | + 0'051                  |                          |
| S <sub>2</sub>           | + 5'259                  | -S <sub>1</sub> - 9'109  | 0                        | 0          | S <sub>1</sub> = nat. sin 15° = '25882 | S <sub>2</sub> = nat. sin 30° = '50000 | B <sub>4</sub> = +0'0336 | A <sub>4</sub> = -0'0051 |                         |                         |                 | B <sub>8</sub> = +0'0001 | A <sub>8</sub> = +0'0043 |
| 12                       | + 13'173                 | 12 - 28'530              | 12 + 0'295               | 12 - 0'512 | S <sub>3</sub> = nat. sin 45° = '70711 | S <sub>4</sub> = nat. sin 60° = '86603 |                          |                          |                         |                         |                 |                          |                          |
| B <sub>2</sub> = +1'0978 | A <sub>2</sub> = -2'3775 | B <sub>6</sub> = +0'0246 | A <sub>6</sub> = -0'0427 |            | S <sub>5</sub> = nat. sin 75° = '96593 | I = nat. sin 90° = 1'00000             |                          |                          |                         |                         |                 |                          |                          |

Compared by

Checked by

Computed by

SHORT-PERIOD TIDES. FORM VII.—VALUES OF  $V_0 + u$  FOR KARACHI, 1883-84.

1883, May 1st, o<sup>h</sup>. N. Lat.  $24^\circ 47'$ , Long.  $66^\circ 58' E.$  ( $= 4^h.4645$ ).

Average Long. Moon's Node for year beginning with 1st May, 1883, Midyear o<sup>h</sup> 31st October, 1883.

Average Long. Moon's Perigee for L Tide (round numbers).

Computation of  $R$  and  $Q$  (round numbers).

Auxiliary Tables  $N = 211^\circ 884$ .

(from below)  $p_0 = 16^\circ 079$   
motion in  $184^d 13\frac{1}{2}^h = 20^\circ 560$

colog.  $[\frac{1}{3} \cot^2 \frac{1}{2} I - \cos 2P] = + 9.23950$

log sin  $2P = + 9.99988$

$P + \xi = 36.639$

log tan.  $R = + 9.23938$

$-\xi = 7.695$

for L Tide,  $R = 9.845$

$P = 44.334$

log tan  $P = + 9.98990$

$\cot \frac{1}{2} I = + 5.891$

log  $\frac{1}{2} = 9.69897$

$\cot^2 \frac{1}{2} I = + 34.704$

log. tan  $Q = + 9.68887$

$\frac{1}{3} \cot^2 \frac{1}{2} I = + 5.784$

$-\cos 2P = - .023$

for  $M_1$  Tide,  $Q = 26.036$

$\frac{1}{3} \cot^2 \frac{1}{2} I - \cos 2P = + 5.761$

Extract from Auxiliary Tables.

$I = 19^\circ 267$ ,  $\nu = -8^\circ 256$ ,  $\xi = -7^\circ 695$ .  
For K Tides  $\nu' = -5^\circ 366$ ,  $2\nu'' = -9^\circ 906$ .

Sid. Time at M. N. }  $h_0$  h m s } Moon's mean long. }  $s_0$   
(Naut. Alm., p. II.) } = 2 36 17.48 } (Naut. Alm.) } =  $332^\circ 32.4$

= 2 36.2913  
= 2.604855  
1.302428

E. Long. Cor. =  $-2.448$

$s_0 = 330.092$

$\xi = -7.695$

Hansen's Tables de la Lune, }  $p_0$   
p. 300.  $\pi$  for 1883, Jan. o. } =  $0.484$

Motion in interval from }  
Jan. o, or 121 days. } =  $13.480$

Constant =  $0.136$

Sum =  $16.100$

E. Long. Cor. =  $-0.021$

$p_0 = 16.079$

$s_0 = 330.092$

$s_0 - p_0 = 314.013$

Extract from }  $p_1$   
Auxiliary Table }  $p_1 = 280.940$

$h_0 = 38.890$

$h_0 - p_1 = 117.950$

E. Long. Cor. =  $-0.183$

$h_0 = 38.890$

$\nu = -8.256$

$h_0 - \nu = 47.146$

$2h_0 = 77.780$

$s_0 - \xi = 337.787$

$2s_0 - 2\xi = 315.574$

$2s_0 = 300.184$

M SERIES.

$h_0 - \nu = 47.146$

$-(s_0 - \xi) = 22.213$

$\S M_1 = 69.359$

( $\times 2$ ) for  $M_2 = 138.718$

( $\times 3$ ) for  $M_3 = 208.077$

( $\times 4$ ) for  $M_4 = 277.436$

( $\times 6$ ) for  $M_6 = 56.154$

( $\times 8$ ) for  $M_8 = 194.872$

$\S$  as above  $M_1 = 69.359$

(see above) +  $Q = 26.036$

$-\frac{1}{2}\pi = 270.000$

for  $M_1 = 5.395$

K<sub>2</sub> SERIES.

$2h_0 = 77.780$

$-2\nu'' = 9.906$

for  $K_2 = 87.686$

K<sub>1</sub> SERIES.

$h_0 = 38.890$

$-\nu' = 5.366$

$-\frac{1}{2}\pi = 270.000$

for  $K_1 = 314.256$

$\lambda$  SERIES.

for  $N = 184.705$

$-2h_0 = 282.220$

+  $2s_0 = 300.184$

+  $\pi = 180.000$

for  $\lambda = 227.109$

$\nu$  SERIES.

for  $M_2 = 138.718$

+  $(s_0 - p_0) = 314.013$

+  $2h_0 = 77.780$

$-2s_0 = 59.816$

for  $\nu = 230.327$

$M_2N$  SERIES.

for  $M_2 = 138.718$

+ for  $N = 184.705$

for  $M_2N = 323.423$

$2M_2K_1$  SERIES.

for  $M_4 = 277.436$

- for  $K_1 = 45.744$

for  $2M_2K_1 = 323.180$

Mm SERIES.

$s_0 - p_0 = 314.013$

Mf SERIES.

$(2s_0 - 2\xi) = 315.574$

COMPUTATIONS.

[PART VI.]

SHORT-PERIOD TIDES.

[KARACHI, 1883-84.]

FORM VIII.—EVALUATION OF SHORT-PERIOD TIDES. SERIES M.

Augmenting Factors.—For  $A_1, B_1 \dots 1'00286$ ,  $A_2, B_2 \dots 1'01152$ ,  $A_3, B_3 \dots 1'02617$ ,  $A_4, B_4 \dots 1'04720$ ,  $A_6, B_6 \dots 1'11072$ ,  $A_8, B_8 \dots 1'20920$ .

|                                                |                                                   |                                                   |                                                  |                                                  |                                                 |
|------------------------------------------------|---------------------------------------------------|---------------------------------------------------|--------------------------------------------------|--------------------------------------------------|-------------------------------------------------|
| $\text{Log } B_1 = +8'67486$                   | $\text{Log } B_2 = +0'04052$                      | $\text{Log } B_3 = +8'28556$                      | $\text{Log } B_4 = +8'52634$                     | $\text{Log } B_6 = +8'39094$                     | $\text{Log } B_8 = +6'00000$                    |
| $\text{Log } A_1 = +8'98677$                   | $\text{Log } A_2 = -0'37612$                      | $\text{Log } A_3 = -8'34830$                      | $\text{Log } A_4 = -7'70757$                     | $\text{Log } A_6 = -8'63043$                     | $\text{Log } A_8 = +7'63347$                    |
| $\text{L. tan } \zeta_1 = +9'68809$            | $\text{L. tan } \zeta_2 = -9'66440$               | $\text{L. tan } \zeta_3 = -9'93726$               | $\text{L. tan } \zeta_4 = -0'81877$              | $\text{L. tan } \zeta_6 = -9'76051$              | $\text{L. tan } \zeta_8 = +8'36653$             |
| $\zeta_1 = 25^{\circ}995$<br>$V_0 + u = 5'395$ | $\zeta_2 = 155^{\circ}215$<br>$V_0 + u = 138'718$ | $\zeta_3 = 139^{\circ}124$<br>$V_0 + u = 208'077$ | $\zeta_4 = 98^{\circ}631$<br>$V_0 + u = 277'436$ | $\zeta_6 = 150^{\circ}053$<br>$V_0 + u = 56'154$ | $\zeta_8 = 1^{\circ}332$<br>$V_0 + u = 194'872$ |
| $\kappa_1 = 31'390$                            | $\kappa_2 = 293'933$                              | $\kappa_3 = 347'201$                              | $\kappa_4 = 16'067$                              | $\kappa_6 = 206'207$                             | $\kappa_8 = 196'204$                            |
| $(B_1)^2 = '002237$                            | $(B_2)^2 = 1'205165$                              | $(B_3)^2 = '000372$                               | $(B_4)^2 = '001129$                              | $(B_6)^2 = '000605$                              | $(B_8)^2 = '000000$                             |
| $(A_1)^2 = '9409$                              | $(A_2)^2 = 5'652506$                              | $(A_3)^2 = '497$                                  | $(A_4)^2 = '26$                                  | $(A_6)^2 = '1823$                                | $(A_8)^2 = '18$                                 |
| $(R_1)^2 = '011646$                            | $(R_2)^2 = 6'857671$                              | $(R_3)^2 = '000869$                               | $(R_4)^2 = '001155$                              | $(R_6)^2 = '002428$                              | $(R_8)^2 = '000018$                             |
| $\text{Aug}^{\text{tn}} R_1 = '1079$<br>$= '3$ | $\text{Aug}^{\text{tn}} R_2 = 2'6187$<br>$= '301$ | $\text{Aug}^{\text{tn}} R_3 = '0295$<br>$= '8$    | $\text{Aug}^{\text{tn}} R_4 = '0340$<br>$= '16$  | $\text{Aug}^{\text{tn}} R_6 = '0493$<br>$= '55$  | $\text{Aug}^{\text{tn}} R_8 = '0043$<br>$= '9$  |
| $\text{Aug}^{\text{td}} R_1 = '1082$           | $\text{Aug}^{\text{td}} R_2 = 2'6488$             | $\text{Aug}^{\text{td}} R_3 = '0303$              | $\text{Aug}^{\text{td}} R_4 = '0356$             | $\text{Aug}^{\text{td}} R_6 = '0548$             | $\text{Aug}^{\text{td}} R_8 = '0052$            |
| $1/f = '7443$                                  | $1/f = '9689$                                     | $1/f = '9537$                                     | $1/f = '9388$                                    | $1/f = '9096$                                    | $1/f = '8813$                                   |
| $H_1 = '0805$                                  | $H_2 = 2'5664$                                    | $H_3 = '0289$                                     | $H_4 = '0334$                                    | $H_6 = '0498$                                    | $H_8 = '0046$                                   |

Computed by \_\_\_\_\_ Checked by \_\_\_\_\_ Compared by \_\_\_\_\_

CHAP. III.]

COMPUTATIONS.

TIDAL OBSERVATIONS.

$$\Sigma dh \sin(\sigma - \omega)t.$$

FORM IX.—LONG-PERIOD TIDES. TIDE Mm.

[KARACHI, 1883-84.]

| Multiplier<br>No. of the day | 1·0       |     | ·9        |     | ·8        |   | ·7        |   | ·6        |   | ·5        |   | ·4        |   | ·3        |   | ·2        |   | ·1        |   | 0   |
|------------------------------|-----------|-----|-----------|-----|-----------|---|-----------|---|-----------|---|-----------|---|-----------|---|-----------|---|-----------|---|-----------|---|-----|
|                              | +         | -   | +         | -   | +         | - | +         | - | +         | - | +         | - | +         | - | +         | - | +         | - | +         | - |     |
| 7 to 0                       | ·27       | ·20 | ·25       | ·22 |           |   |           |   | ·11       |   | ·12       |   |           |   |           |   | ·21       |   |           |   | 15  |
| 8—13                         | ·09       | ·08 | ·19       | ·18 |           |   |           |   | ·18       |   | ·16       |   | ·18       |   |           |   | ·33       |   |           |   |     |
| 34—28                        | ·01       | ·18 | ·23       | ·17 |           |   |           |   | ·52       |   | ·22       |   | ·54       |   |           |   | ·25       |   |           |   |     |
| 35—41                        | ·02       | ·13 | ·27       | ·44 |           |   |           |   |           |   | ·09       |   | ·09       |   |           |   | ·22       |   |           |   |     |
|                              |           | &c. |           |     |           |   |           |   |           |   |           |   |           |   |           |   | &c.       |   |           |   |     |
| 311—316                      | ·16       | ·06 | ·04       | ·07 |           |   |           |   | ·21       |   | ·21       |   | ·21       |   |           |   | ·29       |   |           |   | ·32 |
| 337—331                      | ·20       | ·32 | ·34       | ·38 |           |   |           |   | ·18       |   | ·18       |   | ·18       |   |           |   | ·19       |   |           |   | ·15 |
| 338—344                      | ·16       | ·07 | ·05       | ·08 |           |   |           |   | ·21       |   | ·03       |   | ·03       |   |           |   | ·01       |   |           |   |     |
| 364—359                      | ·25       | ·02 | ·07       |     |           |   |           |   |           |   |           |   |           |   |           |   | ·01       |   |           |   |     |
|                              | 3·10 4·32 |     | 3·19 2·70 |     | 2·46 1·49 |   | 0·87 1·36 |   | 0·94 1·55 |   | 1·28 1·15 |   | 1·25 1·30 |   | 1·47 1·48 |   | 1·88 0·88 |   | 1·44 1·15 |   |     |
| Total a                      | -1·22     |     | +0·49     |     | +0·97     |   | -0·49     |   | -0·61     |   | +0·13     |   | -0·05     |   | -0·01     |   | +1·00     |   | +0·29     |   |     |
| 20 to 14                     | ·10       | ·13 | ·09       | ·03 |           |   |           |   | ·11       |   | ·27       |   | ·27       |   |           |   | ·27       |   |           |   | 39  |
| 21—27                        | ·36       | ·40 | ·35       | ·38 |           |   |           |   | ·43       |   | ·39       |   | ·39       |   |           |   | ·39       |   |           |   |     |
| 48—42                        | ·43       | ·54 | ·58       | ·81 |           |   |           |   | ·98       |   | ·98       |   | ·98       |   |           |   | ·92       |   |           |   |     |
|                              |           | &c. |           |     |           |   |           |   |           |   |           |   |           |   |           |   | &c.       |   |           |   |     |
| 325—330                      | ·19       | ·27 | ·02       | ·10 |           |   |           |   | ·25       |   | ·25       |   | ·25       |   |           |   | ·31       |   |           |   |     |
| 351—345                      | ·05       | ·33 | ·39       | ·38 |           |   |           |   | ·38       |   | ·31       |   | ·31       |   |           |   | ·31       |   |           |   |     |
| 352—358                      | ·02       | ·06 | ·03       | ·02 |           |   |           |   | ·09       |   | ·17       |   | ·17       |   |           |   | ·17       |   |           |   | 11  |
|                              | 2·37 4·13 |     | 2·16 3·84 |     | 0·98 2·02 |   | 1·51 2·15 |   | 1·83 0·99 |   | 0·92 2·60 |   | 2·40 0·71 |   | 0·99 1·78 |   | 1·53 0·56 |   | 1·56 1·32 |   |     |
| Total b                      | -1·76     |     | -1·68     |     | -1·04     |   | -0·64     |   | +0·84     |   | -1·68     |   | +1·69     |   | -0·79     |   | +0·97     |   | +0·24     |   |     |
| a-b                          | +0·54     |     | +2·17     |     | +2·01     |   | +0·15     |   | -1·45     |   | +1·81     |   | -1·74     |   | +0·78     |   | +0·03     |   | +0·05     |   |     |
| (a-b) × Mult                 | +0·54     |     | +1·95     |     | +1·61     |   | +0·11     |   | +         |   | +0·91     |   | +         |   | +0·23     |   | +0·01     |   | +0·01     |   |     |
|                              | -         |     | -         |     | -         |   | -         |   | -0·87     |   | -         |   | -0·70     |   | -         |   | -         |   | -         |   |     |

$$\Sigma(a-b) \times \text{Mult.} = +5\cdot37. \quad \Sigma(a-b) \times \text{Mult.} = -1\cdot57. \quad \Sigma dh \sin(\sigma - \omega)t = \sum_{384}^0 (a-b) \times \text{Mult.} = +3\cdot80.$$

NOTE.—The arrows show the direction of the sequence of the entries of  $dh$  in the columns in which points are inserted, the values being entered under their proper signs—*e.g.*, in the first row (marked 7 to 0) with arrow from right to left, the entry (irrespective of sign) for day 0 is to be entered in column 0, for day 1 in column 2, for day 2 in column 4, and so on. After filling in the first two rows of the upper half, the first two rows of the lower half are to be filled, and so on alternately. The alternation of entry being indicated by the curved arrows. In the spaces containing double points two successive entries are to be filled in—the first entry above, the second below: *e.g.*, in row 7 to 0, column 1·0, there are double points, and the entry of  $dh$  for day 6 is to be made above the line and to left or right according as it is + or -, and for day 7 it is to be made below the line, and to left or right according as it is + or -.

Computed by \_\_\_\_\_ Compared by \_\_\_\_\_

COMPUTATIONS.

[PART VI.]

LONG-PERIOD TIDES.

[KARACHI, 1883-84.

FORM X.—SUMMATIONS AND EVALUATION OF LONG-PERIOD TIDES.—CLEARANCE FROM EFFECTS OF TIDES OF SHORT-PERIOD.

NOTE.—A and B to be extracted from harmonic analysis for Tides of Short-Period.

|                |   | Tide Mm or ( $\sigma - \omega$ ). |            |
|----------------|---|-----------------------------------|------------|
|                |   | Cosine Series.                    | Products.  |
| M <sub>2</sub> | { | A <sub>2</sub> = - 2'378          | } = + 0'13 |
|                |   | multiplier = - 0'0556             |            |
|                | { | B <sub>2</sub> = + 1'098          | } = - 0'19 |
|                |   | multiplier = - 0'1704             |            |
| N              | { | A = - 0'035                       | } = + 0'00 |
|                |   | multiplier = - 0'0588             |            |
|                | { | B = + 0'599                       | } = - 0'05 |
|                |   | multiplier = - 0'0776             |            |
| O              | { | A = - 0'382                       | } = + 0'02 |
|                |   | multiplier = - 0'0649             |            |
|                | { | B = - 0'405                       | } = + 0'14 |
|                |   | multiplier = - 0'3477             |            |
|                |   | Total + = + 0'29                  |            |
|                |   | Total - = - 0'24                  |            |

|  |   | Tide Mf or $2\sigma$ .   |            |
|--|---|--------------------------|------------|
|  |   | Cosine Series.           | Products.  |
|  | { | A <sub>2</sub> = - 2'378 | } = - 0'01 |
|  |   | multiplier = + 0'0030    |            |
|  | { | B <sub>2</sub> = + 1'098 | } = - 0'04 |
|  |   | multiplier = - 0'0377    |            |
|  | { | A = - 0'035              | } = - 0'00 |
|  |   | multiplier = + 0'0368    |            |
|  | { | B = + 0'599              | } = - 0'13 |
|  |   | multiplier = - 0'2234    |            |
|  | { | A = - 0'382              | } = - 0'01 |
|  |   | multiplier = + 0'0167    |            |
|  | { | B = - 0'405              | } = + 0'03 |
|  |   | multiplier = - 0'0779    |            |
|  |   | Total + = + 0'03         |            |
|  |   | Total - = - 0'19         |            |

Total clearance = + 0'05  
 Uncleared  $\sum dh \cos (\sigma - \omega)t = + 1'29$   
 $\sum dh \cos (\sigma - \omega)t = + 1'34$

Total clearance = - 0'16  
 Uncleared  $\sum dh \cos 2\sigma t = + 7'10$   
 $\sum dh \cos 2\sigma t = + 6'94$

Divisor 183'05. 1st approx. A = + 0'007

Divisor 183'18. 1st approx. C = + 0'038

|                |   | Sine Series.             | Products.  |
|----------------|---|--------------------------|------------|
| M <sub>2</sub> | { | A <sub>2</sub> = - 2'378 | } = + 0'41 |
|                |   | multiplier = - 0'1708    |            |
|                | { | B <sub>2</sub> = + 1'098 | } = + 0'05 |
|                |   | multiplier = + 0'0441    |            |
| N              | { | A = - 0'035              | } = + 0'00 |
|                |   | multiplier = - 0'0206    |            |
|                | { | B = + 0'599              | } = + 0'07 |
|                |   | multiplier = + 0'1138    |            |
| O              | { | A = - 0'382              | } = + 0'13 |
|                |   | multiplier = - 0'3452    |            |
|                | { | B = - 0'405              | } = - 0'02 |
|                |   | multiplier = + 0'0405    |            |
|                |   | Total + = + 0'66         |            |
|                |   | Total - = - 0'02         |            |

|  |   | Sine Series.             | Products.  |
|--|---|--------------------------|------------|
|  | { | A <sub>2</sub> = - 2'378 | } = - 0'10 |
|  |   | multiplier = + 0'0417    |            |
|  | { | B <sub>2</sub> = + 1'098 | } = + 0'01 |
|  |   | multiplier = + 0'0105    |            |
|  | { | A = - 0'035              | } = + 0'01 |
|  |   | multiplier = - 0'1525    |            |
|  | { | B = + 0'599              | } = - 0'05 |
|  |   | multiplier = - 0'0854    |            |
|  | { | A = - 0'382              | } = - 0'03 |
|  |   | multiplier = + 0'0842    |            |
|  | { | B = - 0'405              | } = - 0'01 |
|  |   | multiplier = + 0'0338    |            |
|  |   | Total + = + 0'02         |            |
|  |   | Total - = - 0'19         |            |

Total clearance = + 0'64  
 Uncleared  $\sum dh \sin (\sigma - \omega)t = + 3'80$   
 $\sum dh \sin (\sigma - \omega)t = + 4'44$

Total clearance = - 0'17  
 Uncleared  $\sum dh \sin 2\sigma t = + 2'14$   
 $\sum dh \sin 2\sigma t = + 1'97$

Divisor 181'95. 1st approx. B = + 0'024

Divisor 181'82. 1st approx. D = + 0'011

The 1st approximations of the constants for the other three tides are deduced in the same way, and are for MSf, C' = - 0'001, D' = - 0'009; for Sa, E = + 0'089, F = - 0'001; and for Ssa, G = - 0'003, H = + 0'189.

Computed by \_\_\_\_\_ Checked by \_\_\_\_\_ Compared by \_\_\_\_\_

LONG-PERIOD TIDES.

[KARACHI, 1883-84.]

FORM XI.—EVALUATION OF LONG-PERIOD TIDES.

$$\begin{aligned} \sum_0^{360} dh. \cos(\sigma - \omega)t &= +1.34 = +183.05 A + 2.14 B + 0.73 C + 4.29 D + 0.77 C' + 5.04 D' + 4.88 E - 0.43 F + 4.96 G - 0.69 H. \\ &+ 1.34 = 183.05 A + .05 + .03 + .05 - .00 - .05 + .43 + .00 - .01 - .13 \\ &+ 0.97 = 183.05 A \\ &+ 0.005 = A \\ \sum_0^{360} dh. \sin(\sigma - \omega)t &= +4.44 = + 2.14 A + 181.95 B - 4.15 C + 1.02 D - 4.90 C' + 1.07 D' + 3.80 E + 0.34 F + 3.88 G + 0.69 H. \\ &+ 4.44 = + .01 + 181.95 B - .16 + .01 + .00 - .01 + .34 - .00 - .01 + .13 \\ &+ 4.13 = 181.95 B \\ &+ 0.023 = B \\ \sum_0^{360} dh. \cos 2\sigma t &= +6.94 = + 0.73 A - 4.15 B + 183.18 C + 0.88 D + 0.61 C' + 0.92 D' - 1.50 E - 0.10 F - 1.51 G - 0.19 H. \\ &+ 6.94 = + .00 - .10 + 183.18 C + .01 - .00 - .01 - .13 + .00 + .00 - .04 \\ &+ 7.21 = 183.18 C \\ &+ 0.039 = C \\ &\quad \&c. \qquad \qquad \qquad \&c. \qquad \qquad \qquad \&c. \end{aligned}$$

| LUNAR MONTHLY Mm.                                                                                | LUNAR FORTNIGHTLY Mf.                                                                             | LUNI-SOLAR FORTNIGHTLY MSf.                                                                       | SOLAR ANNUAL Sa.                                                                                 | SOLAR SEMI-ANNUAL Ssa.                                                                           |
|--------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------|
| Log B = +8.36173                                                                                 | Log D = +7.95424                                                                                  | Log D' = -8.04139                                                                                 | Log F = -7.00000                                                                                 | Log H = +9.27646                                                                                 |
| Log A = +7.69897                                                                                 | Log C = +8.59106                                                                                  | Log C' = +7.00000                                                                                 | Log E = +8.94448                                                                                 | Log G = -7.60206                                                                                 |
| L. tan $\zeta_1 = +0.66276$                                                                      | L. tan $\zeta_1 = +9.36318$                                                                       | L. tan $\zeta_1 = -1.04139$                                                                       | Tan $\zeta_1 = -8.05552$                                                                         | Tan $\zeta_1 = -1.67440$                                                                         |
| $\zeta_1 = 77.735$                                                                               | $\zeta_1 = 12.995$                                                                                | $\zeta_1 = 275.194$                                                                               | $\zeta_1 = 359.349$                                                                              | $\zeta_1 = 91.212$                                                                               |
| Motion for $\left. \begin{array}{l} \dots \\ 11\frac{1}{2}^h \dots \end{array} \right\} = 6.260$ | Motion for $\left. \begin{array}{l} \dots \\ 11\frac{1}{2}^h \dots \end{array} \right\} = 12.627$ | Motion for $\left. \begin{array}{l} \dots \\ 11\frac{1}{2}^h \dots \end{array} \right\} = 11.683$ | Motion for $\left. \begin{array}{l} \dots \\ 11\frac{1}{2}^h \dots \end{array} \right\} = 0.472$ | Motion for $\left. \begin{array}{l} \dots \\ 11\frac{1}{2}^h \dots \end{array} \right\} = 0.945$ |
| $\zeta = 83.995$                                                                                 | $\zeta = 25.622$                                                                                  | $\zeta = 286.877$                                                                                 | $\zeta = 359.821$                                                                                | $\zeta = 92.157$                                                                                 |
| $V_0 + u = 314.013$                                                                              | $V_0 + u = 315.574$                                                                               | $V_0 + u = 221.282$                                                                               | $V_0 + u = 38.800$                                                                               | $V_0 + u = 77.780$                                                                               |
| $\kappa = 38.008$                                                                                | $\kappa = 351.196$                                                                                | $\kappa = 148.159$                                                                                | $\kappa = 38.711$                                                                                | $\kappa = 169.937$                                                                               |
| $B_2 = 0.000529$                                                                                 | $D^2 = 0.000081$                                                                                  | $(D')^2 = 0.000121$                                                                               | $F^2 = 0.000001$                                                                                 | $H^2 = 0.035721$                                                                                 |
| $A^2 = 0.25$                                                                                     | $C^2 = 0.1521$                                                                                    | $(C')^2 = 0.1$                                                                                    | $E^2 = 1.444$                                                                                    | $G^2 = 16$                                                                                       |
| Sum = $R^2 = 0.000554$                                                                           | Sum = $R^2 = 0.001602$                                                                            | Sum = $R^2 = 0.000122$                                                                            | Sum = $R^2 = 0.001445$                                                                           | Sum = $R^2 = 0.035737$                                                                           |
| R = 0.024                                                                                        | R = 0.040                                                                                         | R = 0.011                                                                                         | R = H = 0.088                                                                                    | R = H = 0.189                                                                                    |
| 1/f = 0.900                                                                                      | 1/f = 1.449                                                                                       | 1/f = 0.969                                                                                       |                                                                                                  |                                                                                                  |
| H = 0.022                                                                                        | H = 0.058                                                                                         | H = 0.011                                                                                         |                                                                                                  |                                                                                                  |

Computed by \_\_\_\_\_ Checked by \_\_\_\_\_ Compared by \_\_\_\_\_



34. These tables have been constructed to facilitate the computations required for the harmonic analysis of the tidal observations. The following explanations and examples are given to illustrate the use of the tables.

Auxiliary tables.

35. This table, for converting decimals of a degree into their corresponding values of minutes and seconds of arc, enters frequently into the computations, more especially in taking out the values of the trigonometrical functions from Shortrede's Logarithm Tables. Its use hardly requires explanation.

Table I.

36. This table is the converse of Table I, but had to be made out somewhat differently. The correct value to three places of decimals of a degree is all that is generally required in the computations. This is given in the 4th column, and it will be observed that the figures in the 4th column are arranged midway between those in the 3rd and also midway between those in the 5th column, where the actual values corresponding to the angles in the 3rd column are given.

Table II.

It will also be observed that the table is divided into six groups. The reason of this is as follows:—6 minutes =  $\cdot 1$  of a degree, 12 minutes =  $\cdot 2$ , 18 minutes =  $\cdot 3$ , and so on. Therefore, any number of seconds added to 6 minutes will give the same figures in the second and third place of decimals (in the equivalent value of a degree expressed in decimals), as the similar number of seconds added to 12 minutes or 18 minutes would give. Similarly, 1 minute, or 7 minutes, or 13 minutes, and so many seconds would each have for the second and third place of decimals the same figures in expressing their corresponding values in decimals of a degree (the first figure of the decimals of course alters).

To use the tables, first look for the minutes in one of the groups of column 1; opposite it in the 2nd column will be found the first figure of the equivalent value in decimals of a degree. *Keeping to the same group*, look in the 3rd column for the seconds (most probably the exact number will not be found), entering this column at the *space* between the number of seconds next less and the number of seconds next greater than that looked for, in the 4th column *opposite this space*

the second and third figures of the corresponding value of the decimals of a degree will be found. If the actual number of seconds looked for is found in the third column, then the corresponding value is obtained in column 4 *opposite the space in column 3 below the number of seconds.*

*Example.*—Required the decimals of a degree corresponding to  $18' 25''$ :

|                                                             |     |     |     |               |           |
|-------------------------------------------------------------|-----|-----|-----|---------------|-----------|
| Column 1, group 1, $18'$                                    | ... | ... | ... | = $\cdot 3$   | in col. 2 |
| „ 3, „ 1, $25''$ (between $23''\cdot 4$ and $27''\cdot 0$ ) |     |     |     | = $\cdot 007$ | in col. 4 |
| Value required =                                            |     |     |     | $\cdot 307$   |           |

Again:—If the decimals of a degree corresponding to  $40' 0''$  are required: then as in the above example,  $40' 3''$  would just equal  $\cdot 668$ , and anything less than  $40' 3''$  must be less than  $\cdot 668$ . Again,  $39' 59''\cdot 4$  would just equal  $\cdot 667$ , and anything greater than  $39' 59''\cdot 4$  must be equal to  $\cdot 667$  or more. Therefore  $40' 0''$  would equal  $\cdot 667$ .

37. This is extracted and deduced from Hansen's *Tables de la Lune*, pages 299 and 300. The values of  $p$ , the mean longitude of the moon's perigee (or  $\pi$  as it is written in the tidal computation forms) is given for what is called January 0 of each year; but in the Preface to Hansen's Tables it will be found that January 0 and December the 31st mean the same date; therefore the values given in the tables are for 0 hours December 31st of the preceding year. But this is not the case for leap-years; the values given in the tables as regards those years are for 0 hours January 1st.

If the value of  $\pi$  for 0 hours January 1st is required for any year which is not leap-year, take the value of  $\pi$  given opposite in the next column, and *add* one day's motion (Table v); if it is leap-year, the one day's motion has not to be added.

*Example.*—Required  $\pi$  for 0 hours January 1st, 1891.

|                                         |     |           |                  |
|-----------------------------------------|-----|-----------|------------------|
| In Table III opposite 1891              | ... | 328°00632 |                  |
| In Table v one day's motion for $\pi$   | ... | 0°11140   |                  |
| . $\pi$ for 0 hours January 1st, 1891 = |     |           | <u>328°11772</u> |

Again :—Required  $\pi$  for 0 hours January 1st, 1892.

Opposite 1892 in Table III is  $8^{\circ}78020$ , which is the value required, for 1892 being leap-year, the one day's motion is not added.

38. These tables are to be employed together ; in using Table v great care must be taken that the proper number of days from January 0 is taken by means of Table IV. The motion of  $p$  or  $\pi$  is given for every day from 1 to 366.

39. This table gives the correction to the value of  $p$  or  $\pi$  on account of difference of longitude from Greenwich. The correction is required only to three places of decimals of a degree, as far as the tidal computations are concerned, and the table is constructed on the principle explained in Table II. Column 3 shows the exact correction for the difference of longitude given immediately opposite in column 1 ; and column 2 gives the correction which has to be used, for all values occurring *between the longitudes* given in column 1.

*Example.*—Required correction for  $\pi$  for  $30^{\circ} 30'$  E. longitude. By Table II  $30' = \cdot 500$  of a degree.

From Table VI.— $30^{\circ} 500$  lies between  $27^{\circ} 468$  and  $30^{\circ} 700$  in column 1, and in column 2 opposite the space between those two longitudes is  $\cdot 009$ , which is the correction required, and the sign is —

40. This table will be found much more convenient than Créle's for the particular multiplications required, and admits of much more rapid computation. The three augmenting factors  $R_3$ ,  $R_6$ , and  $R_8$  occur so seldom that it has not been thought necessary to extend the tables on their account.

The usual multiples of the factors are one integer and three places of decimals, or two figures only preceded by 0 ; for instance,  $1\cdot 412$ , or  $0\cdot 026$ . Sometimes, however, two integers and three places of decimals have to be multiplied by the augmenting factor. The use of the table

hardly requires explanation, care being taken to put down the decimal point correctly. The values in the tables are the products of the factors by whole numbers.

41. These tables require no explanation. They give the value of  $S_1, S_3, S_4,$  and  $S_6$  multiplied by every number between  $\cdot 001$  and  $\cdot 999$ : from which the products of these factors with other numbers can be rapidly obtained, care being taken about the decimal point.

Tables VIII, IX, X, and XI.

42. This table has been made up in order to permit of the natural numbers corresponding to logarithms, with indices 6, 7, and 8, being taken out much more rapidly than could be done with Hutton's Logarithm Tables. The natural number corresponding to the given logarithm is only required correct to three places of decimals; the table has been made up on the same principle as explained in Table 11.

Table XII.

43. This table gives the values of  $N$  (longitude of moon's ascending node) for 0 hour January 1, Greenwich mean time, for each year from 1850 up to 1949.

Table XIII.

44. This table shows the amount to be subtracted from the values given in Table XIII, to obtain  $N$  at any particular date. The mean value of  $N$  to be used in the tidal reductions is the value at mid-year of the observations: and as half a year after 0 hour of the first day under analysis falls at midnight, the values in Table XIV are computed for each midnight.

Table XIV.

45. This table gives the value of the solar perigee for 0 hour January 1, from 1850 to 1949.

Table XV.

46. This table shows the increment to be added to the quantities given in Table XV to obtain the value on certain days of the year, as the value of the solar perigee ( $p_1$ ) is required for mid-year of the observations.

Table XVI.

47. This table gives the values of  $I$  (inclination of the lunar orbit to the equator);  $\nu$  (the right ascension of the intersection of the lunar orbit and the equator) and  $\xi$  (the longitude 'in the moon's orbit' of the intersection) corresponding to each half degree of  $N$  (the longitude of the moon's ascending node) from  $0^\circ$  up to  $180^\circ$ .

Table XVII.

When  $N$  is negative  $I$  has the same value as when  $N$  is positive; but  $\nu$  and  $\xi$  change sign with  $N$ .

The values of  $I$ ,  $\nu$  and  $\xi$  corresponding to  $N$  at mid-year will be easily found by interpolation between the two nearest half degrees.

48. This table is subdivided into seven parts (1), (2), &c., and is used for the determination of the factors  $1/f$  and  $f$  required in calculating  $H$  from  $R$  and *vice versa*. The values are given corresponding to each  $0^\circ.1$  of  $I$ , the inclination of the lunar orbit to the equator. The values required in the computation are those corresponding to  $I$  for mid-year; so that  $I$  is first obtained from Table XVII to correspond to  $N$  at mid-year and then the  $1/f$  or  $f$  will be easily calculated by interpolation from the particular part of this table.

Table XVIII.

49. This table gives the values of  $\nu'$  corresponding to each  $0^\circ.1$  of  $I$ . The  $\nu'$  is required in computing the values of  $h_0 - \nu' - \frac{1}{2}\pi$ , the initial argument for the tide  $K_1$ .

Table XIX.

50. This table is similar to Table XIX, and gives the value of  $2\nu''$  employed in determining the initial argument,  $2h_0 - 2\nu''$ , for the tide  $K_2$ .

Table XX.

Attention should be paid to the notes at the foot of these tables.

COMPUTATIONS.

[PART VI.]

TABLE I.—For converting Decimals of a Degree into Minutes and Seconds.

1

2

| Parts of a Degree | Minutes | Parts of a Degree | Minutes and Seconds | Parts of a Degree | Minutes and Seconds | Parts of a Degree | Minutes and Seconds |
|-------------------|---------|-------------------|---------------------|-------------------|---------------------|-------------------|---------------------|
| .1                | 6       | .001              | 0 3.6               | .034              | 2 2.4               | .067              | 4 1.2               |
| .2                | 12      | 2                 | 0 7.2               | 35                | 2 6.0               | 68                | 4 4.8               |
| .3                | 18      | 3                 | 0 10.8              | 36                | 2 9.6               | 69                | 4 8.4               |
| .4                | 24      | 4                 | 0 14.4              | 37                | 2 13.2              | .070              | 4 12.0              |
| .5                | 30      | 5                 | 0 18.0              | 38                | 2 16.8              | 71                | 4 15.6              |
| .6                | 36      | 6                 | 0 21.6              | 39                | 2 20.4              | 72                | 4 19.2              |
| .7                | 42      | 7                 | 0 25.2              | .040              | 2 24.0              | 73                | 4 22.8              |
| .8                | 48      | 8                 | 0 28.8              | 41                | 2 27.6              | 74                | 4 26.4              |
| .9                | 54      | 9                 | 0 32.4              | 42                | 2 31.2              | 75                | 4 30.0              |
|                   |         | .010              | 0 36.0              | 43                | 2 34.8              | 76                | 4 33.6              |
|                   |         | 11                | 0 39.6              | 44                | 2 38.4              | 77                | 4 37.2              |
|                   |         | 12                | 0 43.2              | 45                | 2 42.0              | 78                | 4 40.8              |
|                   |         | 13                | 0 46.8              | 46                | 2 45.6              | 79                | 4 44.4              |
|                   |         | 14                | 0 50.4              | 47                | 2 49.2              | .080              | 4 48.0              |
|                   |         | 15                | 0 54.0              | 48                | 2 52.8              | 81                | 4 51.6              |
|                   |         | 16                | 0 57.6              | 49                | 2 56.4              | 82                | 4 55.2              |
|                   |         | 17                | 1 1.2               | .050              | 3 0.0               | 83                | 4 58.8              |
|                   |         | 18                | 1 4.8               | 51                | 3 3.6               | 84                | 5 2.4               |
|                   |         | 19                | 1 8.4               | 52                | 3 7.2               | 85                | 5 6.0               |
|                   |         | .020              | 1 12.0              | 53                | 3 10.8              | 86                | 5 9.6               |
|                   |         | 21                | 1 15.6              | 54                | 3 14.4              | 87                | 5 13.2              |
|                   |         | 22                | 1 19.2              | 55                | 3 18.0              | 88                | 5 16.8              |
|                   |         | 23                | 1 22.8              | 56                | 3 21.6              | 89                | 5 20.4              |
|                   |         | 24                | 1 26.4              | 57                | 3 25.2              | .090              | 5 24.0              |
|                   |         | 25                | 1 30.0              | 58                | 3 28.8              | 91                | 5 27.6              |
|                   |         | 26                | 1 33.6              | 59                | 3 32.4              | 92                | 5 31.2              |
|                   |         | 27                | 1 37.2              | .060              | 3 36.0              | 93                | 5 34.8              |
|                   |         | 28                | 1 40.8              | 61                | 3 39.6              | 94                | 5 38.4              |
|                   |         | 29                | 1 44.4              | 62                | 3 43.2              | 95                | 5 42.0              |
|                   |         | .030              | 1 48.0              | 63                | 3 46.8              | 96                | 5 45.6              |
|                   |         | 31                | 1 51.6              | 64                | 3 50.4              | 97                | 5 49.2              |
|                   |         | 32                | 1 55.2              | 65                | 3 54.0              | 98                | 5 52.8              |
|                   |         | 33                | 1 58.8              | 66                | 3 57.6              | 99                | 5 56.4              |

Example.—Required the value of  $0^{\circ}.875$  in minutes and seconds :

From Part 1 ...  $.8 = 48'$

„ 2 ...  $.075 = 4' 30''$

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$.875 = 52' 30''$

TABLE II.—For converting Minutes and Seconds into Decimals of a Degree.

| Minutes | Decimals of a Degree | Seconds and Decimals of Seconds | Decimals of a Degree cor- rect to three places | Actual value of Decimals of a Degree | Minutes | Decimals of a Degree | Seconds and Decimals of Seconds | Decimals of a Degree cor- rect to three places | Actual value of Decimals of a Degree |
|---------|----------------------|---------------------------------|------------------------------------------------|--------------------------------------|---------|----------------------|---------------------------------|------------------------------------------------|--------------------------------------|
| 0       | .0                   | 0.0                             | .000                                           | .0000                                | 3       | .0                   | 1.8                             | .051                                           | .0505                                |
| 6       | .1                   | 1.8                             | .001                                           | .0005                                | 9       | .1                   | 5.4                             | .052                                           | .0515                                |
| 12      | .2                   | 5.4                             | .002                                           | .0015                                | 15      | .2                   | 9.0                             | .053                                           | .0525                                |
| 18      | .3                   | 9.0                             | .003                                           | .0025                                | 21      | .3                   | 12.6                            | .054                                           | .0535                                |
| 24      | .4                   | 12.6                            | .004                                           | .0035                                | 27      | .4                   | 16.2                            | .055                                           | .0545                                |
| 30      | .5                   | 16.2                            | .005                                           | .0045                                | 33      | .5                   | 19.8                            | .056                                           | .0555                                |
| 36      | .6                   | 19.8                            | .006                                           | .0055                                | 39      | .6                   | 23.4                            | .057                                           | .0565                                |
| 42      | .7                   | 23.4                            | .007                                           | .0065                                | 45      | .7                   | 27.0                            | .058                                           | .0575                                |
| 48      | .8                   | 27.0                            | .008                                           | .0075                                | 51      | .8                   | 30.6                            | .059                                           | .0585                                |
| 54      | .9                   | 30.6                            | .009                                           | .0085                                | 57      | .9                   | 34.2                            | .060                                           | .0595                                |
|         |                      | 34.2                            | .010                                           | .0095                                |         |                      | 37.8                            | .061                                           | .0605                                |
|         |                      | 37.8                            | .011                                           | .0105                                |         |                      | 41.4                            | .062                                           | .0615                                |
|         |                      | 41.4                            | .012                                           | .0115                                |         |                      | 45.0                            | .063                                           | .0625                                |
|         |                      | 45.0                            | .013                                           | .0125                                |         |                      | 48.6                            | .064                                           | .0635                                |
|         |                      | 48.6                            | .014                                           | .0135                                |         |                      | 52.2                            | .065                                           | .0645                                |
|         |                      | 52.2                            | .015                                           | .0145                                |         |                      | 55.8                            | .066                                           | .0655                                |
|         |                      | 55.8                            | .016                                           | .0155                                |         |                      | 59.4                            | .067                                           | .0665                                |
|         |                      | 59.4                            | .017                                           | .0165                                |         |                      |                                 |                                                |                                      |
| 1       | .0                   | 3.0                             | .018                                           | .0175                                | 4       | .0                   | 3.0                             | .068                                           | .0675                                |
| 7       | .1                   | 6.6                             | .019                                           | .0185                                | 10      | .1                   | 6.6                             | .069                                           | .0685                                |
| 13      | .2                   | 10.2                            | .020                                           | .0195                                | 16      | .2                   | 10.2                            | .070                                           | .0695                                |
| 19      | .3                   | 13.8                            | .021                                           | .0205                                | 22      | .3                   | 13.8                            | .071                                           | .0705                                |
| 25      | .4                   | 17.4                            | .022                                           | .0215                                | 28      | .4                   | 17.4                            | .072                                           | .0715                                |
| 31      | .5                   | 21.0                            | .023                                           | .0225                                | 34      | .5                   | 21.0                            | .073                                           | .0725                                |
| 37      | .6                   | 24.6                            | .024                                           | .0235                                | 40      | .6                   | 24.6                            | .074                                           | .0735                                |
| 43      | .7                   | 28.2                            | .025                                           | .0245                                | 46      | .7                   | 28.2                            | .075                                           | .0745                                |
| 49      | .8                   | 31.8                            | .026                                           | .0255                                | 52      | .8                   | 31.8                            | .076                                           | .0755                                |
| 55      | .9                   | 35.4                            | .027                                           | .0265                                | 58      | .9                   | 35.4                            | .077                                           | .0765                                |
|         |                      | 39.0                            | .028                                           | .0275                                |         |                      | 39.0                            | .078                                           | .0775                                |
|         |                      | 42.6                            | .029                                           | .0285                                |         |                      | 42.6                            | .079                                           | .0785                                |
|         |                      | 46.2                            | .030                                           | .0295                                |         |                      | 46.2                            | .080                                           | .0795                                |
|         |                      | 49.8                            | .031                                           | .0305                                |         |                      | 49.8                            | .081                                           | .0805                                |
|         |                      | 53.4                            | .032                                           | .0315                                |         |                      | 53.4                            | .082                                           | .0815                                |
|         |                      | 57.0                            | .033                                           | .0325                                |         |                      | 57.0                            | .083                                           | .0825                                |
|         |                      |                                 |                                                |                                      |         |                      |                                 |                                                |                                      |
| 2       | .0                   | 0.6                             | .034                                           | .0335                                | 5       | .0                   | 0.6                             | .084                                           | .0835                                |
| 8       | .1                   | 4.2                             | .035                                           | .0345                                | 11      | .1                   | 4.2                             | .085                                           | .0845                                |
| 14      | .2                   | 7.8                             | .036                                           | .0355                                | 17      | .2                   | 7.8                             | .086                                           | .0855                                |
| 20      | .3                   | 11.4                            | .037                                           | .0365                                | 23      | .3                   | 11.4                            | .087                                           | .0865                                |
| 26      | .4                   | 15.0                            | .038                                           | .0375                                | 29      | .4                   | 15.0                            | .088                                           | .0875                                |
| 32      | .5                   | 18.6                            | .039                                           | .0385                                | 35      | .5                   | 18.6                            | .089                                           | .0885                                |
| 38      | .6                   | 22.2                            | .040                                           | .0395                                | 41      | .6                   | 22.2                            | .090                                           | .0895                                |
| 44      | .7                   | 25.8                            | .041                                           | .0405                                | 47      | .7                   | 25.8                            | .091                                           | .0905                                |
| 50      | .8                   | 29.4                            | .042                                           | .0415                                | 53      | .8                   | 29.4                            | .092                                           | .0915                                |
| 56      | .9                   | 33.0                            | .043                                           | .0425                                | 59      | .9                   | 33.0                            | .093                                           | .0925                                |
|         |                      | 36.6                            | .044                                           | .0435                                |         |                      | 36.6                            | .094                                           | .0935                                |
|         |                      | 40.2                            | .045                                           | .0445                                |         |                      | 40.2                            | .095                                           | .0945                                |
|         |                      | 43.8                            | .046                                           | .0455                                |         |                      | 43.8                            | .096                                           | .0955                                |
|         |                      | 47.4                            | .047                                           | .0465                                |         |                      | 47.4                            | .097                                           | .0965                                |
|         |                      | 51.0                            | .048                                           | .0475                                |         |                      | 51.0                            | .098                                           | .0975                                |
|         |                      | 54.6                            | .049                                           | .0485                                |         |                      | 54.6                            | .099                                           | .0985                                |
|         |                      | 58.2                            | .050                                           | .0495                                |         |                      | 58.2                            | .100                                           | .0995                                |
| 3       | .0                   | 1.8                             |                                                | .0505                                |         | .1                   | 1.8                             |                                                | .1005                                |

## COMPUTATIONS.

[PART VI.]

TABLE III.—Values of  $p$  the Mean Longitude of the Moon's Perigee (the  $\pi$  of the computation forms) for every year from 1850 to 1949.

| Year | $p$ or $\pi$  | Year | $p$ or $\pi$  | Year | $p$ or $\pi$  | Year | $p$ or $\pi$  |
|------|---------------|------|---------------|------|---------------|------|---------------|
|      | °             |      | °             |      | °             |      | °             |
| 1850 | 99° 7' 29.20  | 1875 | 36° 9' 60.88  | 1900 | 334° 19' 13.0 | 1925 | 271° 42' 54.9 |
| 51   | 140° 39' 17.6 | 76   | 77° 73' 47.9  | 1    | 14° 85' 39.6  | 26   | 312° 08' 81.0 |
| 52   | 181° 16' 57.1 | 77   | 118° 39' 7.29 | 2    | 55° 51' 66.1  | 27   | 352° 75' 07.0 |
| 53   | 221° 8' 28.26 | 78   | 159° 05' 97.9 | 3    | 96° 17' 9.26  | 28   | 33° 52' 47.1  |
| 54   | 262° 49' 08.1 | 79   | 199° 7' 22.29 | 4    | 136° 95' 33.1 | 29   | 74° 18' 73.1  |
| 1855 | 303° 15' 33.6 | 1880 | 240° 49' 61.9 | 1905 | 177° 61' 59.6 | 1930 | 114° 8' 49.90 |
| 56   | 343° 92' 73.1 | 81   | 281° 15' 86.8 | 6    | 218° 27' 86.1 | 31   | 155° 51' 25.0 |
| 57   | 24° 58' 98.5  | 82   | 321° 82' 11.8 | 7    | 258° 94' 12.5 | 32   | 196° 28' 64.9 |
| 58   | 65° 25' 23.9  | 83   | 2° 48' 36.7   | 8    | 299° 71' 52.9 | 33   | 236° 94' 90.8 |
| 59   | 105° 91' 49.3 | 84   | 43° 25' 75.6  | 9    | 340° 37' 79.3 | 34   | 277° 61' 16.7 |
| 1860 | 146° 68' 88.7 | 1885 | 83° 92' 00.4  | 1910 | 21° 04' 05.7  | 1935 | 318° 27' 42.6 |
| 61   | 187° 35' 14.0 | 86   | 124° 58' 25.3 | 11   | 61° 70' 32.0  | 36   | 359° 04' 82.5 |
| 62   | 228° 01' 39.3 | 87   | 165° 24' 50.1 | 12   | 102° 47' 72.4 | 37   | 39° 71' 08.3  |
| 63   | 268° 67' 64.6 | 88   | 206° 01' 88.9 | 13   | 143° 13' 98.7 | 38   | 80° 37' 34.1  |
| 64   | 309° 45' 04.0 | 89   | 246° 68' 13.7 | 14   | 183° 80' 25.0 | 39   | 121° 03' 59.9 |
| 1865 | 350° 11' 29.2 | 1890 | 287° 34' 38.5 | 1915 | 224° 46' 51.3 | 1940 | 161° 80' 99.7 |
| 66   | 30° 77' 54.5  | 91   | 328° 00' 63.2 | 16   | 265° 23' 91.5 | 41   | 202° 47' 25.4 |
| 67   | 71° 43' 79.7  | 92   | 8° 78' 02.0   | 17   | 305° 90' 17.8 | 42   | 243° 13' 51.2 |
| 68   | 112° 21' 18.9 | 93   | 49° 44' 26.7  | 18   | 346° 56' 44.0 | 43   | 283° 79' 69.0 |
| 69   | 152° 87' 44.1 | 94   | 90° 105' 14.0 | 19   | 27° 22' 70.2  | 44   | 324° 57' 16.6 |
| 1870 | 193° 53' 69.3 | 1895 | 130° 76' 76.0 | 1920 | 68° 00' 10.4  | 1945 | 5° 23' 42.3   |
| 71   | 234° 19' 94.4 | 96   | 171° 54' 14.7 | 21   | 108° 66' 36.5 | 46   | 45° 89' 67.9  |
| 72   | 274° 97' 33.6 | 97   | 212° 20' 39.3 | 22   | 149° 32' 62.6 | 47   | 86° 55' 93.5  |
| 73   | 315° 63' 58.7 | 98   | 252° 86' 63.9 | 23   | 189° 98' 88.8 | 48   | 127° 33' 33.2 |
| 74   | 356° 29' 83.8 | 99   | 293° 52' 88.5 | 24   | 230° 76' 28.9 | 49   | 167° 99' 58.8 |

These values are for January 0 (*i. e.*, noon December 31st of preceding year), except in the case of leap-years, when the values are for 0 hour January 1st.

The values given in the above table require  $0^{\circ}.136$  to be added to give the true values of  $p$  or  $\pi$  (see page 36, Preface to Hansen's Tables), but as the form for the computation of tidal observations has been constructed showing the constant  $0^{\circ}.136$  to be added, it has been thought advisable not to make this correction in the above table.

TABLE IV.—Number of Days from January 0.

| Month          | Common year | Leap-year | Month           | Common year | Leap-year |
|----------------|-------------|-----------|-----------------|-------------|-----------|
| January 0 ...  | 0           | -1        | July 0 ...      | 181         | 181       |
| February 0 ... | 31          | 30        | August 0 ...    | 212         | 212       |
| March 0 ...    | 59          | 59        | September 0 ... | 243         | 243       |
| April 0 ...    | 90          | 90        | October 0 ...   | 273         | 273       |
| May 0 ...      | 120         | 120       | November 0 ...  | 304         | 304       |
| June 0 ...     | 151         | 151       | December 0 ...  | 334         | 334       |



TABLE V.—Value of Movement of  $p$  or  $\pi$  for 1 to 366 Days.

| Days | $\pi$    | Days | $\pi$     | Days | $\pi$     | Days | $\pi$     | Days | $\pi$     | Days | $\pi$     |
|------|----------|------|-----------|------|-----------|------|-----------|------|-----------|------|-----------|
| 1    | 0° 11140 | 62   | 6° 90708  | 123  | 13° 70276 | 184  | 20° 49844 | 245  | 27° 29411 | 306  | 34° 08979 |
| 2    | 0° 22281 | 63   | 7° 01849  | 124  | 13° 81416 | 185  | 20° 60984 | 246  | 27° 40552 | 307  | 34° 20119 |
| 3    | 0° 33421 | 64   | 7° 12989  | 125  | 13° 92557 | 186  | 20° 72124 | 247  | 27° 51692 | 308  | 34° 31260 |
| 4    | 0° 44562 | 65   | 7° 24130  | 126  | 14° 03697 | 187  | 20° 83265 | 248  | 27° 62833 | 309  | 34° 42400 |
| 5    | 0° 55702 | 66   | 7° 35270  | 127  | 14° 14838 | 188  | 20° 94405 | 249  | 27° 73973 | 310  | 34° 53541 |
| 6    | 0° 66843 | 67   | 7° 46410  | 128  | 14° 25978 | 189  | 21° 05546 | 250  | 27° 85113 | 311  | 34° 64681 |
| 7    | 0° 77983 | 68   | 7° 57551  | 129  | 14° 37119 | 190  | 21° 16686 | 251  | 27° 96254 | 312  | 34° 75822 |
| 8    | 0° 89124 | 69   | 7° 68691  | 130  | 14° 48259 | 191  | 21° 27827 | 252  | 28° 07394 | 313  | 34° 86962 |
| 9    | 1° 00264 | 70   | 7° 79832  | 131  | 14° 59399 | 192  | 21° 38967 | 253  | 28° 18535 | 314  | 34° 98102 |
| 10   | 1° 11405 | 71   | 7° 90972  | 132  | 14° 70540 | 193  | 21° 50108 | 254  | 28° 29675 | 315  | 35° 09243 |
| 11   | 1° 22545 | 72   | 8° 02113  | 133  | 14° 81680 | 194  | 21° 61248 | 255  | 28° 40816 | 316  | 35° 20383 |
| 12   | 1° 33685 | 73   | 8° 13253  | 134  | 14° 92821 | 195  | 21° 72389 | 256  | 28° 51956 | 317  | 35° 31524 |
| 13   | 1° 44826 | 74   | 8° 24394  | 135  | 15° 03961 | 196  | 21° 83529 | 257  | 28° 63097 | 318  | 35° 42664 |
| 14   | 1° 55966 | 75   | 8° 35534  | 136  | 15° 15102 | 197  | 21° 94669 | 258  | 28° 74237 | 319  | 35° 53805 |
| 15   | 1° 67107 | 76   | 8° 46675  | 137  | 15° 26242 | 198  | 22° 05810 | 259  | 28° 85378 | 320  | 35° 64945 |
| 16   | 1° 78247 | 77   | 8° 57815  | 138  | 15° 37383 | 199  | 22° 16950 | 260  | 28° 96518 | 321  | 35° 76086 |
| 17   | 1° 89388 | 78   | 8° 68955  | 139  | 15° 48523 | 200  | 22° 28091 | 261  | 29° 07658 | 322  | 35° 87226 |
| 18   | 2° 00528 | 79   | 8° 80096  | 140  | 15° 59664 | 201  | 22° 39231 | 262  | 29° 18799 | 323  | 35° 98367 |
| 19   | 2° 11669 | 80   | 8° 91236  | 141  | 15° 70804 | 202  | 22° 50372 | 263  | 29° 29939 | 324  | 36° 09507 |
| 20   | 2° 22809 | 81   | 9° 02377  | 142  | 15° 81944 | 203  | 22° 61512 | 264  | 29° 41080 | 325  | 36° 20647 |
| 21   | 2° 33950 | 82   | 9° 13517  | 143  | 15° 93085 | 204  | 22° 72653 | 265  | 29° 52220 | 326  | 36° 31788 |
| 22   | 2° 45090 | 83   | 9° 24658  | 144  | 16° 04225 | 205  | 22° 83793 | 266  | 29° 63361 | 327  | 36° 42928 |
| 23   | 2° 56230 | 84   | 9° 35798  | 145  | 16° 15366 | 206  | 22° 94933 | 267  | 29° 74501 | 328  | 36° 54069 |
| 24   | 2° 67371 | 85   | 9° 46939  | 146  | 16° 26506 | 207  | 23° 06074 | 268  | 29° 85642 | 329  | 36° 65209 |
| 25   | 2° 78511 | 86   | 9° 58079  | 147  | 16° 37647 | 208  | 23° 17214 | 269  | 29° 96782 | 330  | 36° 76350 |
| 26   | 2° 89652 | 87   | 9° 69219  | 148  | 16° 48787 | 209  | 23° 28355 | 270  | 30° 07923 | 331  | 36° 87490 |
| 27   | 3° 00792 | 88   | 9° 80360  | 149  | 16° 59928 | 210  | 23° 39495 | 271  | 30° 19063 | 332  | 36° 98631 |
| 28   | 3° 11933 | 89   | 9° 91500  | 150  | 16° 71068 | 211  | 23° 50636 | 272  | 30° 30203 | 333  | 37° 09771 |
| 29   | 3° 23073 | 90   | 10° 02641 | 151  | 16° 82209 | 212  | 23° 61776 | 273  | 30° 41344 | 334  | 37° 20912 |
| 30   | 3° 34214 | 91   | 10° 13781 | 152  | 16° 93349 | 213  | 23° 72917 | 274  | 30° 52484 | 335  | 37° 32052 |
| 31   | 3° 45354 | 92   | 10° 24922 | 153  | 17° 04489 | 214  | 23° 84057 | 275  | 30° 63625 | 336  | 37° 43192 |
| 32   | 3° 56495 | 93   | 10° 36062 | 154  | 17° 15630 | 215  | 23° 95198 | 276  | 30° 74765 | 337  | 37° 54333 |
| 33   | 3° 67635 | 94   | 10° 47203 | 155  | 17° 26770 | 216  | 24° 06338 | 277  | 30° 85906 | 338  | 37° 65473 |
| 34   | 3° 78775 | 95   | 10° 58343 | 156  | 17° 37911 | 217  | 24° 17478 | 278  | 30° 97046 | 339  | 37° 76614 |
| 35   | 3° 89916 | 96   | 10° 69484 | 157  | 17° 49051 | 218  | 24° 28619 | 279  | 31° 08187 | 340  | 37° 87754 |
| 36   | 4° 01056 | 97   | 10° 80624 | 158  | 17° 60192 | 219  | 24° 39759 | 280  | 31° 19327 | 341  | 37° 98895 |
| 37   | 4° 12197 | 98   | 10° 91765 | 159  | 17° 71332 | 220  | 24° 50900 | 281  | 31° 30468 | 342  | 38° 10035 |
| 38   | 4° 23337 | 99   | 11° 02905 | 160  | 17° 82473 | 221  | 24° 62040 | 282  | 31° 41608 | 343  | 38° 21176 |
| 39   | 4° 34478 | 100  | 11° 14045 | 161  | 17° 93613 | 222  | 24° 73181 | 283  | 31° 52748 | 344  | 38° 32316 |
| 40   | 4° 45618 | 101  | 11° 25186 | 162  | 18° 04754 | 223  | 24° 84321 | 284  | 31° 63889 | 345  | 38° 43457 |
| 41   | 4° 56759 | 102  | 11° 36326 | 163  | 18° 15894 | 224  | 24° 95462 | 285  | 31° 75029 | 346  | 38° 54597 |
| 42   | 4° 67899 | 103  | 11° 47467 | 164  | 18° 27034 | 225  | 25° 06602 | 286  | 31° 86170 | 347  | 38° 65737 |
| 43   | 4° 79040 | 104  | 11° 58607 | 165  | 18° 38175 | 226  | 25° 17743 | 287  | 31° 97310 | 348  | 38° 76878 |
| 44   | 4° 90180 | 105  | 11° 69748 | 166  | 18° 49315 | 227  | 25° 28883 | 288  | 32° 08451 | 349  | 38° 88018 |
| 45   | 5° 01320 | 106  | 11° 80888 | 167  | 18° 60456 | 228  | 25° 40023 | 289  | 32° 19591 | 350  | 38° 99159 |
| 46   | 5° 12461 | 107  | 11° 92029 | 168  | 18° 71596 | 229  | 25° 51164 | 290  | 32° 30732 | 351  | 39° 10299 |
| 47   | 5° 23601 | 108  | 12° 03169 | 169  | 18° 82737 | 230  | 25° 62304 | 291  | 32° 41872 | 352  | 39° 21440 |
| 48   | 5° 34742 | 109  | 12° 14309 | 170  | 18° 93877 | 231  | 25° 73445 | 292  | 32° 53012 | 353  | 39° 32580 |
| 49   | 5° 45882 | 110  | 12° 25450 | 171  | 19° 05018 | 232  | 25° 84585 | 293  | 32° 64153 | 354  | 39° 43721 |
| 50   | 5° 57023 | 111  | 12° 36590 | 172  | 19° 16158 | 233  | 25° 95726 | 294  | 32° 75293 | 355  | 39° 54861 |
| 51   | 5° 68163 | 112  | 12° 47731 | 173  | 19° 27299 | 234  | 26° 06866 | 295  | 32° 86434 | 356  | 39° 66002 |
| 52   | 5° 79304 | 113  | 12° 58871 | 174  | 19° 38439 | 235  | 26° 18007 | 296  | 32° 97574 | 357  | 39° 77142 |
| 53   | 5° 90444 | 114  | 12° 70012 | 175  | 19° 49579 | 236  | 26° 29147 | 297  | 33° 08715 | 358  | 39° 88282 |
| 54   | 6° 01585 | 115  | 12° 81152 | 176  | 19° 60720 | 237  | 26° 40288 | 298  | 33° 19855 | 359  | 39° 99423 |
| 55   | 6° 12725 | 116  | 12° 92293 | 177  | 19° 71860 | 238  | 26° 51428 | 299  | 33° 30996 | 360  | 40° 10563 |
| 56   | 6° 23865 | 117  | 13° 03433 | 178  | 19° 83001 | 239  | 26° 62568 | 300  | 33° 42136 | 361  | 40° 21704 |
| 57   | 6° 35006 | 118  | 13° 14574 | 179  | 19° 94141 | 240  | 26° 73709 | 301  | 33° 53277 | 362  | 40° 32844 |
| 58   | 6° 46146 | 119  | 13° 25714 | 180  | 20° 05282 | 241  | 26° 84849 | 302  | 33° 64417 | 363  | 40° 43985 |
| 59   | 6° 57287 | 120  | 13° 36854 | 181  | 20° 16422 | 242  | 26° 95990 | 303  | 33° 75557 | 364  | 40° 55125 |
| 60   | 6° 68427 | 121  | 13° 47995 | 182  | 20° 27563 | 243  | 27° 07130 | 304  | 33° 86698 | 365  | 40° 66266 |

## COMPUTATIONS.

[PART VI.]

TABLE VI.—Value of the Movement of  $p$  or  $\pi$  for differences of Longitude Greenwich.

| Difference of Longitude | Value of $\pi$ | Actual values of $\pi$ corresponding to Degrees in Column I | Difference of Longitude | Value of $\pi$ | Actual values of $\pi$ corresponding to Degrees in Column I |
|-------------------------|----------------|-------------------------------------------------------------|-------------------------|----------------|-------------------------------------------------------------|
| °                       |                | °                                                           | °                       |                | °                                                           |
| 0° 000                  | °              | °0000                                                       | 88° 865                 | °028           | °0275                                                       |
| 1° 616                  | °000           | °0005                                                       | 92° 097                 | °029           | °0285                                                       |
| 4° 847                  | °001           | °0015                                                       | 95° 328                 | °030           | °0295                                                       |
| 8° 079                  | °002           | °0025                                                       | 98° 560                 | °031           | °0305                                                       |
| 11° 310                 | °003           | °0035                                                       | 101° 791                | °032           | °0315                                                       |
| 14° 542                 | °004           | °0045                                                       | 105° 023                | °033           | °0325                                                       |
| 17° 773                 | °005           | °0055                                                       | 108° 254                | °034           | °0335                                                       |
| 21° 005                 | °006           | °0065                                                       | 111° 486                | °035           | °0345                                                       |
| 24° 236                 | °007           | °0075                                                       | 114° 717                | °036           | °0355                                                       |
| 27° 467                 | °008           | °0085                                                       | 117° 949                | °037           | °0365                                                       |
| 30° 699                 | °009           | °0095                                                       | 121° 180                | °038           | °0375                                                       |
| 33° 930                 | °010           | °0105                                                       | 124° 411                | °039           | °0385                                                       |
| 37° 162                 | °011           | °0115                                                       | 127° 643                | °040           | °0395                                                       |
| 40° 393                 | °012           | °0125                                                       | 130° 874                | °041           | °0405                                                       |
| 43° 625                 | °013           | °0135                                                       | 134° 106                | °042           | °0415                                                       |
| 46° 856                 | °014           | °0145                                                       | 137° 337                | °043           | °0425                                                       |
| 50° 088                 | °015           | °0155                                                       | 140° 569                | °044           | °0435                                                       |
| 53° 319                 | °016           | °0165                                                       | 143° 800                | °045           | °0445                                                       |
| 56° 551                 | °017           | °0175                                                       | 147° 032                | °046           | °0455                                                       |
| 59° 782                 | °018           | °0185                                                       | 150° 263                | °047           | °0465                                                       |
| 63° 014                 | °019           | °0195                                                       | 153° 495                | °048           | °0475                                                       |
| 66° 245                 | °020           | °0205                                                       | 156° 726                | °049           | °0485                                                       |
| 69° 477                 | °021           | °0215                                                       | 159° 958                | °050           | °0495                                                       |
| 72° 708                 | °022           | °0225                                                       | 163° 189                | °051           | °0505                                                       |
| 75° 939                 | °023           | °0235                                                       | 166° 421                | °052           | °0515                                                       |
| 79° 171                 | °024           | °0245                                                       | 169° 652                | °053           | °0525                                                       |
| 82° 402                 | °025           | °0255                                                       | 172° 884                | °054           | °0535                                                       |
| 85° 634                 | °026           | °0265                                                       | 176° 115                | °055           | °0545                                                       |
| 88° 865                 | °027           | °0275                                                       | 179° 346                | °056           | °0555                                                       |
|                         |                |                                                             | 182° 578                |                | °0565                                                       |

Correction for E. Longitude — , for W. Longitude + .

TABLE VII.—Products of Augmenting Factors  $R_1$ ,  $R_2$ , and  $R_4$  multiplied by 1 to 99.

| $R_1 = \cdot 0028.$ |       |       |        |        |        |        |        |        |        |        |   |
|---------------------|-------|-------|--------|--------|--------|--------|--------|--------|--------|--------|---|
|                     | 0     | 10    | 20     | 30     | 40     | 50     | 60     | 70     | 80     | 90     |   |
| 0                   | ·0000 | ·0280 | ·0560  | ·0840  | ·1120  | ·1400  | ·1680  | ·1960  | ·2240  | ·2520  | 0 |
| 1                   | ·0028 | ·0308 | ·0588  | ·0868  | ·1148  | ·1428  | ·1708  | ·1988  | ·2268  | ·2548  | 1 |
| 2                   | ·0056 | ·0336 | ·0616  | ·0896  | ·1176  | ·1456  | ·1736  | ·2016  | ·2296  | ·2576  | 2 |
| 3                   | ·0084 | ·0364 | ·0644  | ·0924  | ·1204  | ·1484  | ·1764  | ·2044  | ·2324  | ·2604  | 3 |
| 4                   | ·0112 | ·0392 | ·0672  | ·0952  | ·1232  | ·1512  | ·1792  | ·2072  | ·2352  | ·2632  | 4 |
| 5                   | ·0140 | ·0420 | ·0700  | ·0980  | ·1260  | ·1540  | ·1820  | ·2100  | ·2380  | ·2660  | 5 |
| 6                   | ·0168 | ·0448 | ·0728  | ·1008  | ·1288  | ·1568  | ·1848  | ·2128  | ·2408  | ·2688  | 6 |
| 7                   | ·0196 | ·0476 | ·0756  | ·1036  | ·1316  | ·1596  | ·1876  | ·2156  | ·2436  | ·2716  | 7 |
| 8                   | ·0224 | ·0504 | ·0784  | ·1064  | ·1344  | ·1624  | ·1904  | ·2184  | ·2464  | ·2744  | 8 |
| 9                   | ·0252 | ·0532 | ·0812  | ·1092  | ·1372  | ·1652  | ·1932  | ·2212  | ·2492  | ·2772  | 9 |
| $R_2 = \cdot 0115.$ |       |       |        |        |        |        |        |        |        |        |   |
|                     | 0     | 10    | 20     | 30     | 40     | 50     | 60     | 70     | 80     | 90     |   |
| 0                   | ·0000 | ·1150 | ·2300  | ·3450  | ·4600  | ·5750  | ·6900  | ·8050  | ·9200  | 1·0350 | 0 |
| 1                   | ·0115 | ·1265 | ·2415  | ·3565  | ·4715  | ·5865  | ·7015  | ·8165  | ·9315  | 1·0465 | 1 |
| 2                   | ·0230 | ·1380 | ·2530  | ·3680  | ·4830  | ·5980  | ·7130  | ·8280  | ·9430  | 1·0580 | 2 |
| 3                   | ·0345 | ·1495 | ·2645  | ·3795  | ·4945  | ·6095  | ·7245  | ·8395  | ·9545  | 1·0695 | 3 |
| 4                   | ·0460 | ·1610 | ·2760  | ·3910  | ·5060  | ·6210  | ·7360  | ·8510  | ·9660  | 1·0810 | 4 |
| 5                   | ·0575 | ·1725 | ·2875  | ·4025  | ·5175  | ·6325  | ·7475  | ·8625  | ·9775  | 1·0925 | 5 |
| 6                   | ·0690 | ·1840 | ·2990  | ·4140  | ·5290  | ·6440  | ·7590  | ·8740  | ·9890  | 1·1040 | 6 |
| 7                   | ·0805 | ·1955 | ·3105  | ·4255  | ·5405  | ·6555  | ·7705  | ·8855  | 1·0005 | 1·1155 | 7 |
| 8                   | ·0920 | ·2070 | ·3220  | ·4370  | ·5520  | ·6670  | ·7820  | ·8970  | 1·0120 | 1·1270 | 8 |
| 9                   | ·1035 | ·2185 | ·3335  | ·4485  | ·5635  | ·6785  | ·7935  | ·9085  | 1·0235 | 1·1385 | 9 |
| $R_4 = \cdot 0472.$ |       |       |        |        |        |        |        |        |        |        |   |
|                     | 0     | 10    | 20     | 30     | 40     | 50     | 60     | 70     | 80     | 90     |   |
| 0                   | ·0000 | ·4720 | ·9440  | 1·4160 | 1·8880 | 2·3600 | 2·8320 | 3·3040 | 3·7760 | 4·2480 | 0 |
| 1                   | ·0472 | ·5192 | ·9912  | 1·4632 | 1·9352 | 2·4072 | 2·8792 | 3·3512 | 3·8232 | 4·2952 | 1 |
| 2                   | ·0944 | ·5664 | 1·0384 | 1·5104 | 1·9824 | 2·4544 | 2·9264 | 3·3984 | 3·8704 | 4·3424 | 2 |
| 3                   | ·1416 | ·6136 | 1·0856 | 1·5576 | 2·0296 | 2·5016 | 2·9736 | 3·4456 | 3·9176 | 4·3896 | 3 |
| 4                   | ·1888 | ·6608 | 1·1328 | 1·6048 | 2·0768 | 2·5488 | 3·0208 | 3·4928 | 3·9648 | 4·4368 | 4 |
| 5                   | ·2360 | ·7080 | 1·1800 | 1·6520 | 2·1240 | 2·5960 | 3·0680 | 3·5400 | 4·0120 | 4·4840 | 5 |
| 6                   | ·2832 | ·7552 | 1·2272 | 1·6992 | 2·1712 | 2·6432 | 3·1152 | 3·5872 | 4·0592 | 4·5312 | 6 |
| 7                   | ·3304 | ·8024 | 1·2744 | 1·7464 | 2·2184 | 2·6904 | 3·1624 | 3·6344 | 4·1064 | 4·5784 | 7 |
| 8                   | ·3776 | ·8496 | 1·3216 | 1·7936 | 2·2656 | 2·7376 | 3·2096 | 3·6816 | 4·1536 | 4·6256 | 8 |
| 9                   | ·4248 | ·8968 | 1·3688 | 1·8408 | 2·3128 | 2·7848 | 3·2568 | 3·7288 | 4·2008 | 4·6728 | 9 |

COMPUTATIONS.

[PART VI.]

TABLE VIII.—Products of  $S_1 \times \cdot 001$  up to  $S_1 \times 1\cdot 000$ .

$S_1 = \sin 15^\circ = \cdot 25882$ .

| No. | ·000    | ·001    | ·002    | ·003    | ·004    | ·005    | ·006    | ·007    | ·008    | ·009    | No. |
|-----|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|-----|
| ·00 | ·000000 | ·000259 | ·000518 | ·000776 | ·001035 | ·001294 | ·001553 | ·001812 | ·002071 | ·002329 | ·00 |
| ·01 | ·002588 | ·002847 | ·003106 | ·003365 | ·003623 | ·003882 | ·004141 | ·004400 | ·004659 | ·004918 | ·01 |
| ·02 | ·005176 | ·005435 | ·005694 | ·005953 | ·006212 | ·006471 | ·006729 | ·006988 | ·007247 | ·007506 | ·02 |
| ·03 | ·007765 | ·008023 | ·008282 | ·008541 | ·008800 | ·009059 | ·009318 | ·009576 | ·009835 | ·010094 | ·03 |
| ·04 | ·010353 | ·010612 | ·010870 | ·011129 | ·011388 | ·011647 | ·011906 | ·012165 | ·012423 | ·012682 | ·04 |
| ·05 | ·012941 | ·013200 | ·013459 | ·013717 | ·013976 | ·014235 | ·014494 | ·014753 | ·015012 | ·015270 | ·05 |
| ·06 | ·015529 | ·015788 | ·016047 | ·016306 | ·016564 | ·016823 | ·017082 | ·017341 | ·017600 | ·017859 | ·06 |
| ·07 | ·018117 | ·018376 | ·018635 | ·018894 | ·019153 | ·019412 | ·019670 | ·019929 | ·020188 | ·020447 | ·07 |
| ·08 | ·020706 | ·020964 | ·021223 | ·021482 | ·021741 | ·022000 | ·022259 | ·022517 | ·022776 | ·023035 | ·08 |
| ·09 | ·023294 | ·023553 | ·023811 | ·024070 | ·024329 | ·024588 | ·024847 | ·025106 | ·025364 | ·025623 | ·09 |
| ·10 | ·025882 | ·026141 | ·026400 | ·026658 | ·026917 | ·027176 | ·027435 | ·027694 | ·027953 | ·028211 | ·10 |
| ·11 | ·028470 | ·028729 | ·028988 | ·029247 | ·029505 | ·029764 | ·030023 | ·030282 | ·030541 | ·030800 | ·11 |
| ·12 | ·031058 | ·031317 | ·031576 | ·031835 | ·032094 | ·032353 | ·032611 | ·032870 | ·033129 | ·033388 | ·12 |
| ·13 | ·033647 | ·033905 | ·034164 | ·034423 | ·034682 | ·034941 | ·035200 | ·035458 | ·035717 | ·035976 | ·13 |
| ·14 | ·036235 | ·036494 | ·036752 | ·037011 | ·037270 | ·037529 | ·037788 | ·038047 | ·038305 | ·038564 | ·14 |
| ·15 | ·038823 | ·039082 | ·039341 | ·039599 | ·039858 | ·040117 | ·040376 | ·040635 | ·040894 | ·041152 | ·15 |
| ·16 | ·041411 | ·041670 | ·041929 | ·042188 | ·042446 | ·042705 | ·042964 | ·043223 | ·043482 | ·043741 | ·16 |
| ·17 | ·043999 | ·044258 | ·044517 | ·044776 | ·045035 | ·045294 | ·045552 | ·045811 | ·046070 | ·046329 | ·17 |
| ·18 | ·046588 | ·046846 | ·047105 | ·047364 | ·047623 | ·047882 | ·048141 | ·048399 | ·048658 | ·048917 | ·18 |
| ·19 | ·049176 | ·049435 | ·049693 | ·049952 | ·050211 | ·050470 | ·050729 | ·050988 | ·051246 | ·051505 | ·19 |
| ·20 | ·051764 | ·052023 | ·052282 | ·052540 | ·052799 | ·053058 | ·053317 | ·053576 | ·053835 | ·054093 | ·20 |
| ·21 | ·054352 | ·054611 | ·054870 | ·055129 | ·055387 | ·055646 | ·055905 | ·056164 | ·056423 | ·056682 | ·21 |
| ·22 | ·056940 | ·057199 | ·057458 | ·057717 | ·057976 | ·058235 | ·058493 | ·058752 | ·059011 | ·059270 | ·22 |
| ·23 | ·059529 | ·059787 | ·060046 | ·060305 | ·060564 | ·060823 | ·061082 | ·061340 | ·061599 | ·061858 | ·23 |
| ·24 | ·062117 | ·062376 | ·062634 | ·062893 | ·063152 | ·063411 | ·063670 | ·063929 | ·064187 | ·064446 | ·24 |
| ·25 | ·064705 | ·064964 | ·065223 | ·065481 | ·065740 | ·065999 | ·066258 | ·066517 | ·066776 | ·067034 | ·25 |
| ·26 | ·067203 | ·067522 | ·067811 | ·068070 | ·068328 | ·068587 | ·068846 | ·069105 | ·069364 | ·069623 | ·26 |
| ·27 | ·069881 | ·070140 | ·070399 | ·070658 | ·070917 | ·071176 | ·071434 | ·071693 | ·071952 | ·072211 | ·27 |
| ·28 | ·072470 | ·072728 | ·072987 | ·073246 | ·073505 | ·073764 | ·074023 | ·074281 | ·074540 | ·074799 | ·28 |
| ·29 | ·075058 | ·075317 | ·075575 | ·075834 | ·076093 | ·076352 | ·076611 | ·076870 | ·077128 | ·077387 | ·29 |
| ·30 | ·077646 | ·077905 | ·078164 | ·078422 | ·078681 | ·078940 | ·079199 | ·079458 | ·079717 | ·079975 | ·30 |
| ·31 | ·080234 | ·080493 | ·080752 | ·081011 | ·081269 | ·081528 | ·081787 | ·082046 | ·082305 | ·082564 | ·31 |
| ·32 | ·082822 | ·083081 | ·083340 | ·083599 | ·083858 | ·084117 | ·084375 | ·084634 | ·084893 | ·085152 | ·32 |
| ·33 | ·085411 | ·085669 | ·085928 | ·086187 | ·086446 | ·086705 | ·086964 | ·087222 | ·087481 | ·087740 | ·33 |
| ·34 | ·087999 | ·088258 | ·088516 | ·088775 | ·089034 | ·089293 | ·089552 | ·089811 | ·090069 | ·090328 | ·34 |
| ·35 | ·090587 | ·090846 | ·091105 | ·091363 | ·091622 | ·091881 | ·092140 | ·092399 | ·092658 | ·092916 | ·35 |
| ·36 | ·093175 | ·093434 | ·093693 | ·093952 | ·094210 | ·094469 | ·094728 | ·094987 | ·095246 | ·095505 | ·36 |
| ·37 | ·095763 | ·096022 | ·096281 | ·096540 | ·096799 | ·097058 | ·097316 | ·097575 | ·097834 | ·098093 | ·37 |
| ·38 | ·098352 | ·098610 | ·098869 | ·099128 | ·099387 | ·099646 | ·099905 | ·100163 | ·100422 | ·100681 | ·38 |
| ·39 | ·100940 | ·101199 | ·101457 | ·101716 | ·101975 | ·102234 | ·102493 | ·102752 | ·103010 | ·103269 | ·39 |
| ·40 | ·103528 | ·103787 | ·104046 | ·104304 | ·104563 | ·104822 | ·105081 | ·105340 | ·105599 | ·105857 | ·40 |
| ·41 | ·106116 | ·106375 | ·106634 | ·106893 | ·107151 | ·107410 | ·107669 | ·107928 | ·108187 | ·108446 | ·41 |
| ·42 | ·108704 | ·108963 | ·109222 | ·109481 | ·109740 | ·109999 | ·110257 | ·110516 | ·110775 | ·111034 | ·42 |
| ·43 | ·111293 | ·111551 | ·111810 | ·112069 | ·112328 | ·112587 | ·112846 | ·113104 | ·113363 | ·113622 | ·43 |
| ·44 | ·113881 | ·114140 | ·114398 | ·114657 | ·114916 | ·115175 | ·115434 | ·115693 | ·115951 | ·116210 | ·44 |
| ·45 | ·116469 | ·116728 | ·116987 | ·117245 | ·117504 | ·117763 | ·118022 | ·118281 | ·118540 | ·118798 | ·45 |
| ·46 | ·119057 | ·119316 | ·119575 | ·119834 | ·120092 | ·120351 | ·120610 | ·120869 | ·121128 | ·121387 | ·46 |
| ·47 | ·121645 | ·121904 | ·122163 | ·122422 | ·122681 | ·122940 | ·123198 | ·123457 | ·123716 | ·123975 | ·47 |
| ·48 | ·124234 | ·124492 | ·124751 | ·125010 | ·125269 | ·125528 | ·125787 | ·126045 | ·126304 | ·126563 | ·48 |
| ·49 | ·126822 | ·127081 | ·127339 | ·127598 | ·127857 | ·128116 | ·128375 | ·128634 | ·128892 | ·129151 | ·49 |
| ·50 | ·129410 | ·129669 | ·129928 | ·130186 | ·130445 | ·130704 | ·130963 | ·131222 | ·131481 | ·131739 | ·50 |



COMPUTATIONS.

[PART VI.]

TABLE IX.—Products of  $S_3 \times \cdot 001$  up to  $S_3 \times 1\cdot 000$ .  
 $S_3 = \sin 45^\circ = \cdot 70711$ .

| No. | ·000    | ·001    | ·002    | ·003    | ·004    | ·005    | ·006    | ·007    | ·008    | ·009    | No. |
|-----|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|-----|
| ·00 | ·000000 | ·000707 | ·001414 | ·002121 | ·002828 | ·003536 | ·004243 | ·004950 | ·005657 | ·006364 | ·00 |
| ·01 | ·007071 | ·007778 | ·008485 | ·009192 | ·009900 | ·010607 | ·011314 | ·012021 | ·012728 | ·013435 | ·01 |
| ·02 | ·014142 | ·014849 | ·015556 | ·016264 | ·016971 | ·017678 | ·018385 | ·019092 | ·019799 | ·020506 | ·02 |
| ·03 | ·021213 | ·021920 | ·022628 | ·023335 | ·024042 | ·024749 | ·025456 | ·026163 | ·026870 | ·027577 | ·03 |
| ·04 | ·028284 | ·028992 | ·029699 | ·030406 | ·031113 | ·031820 | ·032527 | ·033234 | ·033941 | ·034648 | ·04 |
| ·05 | ·035356 | ·036063 | ·036770 | ·037477 | ·038184 | ·038891 | ·039598 | ·040305 | ·041012 | ·041719 | ·05 |
| ·06 | ·042427 | ·043134 | ·043841 | ·044548 | ·045255 | ·045962 | ·046669 | ·047376 | ·048083 | ·048791 | ·06 |
| ·07 | ·049498 | ·050205 | ·050912 | ·051619 | ·052326 | ·053033 | ·053740 | ·054447 | ·055155 | ·055862 | ·07 |
| ·08 | ·056569 | ·057276 | ·057983 | ·058690 | ·059397 | ·060104 | ·060811 | ·061519 | ·062226 | ·062933 | ·08 |
| ·09 | ·063640 | ·064347 | ·065054 | ·065761 | ·066468 | ·067175 | ·067883 | ·068590 | ·069297 | ·070004 | ·09 |
| ·10 | ·070711 | ·071418 | ·072125 | ·072832 | ·073539 | ·074247 | ·074954 | ·075661 | ·076368 | ·077075 | ·10 |
| ·11 | ·077782 | ·078489 | ·079196 | ·079903 | ·080611 | ·081318 | ·082025 | ·082732 | ·083439 | ·084146 | ·11 |
| ·12 | ·084853 | ·085560 | ·086267 | ·086975 | ·087682 | ·088389 | ·089096 | ·089803 | ·090510 | ·091217 | ·12 |
| ·13 | ·091924 | ·092631 | ·093339 | ·094046 | ·094753 | ·095460 | ·096167 | ·096874 | ·097581 | ·098288 | ·13 |
| ·14 | ·098995 | ·099703 | ·100410 | ·101117 | ·101824 | ·102531 | ·103238 | ·103945 | ·104652 | ·105359 | ·14 |
| ·15 | ·106067 | ·106774 | ·107481 | ·108188 | ·108895 | ·109602 | ·110309 | ·111016 | ·111723 | ·112430 | ·15 |
| ·16 | ·113138 | ·113845 | ·114552 | ·115259 | ·115966 | ·116673 | ·117380 | ·118087 | ·118794 | ·119502 | ·16 |
| ·17 | ·120209 | ·120916 | ·121623 | ·122330 | ·123037 | ·123744 | ·124451 | ·125158 | ·125866 | ·126573 | ·17 |
| ·18 | ·127280 | ·127987 | ·128694 | ·129401 | ·130108 | ·130815 | ·131522 | ·132230 | ·132937 | ·133644 | ·18 |
| ·19 | ·134351 | ·135058 | ·135765 | ·136472 | ·137179 | ·137886 | ·138594 | ·139301 | ·140008 | ·140715 | ·19 |
| ·20 | ·141422 | ·142129 | ·142836 | ·143543 | ·144250 | ·144958 | ·145665 | ·146372 | ·147079 | ·147786 | ·20 |
| ·21 | ·148493 | ·149200 | ·149907 | ·150614 | ·151322 | ·152029 | ·152736 | ·153443 | ·154150 | ·154857 | ·21 |
| ·22 | ·155564 | ·156271 | ·156978 | ·157686 | ·158393 | ·159100 | ·159807 | ·160514 | ·161221 | ·161928 | ·22 |
| ·23 | ·162635 | ·163342 | ·164050 | ·164757 | ·165464 | ·166171 | ·166878 | ·167585 | ·168292 | ·168999 | ·23 |
| ·24 | ·169706 | ·170414 | ·171121 | ·171828 | ·172535 | ·173242 | ·173949 | ·174656 | ·175363 | ·176070 | ·24 |
| ·25 | ·176778 | ·177485 | ·178192 | ·178899 | ·179606 | ·180313 | ·181020 | ·181727 | ·182434 | ·183141 | ·25 |
| ·26 | ·183849 | ·184556 | ·185263 | ·185970 | ·186677 | ·187384 | ·188091 | ·188798 | ·189505 | ·190213 | ·26 |
| ·27 | ·190920 | ·191627 | ·192334 | ·193041 | ·193748 | ·194455 | ·195162 | ·195869 | ·196577 | ·197284 | ·27 |
| ·28 | ·197991 | ·198698 | ·199405 | ·200112 | ·200819 | ·201526 | ·202233 | ·202941 | ·203648 | ·204355 | ·28 |
| ·29 | ·205062 | ·205769 | ·206476 | ·207183 | ·207890 | ·208597 | ·209305 | ·210012 | ·210719 | ·211426 | ·29 |
| ·30 | ·212133 | ·212840 | ·213547 | ·214254 | ·214961 | ·215669 | ·216376 | ·217083 | ·217790 | ·218497 | ·30 |
| ·31 | ·219204 | ·219911 | ·220618 | ·221325 | ·222033 | ·222740 | ·223447 | ·224154 | ·224861 | ·225568 | ·31 |
| ·32 | ·226275 | ·226982 | ·227689 | ·228397 | ·229104 | ·229811 | ·230518 | ·231225 | ·231932 | ·232639 | ·32 |
| ·33 | ·233340 | ·234053 | ·234761 | ·235468 | ·236175 | ·236882 | ·237589 | ·238296 | ·239003 | ·239710 | ·33 |
| ·34 | ·240417 | ·241125 | ·241832 | ·242539 | ·243246 | ·243953 | ·244660 | ·245367 | ·246074 | ·246781 | ·34 |
| ·35 | ·247489 | ·248196 | ·248903 | ·249610 | ·250317 | ·251024 | ·251731 | ·252438 | ·253145 | ·253852 | ·35 |
| ·36 | ·254560 | ·255267 | ·255974 | ·256681 | ·257388 | ·258095 | ·258802 | ·259509 | ·260216 | ·260924 | ·36 |
| ·37 | ·261631 | ·262338 | ·263045 | ·263752 | ·264459 | ·265166 | ·265873 | ·266580 | ·267288 | ·267995 | ·37 |
| ·38 | ·268702 | ·269409 | ·270116 | ·270823 | ·271530 | ·272237 | ·272944 | ·273652 | ·274359 | ·275066 | ·38 |
| ·39 | ·275773 | ·276480 | ·277187 | ·277894 | ·278601 | ·279308 | ·280016 | ·280723 | ·281430 | ·282137 | ·39 |
| ·40 | ·282844 | ·283551 | ·284258 | ·284965 | ·285672 | ·286380 | ·287087 | ·287794 | ·288501 | ·289208 | ·40 |
| ·41 | ·289915 | ·290622 | ·291329 | ·292036 | ·292744 | ·293451 | ·294158 | ·294865 | ·295572 | ·296279 | ·41 |
| ·42 | ·296986 | ·297693 | ·298400 | ·299108 | ·299815 | ·300522 | ·301229 | ·301936 | ·302643 | ·303350 | ·42 |
| ·43 | ·304057 | ·304764 | ·305472 | ·306179 | ·306886 | ·307593 | ·308300 | ·309007 | ·309714 | ·310421 | ·43 |
| ·44 | ·311128 | ·311836 | ·312543 | ·313250 | ·313957 | ·314664 | ·315371 | ·316078 | ·316785 | ·317492 | ·44 |
| ·45 | ·318200 | ·318907 | ·319614 | ·320321 | ·321028 | ·321735 | ·322442 | ·323149 | ·323856 | ·324563 | ·45 |
| ·46 | ·325271 | ·325978 | ·326685 | ·327392 | ·328099 | ·328806 | ·329513 | ·330220 | ·330927 | ·331635 | ·46 |
| ·47 | ·332342 | ·333049 | ·333756 | ·334463 | ·335170 | ·335877 | ·336584 | ·337291 | ·337999 | ·338706 | ·47 |
| ·48 | ·339413 | ·340120 | ·340827 | ·341534 | ·342241 | ·342948 | ·343655 | ·344363 | ·345070 | ·345777 | ·48 |
| ·49 | ·346484 | ·347191 | ·347898 | ·348605 | ·349312 | ·350019 | ·350727 | ·351434 | ·352141 | ·352848 | ·49 |
| ·50 | ·353555 | ·354262 | ·354969 | ·355676 | ·356383 | ·357091 | ·357798 | ·358505 | ·359212 | ·359919 | ·50 |



## COMPUTATIONS.

[PART VI.]

TABLE X.—Products of  $S_4 \times \cdot 001$  up to  $S_4 \times 1\cdot 000$ .

$$S_4 = \sin 60^\circ = \cdot 86603.$$

| No. | ·000   | ·001   | ·002   | ·003   | ·004   | ·005   | ·006   | ·007   | ·008   | ·009   | No. |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|-----|
| ·00 | 000000 | 000866 | 001732 | 002598 | 003464 | 004330 | 005196 | 006062 | 006928 | 007794 | ·00 |
| ·01 | 008660 | 009526 | 010392 | 011258 | 012124 | 012990 | 013856 | 014723 | 015589 | 016455 | ·01 |
| ·02 | 017321 | 018187 | 019053 | 019919 | 020785 | 021651 | 022517 | 023383 | 024249 | 025115 | ·02 |
| ·03 | 025981 | 026847 | 027713 | 028579 | 029445 | 030311 | 031177 | 032043 | 032909 | 033775 | ·03 |
| ·04 | 034641 | 035507 | 036373 | 037239 | 038105 | 038971 | 039837 | 040703 | 041569 | 042435 | ·04 |
| ·05 | 043302 | 044168 | 045034 | 045900 | 046766 | 047632 | 048498 | 049364 | 050230 | 051096 | ·05 |
| ·06 | 051962 | 052828 | 053694 | 054560 | 055426 | 056292 | 057158 | 058024 | 058890 | 059756 | ·06 |
| ·07 | 060622 | 061488 | 062354 | 063220 | 064086 | 064952 | 065818 | 066684 | 067550 | 068416 | ·07 |
| ·08 | 069282 | 070148 | 071014 | 071880 | 072747 | 073613 | 074479 | 075345 | 076211 | 077077 | ·08 |
| ·09 | 077943 | 078809 | 079675 | 080541 | 081407 | 082273 | 083139 | 084005 | 084871 | 085737 | ·09 |
| ·10 | 086603 | 087469 | 088335 | 089201 | 090067 | 090933 | 091799 | 092665 | 093531 | 094397 | ·10 |
| ·11 | 095263 | 096129 | 096995 | 097861 | 098727 | 099593 | 100459 | 101326 | 102192 | 103058 | ·11 |
| ·12 | 103924 | 104790 | 105656 | 106522 | 107388 | 108254 | 109120 | 109986 | 110852 | 111718 | ·12 |
| ·13 | 112584 | 113450 | 114316 | 115182 | 116048 | 116914 | 117780 | 118646 | 119512 | 120378 | ·13 |
| ·14 | 121244 | 122110 | 122976 | 123842 | 124708 | 125574 | 126440 | 127306 | 128172 | 129038 | ·14 |
| ·15 | 129905 | 130771 | 131637 | 132503 | 133369 | 134235 | 135101 | 135967 | 136833 | 137699 | ·15 |
| ·16 | 138565 | 139431 | 140297 | 141163 | 142029 | 142895 | 143761 | 144627 | 145493 | 146359 | ·16 |
| ·17 | 147225 | 148091 | 148957 | 149823 | 150689 | 151555 | 152421 | 153287 | 154153 | 155019 | ·17 |
| ·18 | 155885 | 156751 | 157617 | 158483 | 159350 | 160216 | 161082 | 161948 | 162814 | 163680 | ·18 |
| ·19 | 164546 | 165412 | 166278 | 167144 | 168010 | 168876 | 169742 | 170608 | 171474 | 172340 | ·19 |
| ·20 | 173206 | 174072 | 174938 | 175804 | 176670 | 177536 | 178402 | 179268 | 180134 | 181000 | ·20 |
| ·21 | 181866 | 182732 | 183598 | 184464 | 185330 | 186196 | 187062 | 187929 | 188795 | 189661 | ·21 |
| ·22 | 190527 | 191393 | 192259 | 193125 | 193991 | 194857 | 195723 | 196589 | 197455 | 198321 | ·22 |
| ·23 | 199187 | 200053 | 200919 | 201785 | 202651 | 203517 | 204383 | 205249 | 206115 | 206981 | ·23 |
| ·24 | 207847 | 208713 | 209579 | 210445 | 211311 | 212177 | 213043 | 213909 | 214775 | 215641 | ·24 |
| ·25 | 216508 | 217374 | 218240 | 219106 | 219972 | 220838 | 221704 | 222570 | 223436 | 224302 | ·25 |
| ·26 | 225168 | 226034 | 226900 | 227766 | 228632 | 229498 | 230364 | 231230 | 232096 | 232962 | ·26 |
| ·27 | 233828 | 234694 | 235560 | 236426 | 237292 | 238158 | 239024 | 239890 | 240756 | 241622 | ·27 |
| ·28 | 242488 | 243354 | 244220 | 245086 | 245953 | 246819 | 247685 | 248551 | 249417 | 250283 | ·28 |
| ·29 | 251149 | 252015 | 252881 | 253747 | 254613 | 255479 | 256345 | 257211 | 258077 | 258943 | ·29 |
| ·30 | 259809 | 260675 | 261541 | 262407 | 263273 | 264139 | 265005 | 265871 | 266737 | 267603 | ·30 |
| ·31 | 268469 | 269335 | 270201 | 271067 | 271933 | 272799 | 273665 | 274532 | 275398 | 276264 | ·31 |
| ·32 | 277130 | 277996 | 278862 | 279728 | 280594 | 281460 | 282326 | 283192 | 284058 | 284924 | ·32 |
| ·33 | 285790 | 286656 | 287522 | 288388 | 289254 | 290120 | 290986 | 291852 | 292718 | 293584 | ·33 |
| ·34 | 294450 | 295316 | 296182 | 297048 | 297914 | 298780 | 299646 | 300512 | 301378 | 302244 | ·34 |
| ·35 | 303111 | 303977 | 304843 | 305709 | 306575 | 307441 | 308307 | 309173 | 310039 | 310905 | ·35 |
| ·36 | 311771 | 312637 | 313503 | 314369 | 315235 | 316101 | 316967 | 317833 | 318699 | 319565 | ·36 |
| ·37 | 320431 | 321297 | 322163 | 323029 | 323895 | 324761 | 325627 | 326493 | 327359 | 328225 | ·37 |
| ·38 | 329091 | 329957 | 330823 | 331689 | 332555 | 333422 | 334288 | 335154 | 336020 | 336886 | ·38 |
| ·39 | 337752 | 338618 | 339484 | 340350 | 341216 | 342082 | 342948 | 343814 | 344680 | 345546 | ·39 |
| ·40 | 346412 | 347278 | 348144 | 349010 | 349876 | 350742 | 351608 | 352474 | 353340 | 354206 | ·40 |
| ·41 | 355072 | 355938 | 356804 | 357670 | 358536 | 359402 | 360268 | 361135 | 362001 | 362867 | ·41 |
| ·42 | 363733 | 364599 | 365465 | 366331 | 367197 | 368063 | 368929 | 369795 | 370661 | 371527 | ·42 |
| ·43 | 372393 | 373259 | 374125 | 374991 | 375857 | 376723 | 377589 | 378455 | 379321 | 380187 | ·43 |
| ·44 | 381053 | 381919 | 382785 | 383651 | 384517 | 385383 | 386249 | 387115 | 387981 | 388847 | ·44 |
| ·45 | 389714 | 390580 | 391446 | 392312 | 393178 | 394044 | 394910 | 395776 | 396642 | 397508 | ·45 |
| ·46 | 398374 | 399240 | 400106 | 400972 | 401838 | 402704 | 403570 | 404436 | 405302 | 406168 | ·46 |
| ·47 | 407034 | 407900 | 408766 | 409632 | 410498 | 411364 | 412230 | 413096 | 413962 | 414828 | ·47 |
| ·48 | 415694 | 416560 | 417426 | 418292 | 419159 | 420025 | 420891 | 421757 | 422623 | 423489 | ·48 |
| ·49 | 424355 | 425221 | 426087 | 426953 | 427819 | 428685 | 429551 | 430417 | 431283 | 432149 | ·49 |
| ·50 | 433015 | 433881 | 434747 | 435613 | 436479 | 437345 | 438211 | 439077 | 439943 | 440809 | ·50 |





COMPUTATIONS.

[PART VI.]

TABLE XI.—Products of  $S_5 \times \cdot 001$  up to  $S_5 \times 1\cdot000$ .

$$S_5 = \sin 75^\circ = \cdot 96593.$$

| No. | ·000    | ·001    | ·002    | ·003    | ·004    | ·005    | ·006    | ·007    | ·008    | ·009    | No. |
|-----|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|-----|
| ·00 | ·000000 | ·000966 | ·001932 | ·002898 | ·003864 | ·004830 | ·005796 | ·006762 | ·007727 | ·008639 | ·00 |
| ·01 | ·009659 | ·010625 | ·011591 | ·012557 | ·013523 | ·014489 | ·015455 | ·016421 | ·017387 | ·018353 | ·01 |
| ·02 | ·019319 | ·020285 | ·021250 | ·022216 | ·023182 | ·024148 | ·025114 | ·026080 | ·027046 | ·028012 | ·02 |
| ·03 | ·028978 | ·029944 | ·030910 | ·031876 | ·032842 | ·033808 | ·034773 | ·035739 | ·036705 | ·037671 | ·03 |
| ·04 | ·038637 | ·039603 | ·040569 | ·041535 | ·042501 | ·043467 | ·044433 | ·045399 | ·046365 | ·047331 | ·04 |
| ·05 | ·048297 | ·049262 | ·050228 | ·051194 | ·052160 | ·053126 | ·054092 | ·055058 | ·056024 | ·056990 | ·05 |
| ·06 | ·057956 | ·058922 | ·059888 | ·060854 | ·061820 | ·062785 | ·063751 | ·064717 | ·065683 | ·066649 | ·06 |
| ·07 | ·067615 | ·068581 | ·069547 | ·070513 | ·071479 | ·072445 | ·073411 | ·074377 | ·075343 | ·076308 | ·07 |
| ·08 | ·077274 | ·078240 | ·079206 | ·080172 | ·081138 | ·082104 | ·083070 | ·084036 | ·085002 | ·085968 | ·08 |
| ·09 | ·086934 | ·087900 | ·088866 | ·089831 | ·090797 | ·091763 | ·092729 | ·093695 | ·094661 | ·095627 | ·09 |
| ·10 | ·096593 | ·097559 | ·098525 | ·099491 | ·100457 | ·101423 | ·102389 | ·103355 | ·104320 | ·105286 | ·10 |
| ·11 | ·106252 | ·107218 | ·108184 | ·109150 | ·110116 | ·111082 | ·112048 | ·113014 | ·113980 | ·114946 | ·11 |
| ·12 | ·115912 | ·116878 | ·117843 | ·118809 | ·119775 | ·120741 | ·121707 | ·122673 | ·123639 | ·124605 | ·12 |
| ·13 | ·125571 | ·126537 | ·127503 | ·128469 | ·129435 | ·130401 | ·131366 | ·132332 | ·133298 | ·134264 | ·13 |
| ·14 | ·135230 | ·136196 | ·137162 | ·138128 | ·139094 | ·140060 | ·141026 | ·141992 | ·142958 | ·143924 | ·14 |
| ·15 | ·144890 | ·145855 | ·146821 | ·147787 | ·148753 | ·149719 | ·150685 | ·151651 | ·152617 | ·153583 | ·15 |
| ·16 | ·154549 | ·155515 | ·156481 | ·157447 | ·158413 | ·159378 | ·160344 | ·161310 | ·162276 | ·163242 | ·16 |
| ·17 | ·164208 | ·165174 | ·166140 | ·167106 | ·168072 | ·169038 | ·170004 | ·170970 | ·171936 | ·172901 | ·17 |
| ·18 | ·173867 | ·174833 | ·175799 | ·176765 | ·177731 | ·178697 | ·179663 | ·180629 | ·181595 | ·182561 | ·18 |
| ·19 | ·183527 | ·184493 | ·185459 | ·186424 | ·187390 | ·188356 | ·189322 | ·190288 | ·191254 | ·192220 | ·19 |
| ·20 | ·193186 | ·194152 | ·195118 | ·196084 | ·197050 | ·198016 | ·198982 | ·199948 | ·200913 | ·201879 | ·20 |
| ·21 | ·202845 | ·203811 | ·204777 | ·205743 | ·206709 | ·207675 | ·208641 | ·209607 | ·210573 | ·211539 | ·21 |
| ·22 | ·212505 | ·213471 | ·214436 | ·215402 | ·216368 | ·217334 | ·218300 | ·219266 | ·220232 | ·221198 | ·22 |
| ·23 | ·222164 | ·223130 | ·224096 | ·225062 | ·226028 | ·226994 | ·227959 | ·228925 | ·229891 | ·230857 | ·23 |
| ·24 | ·231823 | ·232789 | ·233755 | ·234721 | ·235687 | ·236653 | ·237619 | ·238585 | ·239551 | ·240517 | ·24 |
| ·25 | ·241483 | ·242448 | ·243414 | ·244380 | ·245346 | ·246312 | ·247278 | ·248244 | ·249210 | ·250176 | ·25 |
| ·26 | ·251142 | ·252108 | ·253074 | ·254040 | ·255006 | ·255971 | ·256937 | ·257903 | ·258869 | ·259835 | ·26 |
| ·27 | ·260801 | ·261767 | ·262733 | ·263699 | ·264665 | ·265631 | ·266597 | ·267563 | ·268529 | ·269494 | ·27 |
| ·28 | ·270460 | ·271426 | ·272392 | ·273358 | ·274324 | ·275290 | ·276256 | ·277222 | ·278188 | ·279154 | ·28 |
| ·29 | ·280120 | ·281086 | ·282052 | ·283017 | ·283983 | ·284949 | ·285915 | ·286881 | ·287847 | ·288813 | ·29 |
| ·30 | ·289779 | ·290745 | ·291711 | ·292677 | ·293643 | ·294609 | ·295575 | ·296541 | ·297506 | ·298472 | ·30 |
| ·31 | ·299438 | ·300404 | ·301370 | ·302336 | ·303302 | ·304268 | ·305234 | ·306200 | ·307166 | ·308132 | ·31 |
| ·32 | ·309098 | ·310064 | ·311029 | ·311995 | ·312961 | ·313927 | ·314893 | ·315859 | ·316825 | ·317791 | ·32 |
| ·33 | ·318757 | ·319723 | ·320689 | ·321655 | ·322621 | ·323587 | ·324552 | ·325518 | ·326484 | ·327450 | ·33 |
| ·34 | ·328416 | ·329382 | ·330348 | ·331314 | ·332280 | ·333246 | ·334212 | ·335178 | ·336144 | ·337110 | ·34 |
| ·35 | ·338076 | ·339041 | ·340007 | ·340973 | ·341939 | ·342905 | ·343871 | ·344837 | ·345803 | ·346769 | ·35 |
| ·36 | ·347735 | ·348701 | ·349667 | ·350633 | ·351599 | ·352564 | ·353530 | ·354496 | ·355462 | ·356428 | ·36 |
| ·37 | ·357394 | ·358360 | ·359326 | ·360292 | ·361258 | ·362224 | ·363190 | ·364156 | ·365122 | ·366087 | ·37 |
| ·38 | ·367053 | ·368019 | ·368985 | ·369951 | ·370917 | ·371883 | ·372849 | ·373815 | ·374781 | ·375747 | ·38 |
| ·39 | ·376713 | ·377679 | ·378645 | ·379610 | ·380576 | ·381542 | ·382508 | ·383474 | ·384440 | ·385406 | ·39 |
| ·40 | ·386372 | ·387338 | ·388304 | ·389270 | ·390236 | ·391202 | ·392168 | ·393134 | ·394099 | ·395065 | ·40 |
| ·41 | ·396031 | ·396997 | ·397963 | ·398929 | ·399895 | ·400861 | ·401827 | ·402793 | ·403759 | ·404725 | ·41 |
| ·42 | ·405691 | ·406657 | ·407622 | ·408588 | ·409554 | ·410520 | ·411486 | ·412452 | ·413418 | ·414384 | ·42 |
| ·43 | ·415350 | ·416316 | ·417282 | ·418248 | ·419214 | ·420180 | ·421145 | ·422111 | ·423077 | ·424043 | ·43 |
| ·44 | ·425009 | ·425975 | ·426941 | ·427907 | ·428873 | ·429839 | ·430805 | ·431771 | ·432737 | ·433703 | ·44 |
| ·45 | ·434669 | ·435634 | ·436600 | ·437566 | ·438532 | ·439498 | ·440464 | ·441430 | ·442396 | ·443362 | ·45 |
| ·46 | ·444328 | ·445294 | ·446260 | ·447226 | ·448192 | ·449157 | ·450123 | ·451089 | ·452055 | ·453021 | ·46 |
| ·47 | ·453987 | ·454953 | ·455919 | ·456885 | ·457851 | ·458817 | ·459783 | ·460749 | ·461715 | ·462680 | ·47 |
| ·48 | ·463646 | ·464612 | ·465578 | ·466544 | ·467510 | ·468476 | ·469442 | ·470408 | ·471374 | ·472340 | ·48 |
| ·49 | ·473306 | ·474272 | ·475238 | ·476203 | ·477169 | ·478135 | ·479101 | ·480067 | ·481033 | ·481999 | ·49 |
| ·50 | ·482065 | ·483031 | ·484097 | ·485063 | ·486029 | ·487095 | ·488061 | ·489027 | ·490092 | ·491658 | ·50 |



COMPUTATIONS.

[PART VI.

TABLE XII.—Natural Numbers to three places of Decimals corresponding to Logarithms with Indices 6, 7 and 8.

Logarithms with Index 6 or  $\bar{4}$ .

| Natural No. | Natural No. to 3 places of Decimals | Logarithms |
|-------------|-------------------------------------|------------|
| •00000      | •000                                | •0000000   |
| •00050      | •001                                | •6989700   |
| •00099      |                                     | •9999999   |

Logarithms with Index 7 or  $\bar{3}$ .

| Nat. No. | Nat. No. to 3 places of Decimals | Logarithms | Nat. No. | Nat. No. to 3 places of Decimals | Logarithms | Nat. No. | Nat. No. to 3 places of Decimals | Logarithms |
|----------|----------------------------------|------------|----------|----------------------------------|------------|----------|----------------------------------|------------|
| •0010    |                                  | •0000000   | •0045    |                                  | •6532125   | •0085    |                                  | •9294189   |
| •0015    | •001                             | •1760913   | •0055    | •005                             | •7403627   | •0095    | •009                             | •9777236   |
| •0025    | •002                             | •3979400   | •0065    | •006                             | •8129134   | •0099    | •010                             | •9999999   |
| •0035    | •003                             | •5440680   | •0075    | •007                             | •8750613   |          |                                  |            |
| •0045    | •004                             | •6532125   | •0085    | •008                             | •9294189   |          |                                  |            |

Logarithms with Index 8 or  $\bar{2}$ .

| Nat. No. | Nat. No. to 3 places of Decimals | Logarithms | Nat. No. | Nat. No. to 3 places of Decimals | Logarithms | Nat. No. | Nat. No. to 3 places of Decimals | Logarithms | Nat. No. | Nat. No. to 3 places of Decimals | Logarithms |
|----------|----------------------------------|------------|----------|----------------------------------|------------|----------|----------------------------------|------------|----------|----------------------------------|------------|
| •0100    | •010                             | •0000000   | •0325    | •033                             | •5118834   | •0555    | •056                             | •7442930   | •0785    | •079                             | •8948697   |
| •0105    | •011                             | •0211893   | •0335    | •034                             | •5250448   | •0565    | •057                             | •7520484   | •0795    | •080                             | •9003671   |
| •0115    | •012                             | •0606978   | •0345    | •035                             | •5378191   | •0575    | •058                             | •7596678   | •0805    | •081                             | •9057959   |
| •0125    | •013                             | •0969100   | •0355    | •036                             | •5502284   | •0585    | •059                             | •7671559   | •0815    | •082                             | •9111576   |
| •0135    | •014                             | •1303338   | •0365    | •037                             | •5622929   | •0595    | •060                             | •7745170   | •0825    | •083                             | •9164539   |
| •0145    | •015                             | •1613680   | •0375    | •038                             | •5740313   | •0605    | •061                             | •7817554   | •0835    | •084                             | •9216865   |
| •0155    | •016                             | •1903317   | •0385    | •039                             | •5854607   | •0615    | •062                             | •7888751   | •0845    | •085                             | •9268567   |
| •0165    | •017                             | •2174839   | •0395    | •040                             | •5965971   | •0625    | •063                             | •7958800   | •0855    | •086                             | •9319661   |
| •0175    | •018                             | •2430380   | •0405    | •041                             | •6074550   | •0635    | •064                             | •8027737   | •0865    | •087                             | •9370161   |
| •0185    | •019                             | •2671717   | •0415    | •042                             | •6180481   | •0645    | •065                             | •8095597   | •0875    | •088                             | •9420081   |
| •0195    | •020                             | •2900346   | •0425    | •043                             | •6283889   | •0655    | •066                             | •8162413   | •0885    | •089                             | •9469433   |
| •0205    | •021                             | •3117539   | •0435    | •044                             | •6384893   | •0665    | •067                             | •8228216   | •0895    | •090                             | •9518230   |
| •0215    | •022                             | •3324385   | •0445    | •045                             | •6483600   | •0675    | •068                             | •8293038   | •0905    | •091                             | •9566486   |
| •0225    | •023                             | •3521825   | •0455    | •046                             | •6580114   | •0685    | •069                             | •8356906   | •0915    | •092                             | •9614211   |
| •0235    | •024                             | •3710679   | •0465    | •047                             | •6674530   | •0695    | •070                             | •8419848   | •0925    | •093                             | •9661417   |
| •0245    | •025                             | •3891661   | •0475    | •048                             | •6766936   | •0705    | •071                             | •8481891   | •0935    | •094                             | •9708116   |
| •0255    | •026                             | •4065402   | •0485    | •049                             | •6857417   | •0715    | •072                             | •8543060   | •0945    | •095                             | •9754318   |
| •0265    | •027                             | •4232459   | •0495    | •050                             | •6946052   | •0725    | •073                             | •8603380   | •0955    | •096                             | •9800034   |
| •0275    | •028                             | •4393327   | •0505    | •051                             | •7032914   | •0735    | •074                             | •8662873   | •0965    | •097                             | •9845273   |
| •0285    | •029                             | •4548449   | •0515    | •052                             | •7118072   | •0745    | •075                             | •8721563   | •0975    | •098                             | •9890046   |
| •0295    | •030                             | •4698220   | •0525    | •053                             | •7201593   | •0755    | •076                             | •8779470   | •0985    | •099                             | •9934362   |
| •0305    | •031                             | •4842998   | •0535    | •054                             | •7283538   | •0765    | •077                             | •8836614   | •0995    | •100                             | •9978231   |
| •0315    | •032                             | •4983106   | •0545    | •055                             | •7363965   | •0775    | •078                             | •8893017   | •0999    |                                  | •9999999   |
| •0325    |                                  | •5118834   | •0555    |                                  | •7442930   | •0785    |                                  | •8948697   |          |                                  |            |

TABLE XIII.—Values of *N* (Longitude of Moon's Ascending Node) for 0 hour Greenwich Mean Time, January 1.

Value on 0 hour G. M. T., January 1st, 1880 = 285°·956863.  
 Motion per Julian year in 1880 = 19°·34146248.  
 Motion for 365 days = 19°·32822387 and for 1 day = 0°·052954.

| Year | <i>N</i>    | Year | <i>N</i>    | Year | <i>N</i>    | Year | <i>N</i>    |
|------|-------------|------|-------------|------|-------------|------|-------------|
| 1850 | 146°·174466 | 1875 | 22°·650943  | 1900 | 259°·127704 | 1925 | 135°·604751 |
| 1    | 126°·846228 | 6    | 3°·322717   | 1    | 239°·799490 | 6    | 116°·276548 |
| 2    | 107°·517991 | 7    | 343°·941537 | 2    | 220°·471276 | 7    | 96°·948346  |
| 3    | 88°·136801  | 8    | 324°·613312 | 3    | 201°·143062 | 8    | 77°·620143  |
| 4    | 68°·808565  | 9    | 305°·285087 | 4    | 181°·814849 | 9    | 58°·238987  |
| 1855 | 49°·480329  | 1880 | 285°·956863 | 1905 | 162°·433682 | 1930 | 38°·910786  |
| 6    | 30°·152094  | 1    | 266°·575685 | 6    | 143°·105470 | 1    | 19°·582585  |
| 7    | 10°·770905  | 2    | 247°·247462 | 7    | 123°·777258 | 2    | 0°·254384   |
| 8    | 351°·442671 | 3    | 227°·919239 | 8    | 104°·449046 | 3    | 340°·873230 |
| 9    | 332°·114437 | 4    | 208°·591017 | 9    | 85°·067881  | 4    | 321°·545031 |
| 1860 | 312°·786204 | 1885 | 189°·209841 | 1910 | 65°·739671  | 1935 | 302°·216832 |
| 1    | 293°·405017 | 6    | 169°·881619 | 1    | 46°·411461  | 6    | 282°·888633 |
| 2    | 274°·076784 | 7    | 150°·553398 | 2    | 27°·083251  | 7    | 263°·507481 |
| 3    | 254°·748553 | 8    | 131°·225178 | 3    | 7°·702088   | 8    | 244°·179283 |
| 4    | 235°·420321 | 9    | 111°·844004 | 4    | 348°·373879 | 9    | 224°·851086 |
| 1865 | 216°·039136 | 1890 | 92°·515784  | 1915 | 329°·045671 | 1940 | 205°·522889 |
| 6    | 196°·710905 | 1    | 73°·187565  | 6    | 309°·717463 | 1    | 186°·141739 |
| 7    | 177°·382675 | 2    | 53°·859346  | 7    | 290°·336302 | 2    | 166°·813543 |
| 8    | 158°·054446 | 3    | 34°·478174  | 8    | 271°·008095 | 3    | 147°·485347 |
| 9    | 138°·673262 | 4    | 15°·149956  | 9    | 251°·679888 | 4    | 128°·157152 |
| 1870 | 119°·345034 | 1895 | 355°·821739 | 1920 | 232°·351682 | 1945 | 108°·776004 |
| 1    | 100°·016805 | 6    | 336°·493522 | 1    | 212°·970523 | 6    | 89°·447810  |
| 2    | 80°·688577  | 7    | 317°·112351 | 2    | 193°·642318 | 7    | 70°·119616  |
| 3    | 61°·307396  | 8    | 297°·784135 | 3    | 174°·314113 | 8    | 50°·791423  |
| 4    | 41°·979169  | 9    | 278°·455920 | 4    | 154°·985909 | 9    | 31°·410276  |

TABLE XIV.—Showing the Decrement of *N* (Longitude of Moon's Ascending Node) since 0 hour January 1 up to Midnight of each Day throughout the Year.

Daily Motion = 0°·0529541.

In Leap Years for all dates after February 28—March 1, use a mean value between the particular day and the day following.

| Date  | Decre.  | Date  | Decre.  | Date  | Decre.  | Date  | Decre.  | Date  | Decre.  | Date  | Decre.  |
|-------|---------|-------|---------|-------|---------|-------|---------|-------|---------|-------|---------|
| JAN.  |         | JAN.  |         | JAN.  |         | JAN.  |         | FEB.  |         | FEB.  |         |
| 1-2   | 0°·0265 | 11-12 | 0°·5560 | 21-22 | 1°·0856 | 31-32 | 1°·6151 | 9-10  | 2°·0917 | 19-20 | 2°·6212 |
| 2-3   | 0°·0794 | 12-13 | 0°·6090 | 22-23 | 1°·1385 | FEB.  |         | 10-11 | 2°·1446 | 20-21 | 2°·6742 |
| 3-4   | 0°·1324 | 13-14 | 0°·6619 | 23-24 | 1°·1915 | 1-2   | 1°·6681 | 11-12 | 2°·1976 | 21-22 | 2°·7271 |
| 4-5   | 0°·1853 | 14-15 | 0°·7149 | 24-25 | 1°·2444 | 2-3   | 1°·7210 | 12-13 | 2°·2505 | 22-23 | 2°·7801 |
| 5-6   | 0°·2383 | 15-16 | 0°·7678 | 25-26 | 1°·2974 | 3-4   | 1°·7740 | 13-14 | 2°·3035 | 23-24 | 2°·8330 |
| 6-7   | 0°·2912 | 16-17 | 0°·8208 | 26-27 | 1°·3503 | 4-5   | 1°·8269 | 14-15 | 2°·3565 | 24-25 | 2°·8860 |
| 7-8   | 0°·3442 | 17-18 | 0°·8737 | 27-28 | 1°·4033 | 5-6   | 1°·8799 | 15-16 | 2°·4094 | 25-26 | 2°·9390 |
| 8-9   | 0°·3972 | 18-19 | 0°·9267 | 28-29 | 1°·4562 | 6-7   | 1°·9328 | 16-17 | 2°·4624 | 26-27 | 2°·9919 |
| 9-10  | 0°·4501 | 19-20 | 0°·9797 | 29-30 | 1°·5092 | 7-8   | 1°·9858 | 17-18 | 2°·5153 | 27-28 | 3°·0449 |
| 10-11 | 0°·5031 | 20-21 | 1°·0326 | 30-31 | 1°·5621 | 8-9   | 2°·0387 | 18-19 | 2°·5683 | 28-29 | 3°·0978 |

N.B.—In Table XIV. The middle of the year of observations will occur at noon or midnight according as the 29th February is included in the period of observations or not. If the *midnight* falls on a date in a common year, or before the 29th February in a leap year, then the Decrement for *N* as given in the Table is correct: if, however, the *midnight* falls after the 29th February in a leap year, then take the value as that given for the  *succeeding date* in the Table.

If the *noon* falls in a common year or before the 29th February in a leap year, the value to be taken from

COMPUTATIONS.

[PART VI.]

TABLE XIV.—Showing the Decrement of *N* (Longitude of Moon's Ascending Node) since 0 hour January 1 up to Midnight of each Day throughout the Year—(Continued).

Daily Motion = 0° 05' 29.54".

In Leap Years for all dates after February 28—March 1, use a mean value between the particular day and the day following.

| Date           | Decre.    | Date  | Decre.   | Date  | Decre.    | Date  | Decre.    | Date  | Decre.     | Date  | Decre.    |
|----------------|-----------|-------|----------|-------|-----------|-------|-----------|-------|------------|-------|-----------|
| MAR.           |           |       |          |       |           |       |           |       |            |       |           |
| 1-23           | 3° 15' 08 | 22-23 | 5° 9' 04 | 12-13 | 8° 6' 05  | 2-3   | 11° 3' 05 | 23-24 | 14° 0' 59  | 13-14 | 16° 7' 60 |
| 2-3            | 3° 20' 37 | 23-24 | 5° 9' 57 | 13-14 | 8° 6' 58  | 3-4   | 11° 3' 58 | 24-25 | 14° 1' 12  | 14-15 | 16° 8' 12 |
| 3-4            | 3° 25' 67 | 24-25 | 6° 0' 10 | 14-15 | 8° 7' 10  | 4-5   | 11° 4' 16 | 25-26 | 14° 16' 52 | 15-16 | 16° 8' 65 |
| 4-5            | 3° 30' 96 | 25-26 | 6° 0' 63 | 15-16 | 8° 7' 63  | 5-6   | 11° 4' 64 | 26-27 | 14° 21' 82 | 16-17 | 16° 9' 18 |
| 5-6            | 3° 36' 26 | 26-27 | 6° 1' 16 | 16-17 | 8° 8' 16  | 6-7   | 11° 5' 17 | 27-28 | 14° 27' 11 | 17-18 | 16° 9' 71 |
| 6-7            | 3° 41' 55 | 27-28 | 6° 1' 69 | 17-18 | 8° 8' 69  | 7-8   | 11° 5' 70 | 28-29 | 14° 32' 41 | 18-19 | 17° 0' 24 |
| 7-8            | 3° 46' 85 | 28-29 | 6° 2' 21 | 18-19 | 8° 9' 22  | 8-9   | 11° 6' 23 | 29-30 | 14° 37' 70 | 19-20 | 17° 0' 77 |
| 8-9            | 3° 52' 14 | 29-30 | 6° 2' 75 | 19-20 | 8° 9' 75  | 9-10  | 11° 6' 76 | 30-31 | 14° 43' 00 | 20-21 | 17° 1' 30 |
| 9-10           | 3° 57' 44 | 30-31 | 6° 3' 28 | 20-21 | 9° 0' 28  | 10-11 | 11° 7' 29 | OCT.  |            | 21-22 | 17° 1' 83 |
| 10-11          | 3° 62' 74 | MAY   |          | 21-22 | 9° 0' 81  | 11-12 | 11° 7' 82 | 1-2   | 14° 48' 29 | 22-23 | 17° 2' 36 |
| 11-12          | 3° 68' 03 | 1-2   | 6° 3' 81 | 22-23 | 9° 1' 34  | 12-13 | 11° 8' 35 | 2-3   | 14° 53' 59 | 23-24 | 17° 2' 89 |
| 12-13          | 3° 73' 33 | 2-3   | 6° 4' 33 | 23-24 | 9° 1' 87  | 13-14 | 11° 8' 88 | 3-4   | 14° 58' 89 | 24-25 | 17° 3' 42 |
| 13-14          | 3° 78' 62 | 3-4   | 6° 4' 86 | 24-25 | 9° 2' 40  | 14-15 | 11° 9' 41 | 4-5   | 14° 64' 18 | 25-26 | 17° 3' 95 |
| 14-15          | 3° 83' 92 | 4-5   | 6° 5' 39 | 25-26 | 9° 2' 93  | 15-16 | 11° 9' 94 | 5-6   | 14° 69' 48 | 26-27 | 17° 4' 48 |
| 15-16          | 3° 89' 21 | 5-6   | 6° 6' 52 | 26-27 | 9° 3' 46  | 16-17 | 12° 0' 47 | 6-7   | 14° 74' 77 | 27-28 | 17° 5' 01 |
| 16-17          | 3° 94' 51 | 6-7   | 6° 6' 64 | 27-28 | 9° 3' 99  | 17-18 | 12° 1' 00 | 7-8   | 14° 80' 07 | 28-29 | 17° 5' 54 |
| 17-18          | 3° 99' 80 | 7-8   | 6° 6' 98 | 28-29 | 9° 4' 52  | 18-19 | 12° 1' 53 | 8-9   | 14° 85' 36 | 29-30 | 17° 6' 07 |
| 18-19          | 4° 05' 10 | 8-9   | 6° 7' 51 | 29-30 | 9° 5' 05  | 19-20 | 12° 2' 05 | 9-10  | 14° 90' 66 | 30-31 | 17° 6' 60 |
| 19-20          | 4° 10' 39 | 9-10  | 6° 8' 04 | 30-31 | 9° 5' 58  | 20-21 | 12° 2' 58 | 10-11 | 14° 95' 95 | DEC.  |           |
| 20-21          | 4° 15' 69 | 10-11 | 6° 8' 57 | JULY  |           | 21-22 | 12° 3' 18 | 11-12 | 15° 0' 12  | 1-2   | 17° 7' 13 |
| 21-22          | 4° 20' 99 | 11-12 | 6° 9' 10 | 1-2   | 9° 6' 11  | 22-23 | 12° 3' 64 | 12-13 | 15° 0' 65  | 2-3   | 17° 7' 66 |
| 22-23          | 4° 26' 28 | 12-13 | 6° 9' 63 | 2-3   | 9° 6' 64  | 23-24 | 12° 4' 17 | 13-14 | 15° 1' 18  | 3-4   | 17° 8' 19 |
| 23-24          | 4° 31' 58 | 13-14 | 7° 0' 16 | 3-4   | 9° 7' 17  | 24-25 | 12° 4' 70 | 14-15 | 15° 1' 71  | 4-5   | 17° 8' 72 |
| 24-25          | 4° 36' 87 | 14-15 | 7° 0' 69 | 4-5   | 9° 7' 70  | 25-26 | 12° 5' 23 | 15-16 | 15° 2' 24  | 5-6   | 17° 9' 25 |
| 25-26          | 4° 42' 17 | 15-16 | 7° 1' 22 | 5-6   | 9° 8' 23  | 26-27 | 12° 5' 76 | 16-17 | 15° 2' 77  | 6-7   | 17° 9' 77 |
| 26-27          | 4° 47' 46 | 16-17 | 7° 1' 75 | 6-7   | 9° 8' 75  | 27-28 | 12° 6' 29 | 17-18 | 15° 3' 30  | 7-8   | 18° 0' 30 |
| 27-28          | 4° 52' 76 | 17-18 | 7° 2' 28 | 7-8   | 9° 9' 28  | 28-29 | 12° 6' 82 | 18-19 | 15° 3' 83  | 8-9   | 18° 0' 83 |
| 28-29          | 4° 58' 05 | 18-19 | 7° 2' 81 | 8-9   | 9° 9' 81  | 29-30 | 12° 7' 35 | 19-20 | 15° 4' 36  | 9-10  | 18° 1' 36 |
| 29-30          | 4° 63' 35 | 19-20 | 7° 3' 34 | 9-10  | 10° 0' 34 | 30-31 | 12° 7' 88 | 20-21 | 15° 4' 89  | 10-11 | 18° 1' 89 |
| 30-31          | 4° 68' 64 | 20-21 | 7° 3' 87 | 10-11 | 10° 0' 87 | 31-32 | 12° 8' 41 | 21-22 | 15° 5' 42  | 11-12 | 18° 2' 42 |
| 31-32          | 4° 73' 94 | 21-22 | 7° 4' 40 | 11-12 | 10° 1' 40 | SEPT. |           | 22-23 | 15° 5' 95  | 12-13 | 18° 2' 95 |
| APR.           |           |       |          |       |           |       |           |       |            |       |           |
| 1-2            | 4° 79' 23 | 22-23 | 7° 4' 93 | 12-13 | 10° 1' 93 | 1-2   | 12° 8' 94 | 23-24 | 15° 6' 49  | 13-14 | 18° 3' 48 |
| 2-3            | 4° 84' 53 | 23-24 | 7° 5' 46 | 13-14 | 10° 2' 46 | 2-3   | 12° 9' 47 | 24-25 | 15° 7' 00  | 14-15 | 18° 4' 01 |
| 3-4            | 4° 89' 83 | 24-25 | 7° 5' 99 | 14-15 | 10° 2' 99 | 3-4   | 13° 0' 00 | 25-26 | 15° 7' 53  | 15-16 | 18° 4' 54 |
| 4-5            | 4° 95' 12 | 25-26 | 7° 6' 51 | 15-16 | 10° 3' 52 | 4-5   | 13° 0' 53 | 26-27 | 15° 8' 06  | 16-17 | 18° 5' 07 |
| 5-6            | 5° 00' 42 | 26-27 | 7° 7' 04 | 16-17 | 10° 4' 05 | 5-6   | 13° 1' 06 | 27-28 | 15° 8' 59  | 17-18 | 18° 5' 60 |
| 6-7            | 5° 05' 71 | 27-28 | 7° 7' 57 | 17-18 | 10° 4' 58 | 6-7   | 13° 1' 59 | 28-29 | 15° 9' 12  | 18-19 | 18° 6' 13 |
| 7-8            | 5° 11' 01 | 28-29 | 7° 8' 10 | 18-19 | 10° 5' 11 | 7-8   | 13° 2' 10 | 29-30 | 15° 9' 65  | 19-20 | 18° 6' 66 |
| 8-9            | 5° 16' 30 | 29-30 | 7° 8' 63 | 19-20 | 10° 5' 64 | 8-9   | 13° 2' 65 | 30-31 | 16° 0' 18  | 20-21 | 18° 7' 19 |
| 9-10           | 5° 21' 60 | 30-31 | 7° 9' 16 | 20-21 | 10° 6' 17 | 9-10  | 13° 3' 18 | 31-32 | 16° 0' 71  | 21-22 | 18° 7' 72 |
| 10-11          | 5° 26' 89 | 31-32 | 7° 9' 69 | 21-22 | 10° 6' 70 | 10-11 | 13° 3' 70 | NOV.  |            | 22-23 | 18° 8' 25 |
| MAY            |           |       |          |       |           |       |           |       |            |       |           |
| 11-12          | 5° 32' 19 | 1-2   | 8° 0' 22 | 22-23 | 10° 7' 23 | 11-12 | 13° 4' 23 | 1-2   | 16° 1' 24  | 23-24 | 18° 8' 78 |
| 12-13          | 5° 37' 48 | 2-3   | 8° 0' 75 | 23-24 | 10° 7' 76 | 12-13 | 13° 4' 76 | 2-3   | 16° 1' 77  | 24-25 | 18° 9' 31 |
| 13-14          | 5° 42' 78 | 3-4   | 8° 1' 28 | 24-25 | 10° 8' 29 | 13-14 | 13° 5' 29 | 3-4   | 16° 2' 30  | 25-26 | 18° 9' 84 |
| 14-15          | 5° 48' 07 | 4-5   | 8° 1' 81 | 25-26 | 10° 8' 82 | 14-15 | 13° 5' 82 | 4-5   | 16° 2' 83  | 26-27 | 19° 0' 37 |
| 15-16          | 5° 53' 37 | 5-6   | 8° 2' 34 | 26-27 | 10° 9' 35 | 15-16 | 13° 6' 35 | 5-6   | 16° 3' 36  | 27-28 | 19° 0' 90 |
| 16-17          | 5° 58' 67 | 6-7   | 8° 2' 87 | 27-28 | 10° 9' 88 | 16-17 | 13° 6' 88 | 6-7   | 16° 3' 89  | 28-29 | 19° 1' 42 |
| 17-18          | 5° 63' 96 | 7-8   | 8° 3' 40 | 28-29 | 11° 0' 40 | 17-18 | 13° 7' 41 | 7-8   | 16° 4' 42  | 29-30 | 19° 1' 95 |
| 18-19          | 5° 69' 26 | 8-9   | 8° 3' 93 | 29-30 | 11° 0' 93 | 18-19 | 13° 7' 94 | 8-9   | 16° 4' 95  | 30-31 | 19° 2' 48 |
| 19-20          | 5° 74' 55 | 9-10  | 8° 4' 46 | 30-31 | 11° 1' 46 | 19-20 | 13° 8' 47 | 9-10  | 16° 5' 48  | 31-32 | 19° 3' 01 |
| 20-21          | 5° 79' 85 | 10-11 | 8° 4' 99 | 31-32 | 11° 1' 99 | 20-21 | 13° 9' 00 | 10-11 | 16° 6' 01  |       |           |
| 21-22          | 5° 85' 14 | 11-12 | 8° 5' 52 | AUG.  |           | 21-22 | 13° 9' 53 | 11-12 | 16° 6' 54  |       |           |
| JUNE           |           |       |          |       |           |       |           |       |            |       |           |
| 1-2 11° 25' 27 |           |       |          |       |           |       |           |       |            |       |           |

The Table is the mean between the preceding and succeeding midnights; but if the moon falls after the 29th February in a leap year, the mean between the values for the two midnights immediately following is to be taken.

TABLE XV.—Values of  $p_1$  (Mean Longitude of Solar Perigee) for 0 hour, January 1.

$p_1$  for 0 hour, January 1, 1880 =  $280^\circ.874802$ .

Motion per Julian year =  $0^\circ.01710693$ .

Motion for 365 days =  $0^\circ.01709295$ .

| Year | $p_1$            | Year | $p_1$            | Year | $p_1$            | Year | $p_1$            |
|------|------------------|------|------------------|------|------------------|------|------------------|
| 1850 | $280^\circ.3614$ | 1875 | $280^\circ.7892$ | 1900 | $281^\circ.2171$ | 1925 | $281^\circ.6450$ |
| 51   | .3785            | 76   | .8063            | 1    | .2342            | 26   | .6621            |
| 52   | .3956            | 77   | .8235            | 2    | .2513            | 27   | .6792            |
| 53   | .4125            | 78   | .8406            | 3    | .2684            | 28   | .6963            |
| 54   | .4299            | 79   | .8577            | 4    | .2855            | 29   | .7135            |
| 55   | .4470            | 1880 | .8748            | 5    | .3027            | 1930 | .7306            |
| 56   | .4641            | 81   | .8920            | 6    | .3198            | 31   | .7477            |
| 57   | .4812            | 82   | .9091            | 7    | .3369            | 32   | .7648            |
| 58   | .4983            | 83   | .9262            | 8    | .3540            | 33   | .7820            |
| 59   | .5154            | 84   | .9433            | 9    | .3711            | 34   | .7991            |
| 1860 | .5325            | 85   | .9604            | 1910 | .3882            | 35   | .8162            |
| 61   | .5497            | 86   | .9775            | 11   | .4054            | 36   | .8333            |
| 62   | .5668            | 87   | .9946            | 12   | .4225            | 37   | .8505            |
| 63   | .5839            | 88   | $281^\circ.0117$ | 13   | .4396            | 38   | .8676            |
| 64   | .6010            | 89   | .0289            | 14   | .4567            | 39   | .8847            |
| 65   | .6181            | 1890 | .0460            | 15   | .4738            | 1940 | .9018            |
| 66   | .6352            | 91   | .0631            | 16   | .4909            | 41   | .9189            |
| 67   | .6523            | 92   | .0802            | 17   | .5081            | 42   | .9360            |
| 68   | .6694            | 93   | .0973            | 18   | .5252            | 43   | .9532            |
| 69   | .6866            | 94   | .1144            | 19   | .5423            | 44   | .9703            |
| 1870 | .7037            | 95   | .1315            | 1920 | .5594            | 45   | .9874            |
| 71   | .7208            | 96   | .1486            | 21   | .5766            | 46   | $282^\circ.0045$ |
| 72   | .7379            | 97   | .1658            | 22   | .5937            | 47   | .0216            |
| 73   | .7550            | 98   | .1829            | 23   | .6108            | 48   | .0387            |
| 74   | .7721            | 99   | .2000            | 24   | .6279            | 49   | .0559            |

TABLE XVI.—Increment of  $p_1$  since 0 hour, January 1, for certain Days of the Year.

Motion for 1 day =  $0^\circ.00004683$ .

| Date    | Increment       | Date    | Increment       | Date    | Increment       | Date   | Increment       |
|---------|-----------------|---------|-----------------|---------|-----------------|--------|-----------------|
| Jan. 10 | $0^\circ.00042$ | Apr. 10 | $0^\circ.00464$ | July 9  | $0^\circ.00885$ | Oct. 7 | $0^\circ.01307$ |
| " 20    | .00089          | " 20    | .00510          | " 19    | .00932          | " 17   | .01353          |
| " 30    | .00136          | " 30    | .00557          | " 29    | .00979          | " 27   | .01400          |
| Feb. 9  | .00183          | May 10  | .00604          | Aug. 8  | .01026          | Nov. 6 | .01447          |
| " 19    | .00229          | " 20    | .00651          | " 18    | .01072          | " 16   | .01494          |
| Mar. 1  | .00276          | " 30    | .00698          | " 28    | .01119          | " 26   | .01541          |
| " 11    | .00323          | June 9  | .00745          | Sept. 7 | .01166          | Dec. 6 | .01588          |
| " 21    | .00370          | " 19    | .00791          | " 17    | .01213          | " 16   | .01634          |
| " 31    | .00417          | " 29    | .00838          | " 27    | .01260          | " 26   | .01681          |

COMPUTATIONS.

[PART VI.]

TABLE XVII.—Values of  $I, \nu$  and  $\xi$ , corresponding to  $N$ .

| $N$  | $I$    | $\nu$ | $\xi$ | $N$   | $N$  | $I$    | $\nu$  | $\xi$ | $N$   | $N$  | $I$    | $\nu$  | $\xi$  | $N$   |
|------|--------|-------|-------|-------|------|--------|--------|-------|-------|------|--------|--------|--------|-------|
| 0°   | 28°602 | 0°000 | 0°000 | 360°  | 30°5 | 28°005 | 5°564  | 5°017 | 329°5 | 60°5 | 26°351 | 10°131 | 9°196  | 299°5 |
| 0°5  | 28°602 | 0°094 | 0°084 | 359°5 | 31°0 | 27°985 | 5°651  | 5°095 | 329°0 | 61°0 | 26°316 | 10°194 | 9°255  | 299°0 |
| 1°0  | 28°601 | 0°188 | 0°169 | 359°0 | 31°5 | 27°965 | 5°736  | 5°173 | 328°5 | 61°5 | 26°280 | 10°256 | 9°313  | 298°5 |
| 1°5  | 28°600 | 0°281 | 0°253 | 358°5 | 32°0 | 27°945 | 5°822  | 5°251 | 328°0 | 62°0 | 26°245 | 10°318 | 9°370  | 298°0 |
| 2°0  | 28°599 | 0°375 | 0°337 | 358°0 | 32°5 | 27°925 | 5°907  | 5°328 | 327°5 | 62°5 | 26°209 | 10°379 | 9°427  | 297°5 |
| 2°5  | 28°598 | 0°468 | 0°421 | 357°5 | 33°0 | 27°904 | 5°992  | 5°406 | 327°0 | 63°0 | 26°173 | 10°440 | 9°484  | 297°0 |
| 3°0  | 28°596 | 0°562 | 0°506 | 357°0 | 33°5 | 27°884 | 6°077  | 5°482 | 326°5 | 63°5 | 26°137 | 10°500 | 9°539  | 296°5 |
| 3°5  | 28°594 | 0°656 | 0°590 | 356°5 | 34°0 | 27°862 | 6°162  | 5°559 | 326°0 | 64°0 | 26°101 | 10°560 | 9°595  | 296°0 |
| 4°0  | 28°591 | 0°749 | 0°674 | 356°0 | 34°5 | 27°841 | 6°246  | 5°636 | 325°5 | 64°5 | 26°064 | 10°619 | 9°650  | 295°5 |
| 4°5  | 28°589 | 0°843 | 0°758 | 355°5 | 35°0 | 27°819 | 6°330  | 5°712 | 325°0 | 65°0 | 26°027 | 10°677 | 9°705  | 295°0 |
| 5°0  | 28°585 | 0°936 | 0°842 | 355°0 | 35°5 | 27°797 | 6°414  | 5°788 | 324°5 | 65°5 | 25°990 | 10°735 | 9°759  | 294°5 |
| 5°5  | 28°582 | 1°030 | 0°926 | 354°5 | 36°0 | 27°775 | 6°497  | 5°864 | 324°0 | 66°0 | 25°953 | 10°793 | 9°812  | 294°0 |
| 6°0  | 28°578 | 1°123 | 1°010 | 354°0 | 36°5 | 27°752 | 6°580  | 5°939 | 323°5 | 66°5 | 25°916 | 10°849 | 9°865  | 293°5 |
| 6°5  | 28°574 | 1°217 | 1°094 | 353°5 | 37°0 | 27°729 | 6°663  | 6°015 | 323°0 | 67°0 | 25°878 | 10°906 | 9°918  | 293°0 |
| 7°0  | 28°570 | 1°310 | 1°178 | 353°0 | 37°5 | 27°706 | 6°745  | 6°090 | 322°5 | 67°5 | 25°841 | 10°961 | 9°970  | 292°5 |
| 7°5  | 28°565 | 1°403 | 1°262 | 352°5 | 38°0 | 27°682 | 6°828  | 6°164 | 322°0 | 68°0 | 25°803 | 11°016 | 10°021 | 292°0 |
| 8°0  | 28°560 | 1°496 | 1°346 | 352°0 | 38°5 | 27°658 | 6°909  | 6°239 | 321°5 | 68°5 | 25°765 | 11°070 | 10°072 | 291°5 |
| 8°5  | 28°555 | 1°590 | 1°430 | 351°5 | 39°0 | 27°634 | 6°991  | 6°313 | 321°0 | 69°0 | 25°726 | 11°124 | 10°123 | 291°0 |
| 9°0  | 28°549 | 1°683 | 1°514 | 351°0 | 39°5 | 27°610 | 7°072  | 6°387 | 320°5 | 69°5 | 25°688 | 11°177 | 10°173 | 290°5 |
| 9°5  | 28°543 | 1°776 | 1°598 | 350°5 | 40°0 | 27°585 | 7°153  | 6°461 | 320°0 | 70°0 | 25°649 | 11°230 | 10°222 | 290°0 |
| 10°0 | 28°537 | 1°869 | 1°681 | 350°0 | 40°5 | 27°560 | 7°234  | 6°534 | 319°5 | 70°5 | 25°610 | 11°282 | 10°271 | 289°5 |
| 10°5 | 28°530 | 1°962 | 1°765 | 349°5 | 41°0 | 27°535 | 7°314  | 6°608 | 319°0 | 71°0 | 25°571 | 11°333 | 10°319 | 289°0 |
| 11°0 | 28°523 | 2°054 | 1°848 | 349°0 | 41°5 | 27°510 | 7°394  | 6°680 | 318°5 | 71°5 | 25°532 | 11°383 | 10°366 | 288°5 |
| 11°5 | 28°516 | 2°147 | 1°932 | 348°5 | 42°0 | 27°484 | 7°473  | 6°753 | 318°0 | 72°0 | 25°493 | 11°433 | 10°414 | 288°0 |
| 12°0 | 28°508 | 2°240 | 2°015 | 348°0 | 42°5 | 27°458 | 7°553  | 6°825 | 317°5 | 72°5 | 25°453 | 11°482 | 10°460 | 287°5 |
| 12°5 | 28°500 | 2°332 | 2°099 | 347°5 | 43°0 | 27°432 | 7°631  | 6°897 | 317°0 | 73°0 | 25°413 | 11°531 | 10°506 | 287°0 |
| 13°0 | 28°492 | 2°424 | 2°182 | 347°0 | 43°5 | 27°405 | 7°710  | 6°969 | 316°5 | 73°5 | 25°374 | 11°579 | 10°551 | 286°5 |
| 13°5 | 28°483 | 2°517 | 2°265 | 346°5 | 44°0 | 27°378 | 7°788  | 7°040 | 316°0 | 74°0 | 25°334 | 11°626 | 10°596 | 286°0 |
| 14°0 | 28°475 | 2°609 | 2°348 | 346°0 | 44°5 | 27°351 | 7°866  | 7°111 | 315°5 | 74°5 | 25°293 | 11°673 | 10°640 | 285°5 |
| 14°5 | 28°465 | 2°701 | 2°431 | 345°5 | 45°0 | 27°324 | 7°943  | 7°182 | 315°0 | 75°0 | 25°253 | 11°719 | 10°684 | 285°0 |
| 15°0 | 28°456 | 2°793 | 2°514 | 345°0 | 45°5 | 27°296 | 8°020  | 7°253 | 314°5 | 75°5 | 25°213 | 11°764 | 10°726 | 284°5 |
| 15°5 | 28°446 | 2°885 | 2°596 | 344°5 | 46°0 | 27°268 | 8°097  | 7°323 | 314°0 | 76°0 | 25°172 | 11°809 | 10°769 | 284°0 |
| 16°0 | 28°436 | 2°977 | 2°679 | 344°0 | 46°5 | 27°240 | 8°173  | 7°392 | 313°5 | 76°5 | 25°131 | 11°852 | 10°811 | 283°5 |
| 16°5 | 28°425 | 3°068 | 2°762 | 343°5 | 47°0 | 27°212 | 8°249  | 7°462 | 313°0 | 77°0 | 25°090 | 11°895 | 10°852 | 283°0 |
| 17°0 | 28°414 | 3°160 | 2°844 | 343°0 | 47°5 | 27°183 | 8°324  | 7°531 | 312°5 | 77°5 | 25°049 | 11°938 | 10°892 | 282°5 |
| 17°5 | 28°403 | 3°251 | 2°927 | 342°5 | 48°0 | 27°154 | 8°399  | 7°600 | 312°0 | 78°0 | 25°008 | 11°980 | 10°932 | 282°0 |
| 18°0 | 28°392 | 3°342 | 3°009 | 342°0 | 48°5 | 27°125 | 8°474  | 7°668 | 311°5 | 78°5 | 24°966 | 12°021 | 10°971 | 281°5 |
| 18°5 | 28°380 | 3°433 | 3°091 | 341°5 | 49°0 | 27°095 | 8°548  | 7°736 | 311°0 | 79°0 | 24°925 | 12°061 | 11°009 | 281°0 |
| 19°0 | 28°368 | 3°524 | 3°173 | 341°0 | 49°5 | 27°066 | 8°622  | 7°804 | 310°5 | 79°5 | 24°883 | 12°100 | 11°047 | 280°5 |
| 19°5 | 28°356 | 3°615 | 3°255 | 340°5 | 50°0 | 27°036 | 8°695  | 7°871 | 310°0 | 80°0 | 24°841 | 12°139 | 11°084 | 280°0 |
| 20°0 | 28°343 | 3°705 | 3°337 | 340°0 | 50°5 | 27°006 | 8°768  | 7°938 | 309°5 | 80°5 | 24°800 | 12°177 | 11°121 | 279°5 |
| 20°5 | 28°330 | 3°796 | 3°418 | 339°5 | 51°0 | 26°975 | 8°841  | 8°005 | 309°0 | 81°0 | 24°757 | 12°214 | 11°157 | 279°0 |
| 21°0 | 28°317 | 3°886 | 3°500 | 339°0 | 51°5 | 26°944 | 8°913  | 8°071 | 308°5 | 81°5 | 24°715 | 12°251 | 11°192 | 278°5 |
| 21°5 | 28°303 | 3°976 | 3°581 | 338°5 | 52°0 | 26°913 | 8°985  | 8°137 | 308°0 | 82°0 | 24°673 | 12°287 | 11°227 | 278°0 |
| 22°0 | 28°289 | 4°066 | 3°662 | 338°0 | 52°5 | 26°882 | 9°056  | 8°203 | 307°5 | 82°5 | 24°631 | 12°322 | 11°261 | 277°5 |
| 22°5 | 28°275 | 4°156 | 3°743 | 337°5 | 53°0 | 26°851 | 9°127  | 8°268 | 307°0 | 83°0 | 24°588 | 12°356 | 11°294 | 277°0 |
| 23°0 | 28°260 | 4°245 | 3°824 | 337°0 | 53°5 | 26°819 | 9°197  | 8°333 | 306°5 | 83°5 | 24°545 | 12°389 | 11°326 | 276°5 |
| 23°5 | 28°245 | 4°335 | 3°905 | 336°5 | 54°0 | 26°787 | 9°267  | 8°397 | 306°0 | 84°0 | 24°503 | 12°422 | 11°358 | 276°0 |
| 24°0 | 28°230 | 4°424 | 3°985 | 336°0 | 54°5 | 26°755 | 9°336  | 8°461 | 305°5 | 84°5 | 24°460 | 12°454 | 11°389 | 275°5 |
| 24°5 | 28°215 | 4°513 | 4°066 | 335°5 | 55°0 | 26°723 | 9°405  | 8°525 | 305°0 | 85°0 | 24°417 | 12°485 | 11°419 | 275°0 |
| 25°0 | 28°199 | 4°602 | 4°146 | 335°0 | 55°5 | 26°690 | 9°474  | 8°588 | 304°5 | 85°5 | 24°374 | 12°515 | 11°449 | 274°5 |
| 25°5 | 28°183 | 4°690 | 4°226 | 334°5 | 56°0 | 26°657 | 9°542  | 8°651 | 304°0 | 86°0 | 24°331 | 12°545 | 11°478 | 274°0 |
| 26°0 | 28°166 | 4°779 | 4°306 | 334°0 | 56°5 | 26°624 | 9°609  | 8°713 | 303°5 | 86°5 | 24°287 | 12°573 | 11°506 | 273°5 |
| 26°5 | 28°149 | 4°867 | 4°386 | 333°5 | 57°0 | 26°591 | 9°676  | 8°775 | 303°0 | 87°0 | 24°244 | 12°601 | 11°533 | 273°0 |
| 27°0 | 28°132 | 4°955 | 4°466 | 333°0 | 57°5 | 26°557 | 9°743  | 8°836 | 302°5 | 87°5 | 24°200 | 12°628 | 11°560 | 272°5 |
| 27°5 | 28°115 | 5°043 | 4°545 | 332°5 | 58°0 | 26°523 | 9°809  | 8°897 | 302°0 | 88°0 | 24°157 | 12°654 | 11°586 | 272°0 |
| 28°0 | 28°097 | 5°130 | 4°624 | 332°0 | 58°5 | 26°489 | 9°874  | 8°958 | 301°5 | 88°5 | 24°113 | 12°680 | 11°611 | 271°5 |
| 28°5 | 28°079 | 5°218 | 4°703 | 331°5 | 59°0 | 26°455 | 9°939  | 9°018 | 301°0 | 89°0 | 24°070 | 12°704 | 11°635 | 271°0 |
| 29°0 | 28°061 | 5°305 | 4°782 | 331°0 | 59°5 | 26°421 | 10°004 | 9°078 | 300°5 | 89°5 | 24°026 | 12°728 | 11°659 | 270°5 |
| 29°5 | 28°043 | 5°391 | 4°861 | 330°5 | 60°0 | 26°386 | 10°068 | 9°137 | 300°0 | 90°0 | 23°982 | 12°751 | 11°681 | 270°0 |

N.B.— $I$  is always positive. When  $N$  is between 0° and 180°,  $\nu$  and  $\xi$  are positive; when  $N$  is between 180° and 360°,  $\nu$  and  $\xi$  are negative.



TABLE XVII.—Values of  $I$ ,  $\nu$  and  $\xi$ , corresponding to  $N$ —(Continued).

| $N$   | $I$    | $\nu$  | $\xi$  | $N$   | $N$   | $I$    | $\nu$  | $\xi$  | $N$   | $N$   | $I$    | $\nu$ | $\xi$ | $N$   |
|-------|--------|--------|--------|-------|-------|--------|--------|--------|-------|-------|--------|-------|-------|-------|
| 90°   | 23°938 | 12°772 | 11°704 | 269°5 | 120°  | 21°283 | 12°295 | 11°378 | 239°5 | 150°5 | 19°135 | 7°745 | 7°223 | 209°5 |
| 91°   | 23°894 | 12°793 | 11°725 | 269°0 | 121°  | 21°241 | 12°254 | 11°342 | 239°0 | 151°  | 19°108 | 7°635 | 7°120 | 209°0 |
| 91°5  | 23°850 | 12°814 | 11°745 | 268°5 | 121°5 | 21°199 | 12°211 | 11°304 | 238°5 | 151°5 | 19°082 | 7°523 | 7°017 | 208°5 |
| 92°   | 23°806 | 12°833 | 11°765 | 268°0 | 122°  | 21°157 | 12°168 | 11°266 | 238°0 | 152°  | 19°056 | 7°411 | 6°913 | 208°0 |
| 92°5  | 23°761 | 12°851 | 11°784 | 267°5 | 122°5 | 21°115 | 12°123 | 11°226 | 237°5 | 152°5 | 19°030 | 7°298 | 6°809 | 207°5 |
| 93°   | 23°717 | 12°869 | 11°802 | 267°0 | 123°  | 21°073 | 12°078 | 11°186 | 237°0 | 153°  | 19°005 | 7°184 | 6°703 | 207°0 |
| 93°5  | 23°673 | 12°886 | 11°819 | 266°5 | 123°5 | 21°032 | 12°031 | 11°144 | 236°5 | 153°5 | 18°981 | 7°069 | 6°596 | 206°5 |
| 94°   | 23°628 | 12°901 | 11°835 | 266°0 | 124°  | 20°990 | 11°983 | 11°101 | 236°0 | 154°  | 18°956 | 6°953 | 6°488 | 206°0 |
| 94°5  | 23°584 | 12°916 | 11°851 | 265°5 | 124°5 | 20°949 | 11°933 | 11°057 | 235°5 | 154°5 | 18°933 | 6°836 | 6°380 | 205°5 |
| 95°   | 23°539 | 12°930 | 11°866 | 265°0 | 125°  | 20°908 | 11°883 | 11°012 | 235°0 | 155°  | 18°909 | 6°718 | 6°270 | 205°0 |
| 95°5  | 23°495 | 12°943 | 11°880 | 264°5 | 125°5 | 20°867 | 11°831 | 10°965 | 234°5 | 155°5 | 18°886 | 6°599 | 6°160 | 204°5 |
| 96°   | 23°450 | 12°955 | 11°893 | 264°0 | 126°  | 20°826 | 11°778 | 10°918 | 234°0 | 156°  | 18°863 | 6°480 | 6°049 | 204°0 |
| 96°5  | 23°406 | 12°966 | 11°905 | 263°5 | 126°5 | 20°786 | 11°724 | 10°869 | 233°5 | 156°5 | 18°841 | 6°359 | 5°937 | 203°5 |
| 97°   | 23°361 | 12°976 | 11°916 | 263°0 | 127°  | 20°746 | 11°669 | 10°820 | 233°0 | 157°  | 18°819 | 6°238 | 5°824 | 203°0 |
| 97°5  | 23°316 | 12°985 | 11°927 | 262°5 | 127°5 | 20°705 | 11°612 | 10°769 | 232°5 | 157°5 | 18°798 | 6°116 | 5°710 | 202°5 |
| 98°   | 23°271 | 12°994 | 11°936 | 262°0 | 128°  | 20°666 | 11°555 | 10°717 | 232°0 | 158°  | 18°777 | 5°993 | 5°596 | 202°0 |
| 98°5  | 23°227 | 13°001 | 11°945 | 261°5 | 128°5 | 20°626 | 11°496 | 10°664 | 231°5 | 158°5 | 18°756 | 5°869 | 5°480 | 201°5 |
| 99°   | 23°182 | 13°007 | 11°953 | 261°0 | 129°  | 20°586 | 11°436 | 10°610 | 231°0 | 159°  | 18°736 | 5°744 | 5°364 | 201°0 |
| 99°5  | 23°137 | 13°013 | 11°960 | 260°5 | 129°5 | 20°547 | 11°374 | 10°554 | 230°5 | 159°5 | 18°716 | 5°618 | 5°247 | 200°5 |
| 100°  | 23°092 | 13°017 | 11°966 | 260°0 | 130°  | 20°508 | 11°312 | 10°498 | 230°0 | 160°  | 18°697 | 5°492 | 5°130 | 200°0 |
| 100°5 | 23°047 | 13°021 | 11°971 | 259°5 | 130°5 | 20°469 | 11°248 | 10°440 | 229°5 | 160°5 | 18°678 | 5°365 | 5°011 | 199°5 |
| 101°  | 23°003 | 13°023 | 11°975 | 259°0 | 131°  | 20°430 | 11°184 | 10°382 | 229°0 | 161°  | 18°660 | 5°237 | 4°892 | 199°0 |
| 101°5 | 22°958 | 13°024 | 11°979 | 258°5 | 131°5 | 20°392 | 11°118 | 10°322 | 228°5 | 161°5 | 18°642 | 5°109 | 4°773 | 198°5 |
| 102°  | 22°913 | 13°025 | 11°981 | 258°0 | 132°  | 20°353 | 11°050 | 10°261 | 228°0 | 162°  | 18°624 | 4°980 | 4°652 | 198°0 |
| 102°5 | 22°868 | 13°024 | 11°983 | 257°5 | 132°5 | 20°315 | 10°982 | 10°199 | 227°5 | 162°5 | 18°607 | 4°850 | 4°531 | 197°5 |
| 103°  | 22°823 | 13°023 | 11°983 | 257°0 | 133°  | 20°278 | 10°912 | 10°135 | 227°0 | 163°  | 18°591 | 4°719 | 4°409 | 197°0 |
| 103°5 | 22°778 | 13°020 | 11°983 | 256°5 | 133°5 | 20°240 | 10°841 | 10°071 | 226°5 | 163°5 | 18°575 | 4°588 | 4°287 | 196°5 |
| 104°  | 22°734 | 13°017 | 11°982 | 256°0 | 134°  | 20°203 | 10°769 | 10°005 | 226°0 | 164°  | 18°559 | 4°456 | 4°164 | 196°0 |
| 104°5 | 22°689 | 13°012 | 11°979 | 255°5 | 134°5 | 20°166 | 10°696 | 9°939  | 225°5 | 164°5 | 18°544 | 4°323 | 4°040 | 195°5 |
| 105°  | 22°644 | 13°006 | 11°976 | 255°0 | 135°  | 20°129 | 10°622 | 9°871  | 225°0 | 165°  | 18°529 | 4°190 | 3°916 | 195°0 |
| 105°5 | 22°599 | 13°000 | 11°972 | 254°5 | 135°5 | 20°092 | 10°546 | 9°802  | 224°5 | 165°5 | 18°515 | 4°056 | 3°791 | 194°5 |
| 106°  | 22°554 | 12°992 | 11°967 | 254°0 | 136°  | 20°056 | 10°469 | 9°732  | 224°0 | 166°  | 18°501 | 3°922 | 3°665 | 194°0 |
| 106°5 | 22°510 | 12°983 | 11°961 | 253°5 | 136°5 | 20°020 | 10°391 | 9°660  | 223°5 | 166°5 | 18°487 | 3°787 | 3°539 | 193°5 |
| 107°  | 22°465 | 12°974 | 11°954 | 253°0 | 137°  | 19°984 | 10°312 | 9°588  | 223°0 | 167°  | 18°475 | 3°651 | 3°413 | 193°0 |
| 107°5 | 22°420 | 12°963 | 11°946 | 252°5 | 137°5 | 19°949 | 10°232 | 9°515  | 222°5 | 167°5 | 18°462 | 3°515 | 3°286 | 192°5 |
| 108°  | 22°376 | 12°951 | 11°937 | 252°0 | 138°  | 19°913 | 10°150 | 9°440  | 222°0 | 168°  | 18°450 | 3°379 | 3°158 | 192°0 |
| 108°5 | 22°331 | 12°938 | 11°927 | 251°5 | 138°5 | 19°878 | 10°068 | 9°364  | 221°5 | 168°5 | 18°439 | 3°242 | 3°030 | 191°5 |
| 109°  | 22°287 | 12°924 | 11°916 | 251°0 | 139°  | 19°844 | 9°984  | 9°287  | 221°0 | 169°  | 18°428 | 3°104 | 2°902 | 191°0 |
| 109°5 | 22°242 | 12°909 | 11°904 | 250°5 | 139°5 | 19°809 | 9°899  | 9°209  | 220°5 | 169°5 | 18°417 | 2°966 | 2°773 | 190°5 |
| 110°  | 22°198 | 12°892 | 11°891 | 250°0 | 140°  | 19°775 | 9°813  | 9°130  | 220°0 | 170°  | 18°407 | 2°828 | 2°644 | 190°0 |
| 110°5 | 22°153 | 12°875 | 11°879 | 249°5 | 140°5 | 19°742 | 9°725  | 9°050  | 219°5 | 170°5 | 18°398 | 2°689 | 2°514 | 189°5 |
| 111°  | 22°109 | 12°857 | 11°862 | 249°0 | 141°  | 19°708 | 9°637  | 8°969  | 219°0 | 171°  | 18°388 | 2°550 | 2°384 | 189°0 |
| 111°5 | 22°065 | 12°837 | 11°846 | 248°5 | 141°5 | 19°675 | 9°547  | 8°887  | 218°5 | 171°5 | 18°380 | 2°410 | 2°254 | 188°5 |
| 112°  | 22°021 | 12°817 | 11°829 | 248°0 | 142°  | 19°642 | 9°457  | 8°803  | 218°0 | 172°  | 18°372 | 2°270 | 2°123 | 188°0 |
| 112°5 | 21°976 | 12°795 | 11°811 | 247°5 | 142°5 | 19°610 | 9°365  | 8°719  | 217°5 | 172°5 | 18°364 | 2°130 | 1°992 | 187°5 |
| 113°  | 21°932 | 12°772 | 11°792 | 247°0 | 143°  | 19°577 | 9°272  | 8°633  | 217°0 | 173°  | 18°357 | 1°989 | 1°860 | 187°0 |
| 113°5 | 21°888 | 12°748 | 11°772 | 246°5 | 143°5 | 19°545 | 9°177  | 8°546  | 216°5 | 173°5 | 18°350 | 1°848 | 1°729 | 186°5 |
| 114°  | 21°845 | 12°723 | 11°750 | 246°0 | 144°  | 19°514 | 9°082  | 8°458  | 216°0 | 174°  | 18°344 | 1°707 | 1°597 | 186°0 |
| 114°5 | 21°801 | 12°697 | 11°728 | 245°5 | 144°5 | 19°483 | 8°986  | 8°370  | 215°5 | 174°5 | 18°338 | 1°566 | 1°464 | 185°5 |
| 115°  | 21°757 | 12°670 | 11°705 | 245°0 | 145°  | 19°452 | 8°888  | 8°280  | 215°0 | 175°  | 18°333 | 1°424 | 1°332 | 185°0 |
| 115°5 | 21°713 | 12°642 | 11°681 | 244°5 | 145°5 | 19°421 | 8°790  | 8°189  | 214°5 | 175°5 | 18°328 | 1°282 | 1°199 | 184°5 |
| 116°  | 21°670 | 12°612 | 11°655 | 244°0 | 146°  | 19°391 | 8°690  | 8°097  | 214°0 | 176°  | 18°324 | 1°140 | 1°067 | 184°0 |
| 116°5 | 21°627 | 12°581 | 11°629 | 243°5 | 146°5 | 19°361 | 8°589  | 8°004  | 213°5 | 176°5 | 18°320 | 0°998 | 0°934 | 183°5 |
| 117°  | 21°583 | 12°550 | 11°601 | 243°0 | 147°  | 19°332 | 8°487  | 7°910  | 213°0 | 177°  | 18°317 | 0°856 | 0°801 | 183°0 |
| 117°5 | 21°540 | 12°517 | 11°572 | 242°5 | 147°5 | 19°302 | 8°384  | 7°814  | 212°5 | 177°5 | 18°315 | 0°713 | 0°667 | 182°5 |
| 118°  | 21°497 | 12°483 | 11°543 | 242°0 | 148°  | 19°273 | 8°280  | 7°718  | 212°0 | 178°  | 18°312 | 0°571 | 0°534 | 182°0 |
| 118°5 | 21°454 | 12°447 | 11°512 | 241°5 | 148°5 | 19°245 | 8°175  | 7°621  | 211°5 | 178°5 | 18°311 | 0°428 | 0°401 | 181°5 |
| 119°  | 21°411 | 12°411 | 11°480 | 241°0 | 149°  | 19°217 | 8°069  | 7°523  | 211°0 | 179°  | 18°309 | 0°286 | 0°267 | 181°0 |
| 119°5 | 21°368 | 12°373 | 11°447 | 240°5 | 149°5 | 19°189 | 7°962  | 7°424  | 210°5 | 179°5 | 18°309 | 0°143 | 0°133 | 180°5 |
| 120°  | 21°326 | 12°335 | 11°413 | 240°0 | 150°  | 19°162 | 7°854  | 7°324  | 210°0 | 180°  | 18°308 | 0°000 | 0°000 | 180°0 |

N.B.— $I$  is always positive. When  $N$  is between  $0^\circ$  and  $180^\circ$ ,  $\nu$  and  $\xi$  are positive; when  $N$  is between  $180^\circ$  and  $360^\circ$ ,  $\nu$  and  $\xi$  are negative.

COMPUTATIONS.

[PART VI.]

TABLE XVIII. (1)—Values of  $1/f$  and  $f$  corresponding to various values of  $I$ , to be used in computing H and R for the Tides  $M_2$ ,  $N$ ,  $2N$ ,  $\nu$ ,  $MS$ ,  $2SM$  and Luni-Solar fortnightly.

$$\text{Argument } 1/f = \frac{\text{Cos}^4 \frac{1}{2} \omega \text{Cos}^4 \frac{1}{2} i}{\text{Cos}^4 \frac{1}{2} I}$$

| Values of $I$ | $1/f$   | Differences for $0^{\circ}.1$ of $I$ | $f$     | Differences for $0^{\circ}.1$ of $I$ | Values of $I$ | $1/f$   | Differences for $0^{\circ}.1$ of $I$ | $f$     | Differences for $0^{\circ}.1$ of $I$ |
|---------------|---------|--------------------------------------|---------|--------------------------------------|---------------|---------|--------------------------------------|---------|--------------------------------------|
| 18° 18' 30"   | 0.96354 |                                      | 1.03784 |                                      | 23° 5         | 0.99630 | 73                                   | 1.00371 | 73                                   |
| 18° 4         | .96403  |                                      | .03731  | 59                                   | 6             | .99703  | 72                                   | .00298  | 73                                   |
| 5             | .96458  | 55                                   | .03672  | 59                                   | 7             | .99775  | 74                                   | .00225  | 73                                   |
| 6             | .96513  | 55                                   | .03613  | 60                                   | 8             | .99849  | 73                                   | .00152  | 74                                   |
| 7             | .96569  | 56                                   | .03553  | 59                                   | 9             | .99922  | 74                                   | .00078  | 74                                   |
| 8             | .96624  | 55                                   | .03494  | 59                                   | 24 0          | .99996  | 74                                   | .00004  | 74                                   |
| 9             | .96680  | 56                                   | .03434  | 60                                   | 1             | 1.00070 | 74                                   | 0.99930 | 74                                   |
| 19° 0         | .96736  | 56                                   | .03374  | 61                                   | 2             | .00145  | 75                                   | .99855  | 74                                   |
| 1             | .96793  | 57                                   | .03313  | 61                                   | 3             | .00220  | 75                                   | .99781  | 76                                   |
| 2             | .96850  | 57                                   | .03252  | 61                                   | 4             | .00296  | 76                                   | .99705  | 76                                   |
| 3             | .96908  | 58                                   | .03191  | 62                                   | 5             | .00372  | 76                                   | .99629  | 76                                   |
| 4             | .96965  | 57                                   | .03129  | 62                                   | 6             | .00448  | 76                                   | .99553  | 75                                   |
| 5             | .97023  | 58                                   | .03068  | 61                                   | 7             | .00525  | 76                                   | .99478  | 76                                   |
| 6             | .97082  | 59                                   | .03006  | 62                                   | 8             | .00601  | 78                                   | .99402  | 76                                   |
| 7             | .97141  | 59                                   | .02943  | 63                                   | 9             | .00679  | 78                                   | .99326  | 77                                   |
| 8             | .97199  | 58                                   | .02882  | 61                                   | 25 0          | .00757  | 78                                   | .99249  | 77                                   |
| 9             | .97259  | 60                                   | .02819  | 63                                   | 1             | .00835  | 78                                   | .99172  | 77                                   |
| 20° 0         | .97318  | 59                                   | .02756  | 63                                   | 2             | .00913  | 78                                   | .99095  | 78                                   |
| 1             | .97379  | 61                                   | .02692  | 64                                   | 3             | .00993  | 80                                   | .99017  | 78                                   |
| 2             | .97439  | 60                                   | .02628  | 64                                   | 4             | .01072  | 79                                   | .98939  | 77                                   |
| 3             | .97500  | 61                                   | .02564  | 64                                   | 5             | .01151  | 79                                   | .98862  | 77                                   |
| 4             | .97561  | 61                                   | .02500  | 64                                   | 6             | .01231  | 80                                   | .98783  | 79                                   |
| 5             | .97622  | 61                                   | .02436  | 64                                   | 7             | .01312  | 81                                   | .98705  | 78                                   |
| 6             | .97684  | 62                                   | .02371  | 65                                   | 8             | .01393  | 81                                   | .98626  | 79                                   |
| 7             | .97746  | 62                                   | .02306  | 65                                   | 9             | .01474  | 82                                   | .98547  | 79                                   |
| 8             | .97808  | 62                                   | .02241  | 65                                   | 26 0          | .01556  | 82                                   | .98468  | 80                                   |
| 9             | .97871  | 63                                   | .02175  | 66                                   | 1             | .01638  | 82                                   | .98388  | 80                                   |
| 21° 0         | .97935  | 64                                   | .02109  | 66                                   | 2             | .01721  | 83                                   | .98308  | 80                                   |
| 1             | .97999  | 64                                   | .02042  | 67                                   | 3             | .01803  | 82                                   | .98229  | 79                                   |
| 2             | .98062  | 63                                   | .01976  | 66                                   | 4             | .01887  | 84                                   | .98148  | 81                                   |
| 3             | .98127  | 65                                   | .01909  | 67                                   | 5             | .01970  | 83                                   | .98068  | 80                                   |
| 4             | .98191  | 64                                   | .01842  | 67                                   | 6             | .02054  | 84                                   | .97988  | 80                                   |
| 5             | .98256  | 65                                   | .01775  | 67                                   | 7             | .02138  | 84                                   | .97907  | 81                                   |
| 6             | .98321  | 65                                   | .01707  | 68                                   | 8             | .02223  | 85                                   | .97826  | 81                                   |
| 7             | .98387  | 66                                   | .01639  | 68                                   | 9             | .02308  | 85                                   | .97744  | 82                                   |
| 8             | .98453  | 66                                   | .01572  | 67                                   | 27 0          | .02394  | 86                                   | .97662  | 82                                   |
| 9             | .98519  | 66                                   | .01503  | 69                                   | 1             | .02480  | 86                                   | .97580  | 82                                   |
| 22° 0         | .98586  | 67                                   | .01434  | 69                                   | 2             | .02566  | 86                                   | .97498  | 82                                   |
| 1             | .98653  | 67                                   | .01365  | 69                                   | 3             | .02653  | 87                                   | .97416  | 82                                   |
| 2             | .98721  | 68                                   | .01296  | 69                                   | 4             | .02740  | 87                                   | .97333  | 83                                   |
| 3             | .98788  | 67                                   | .01226  | 70                                   | 5             | .02828  | 88                                   | .97250  | 83                                   |
| 4             | .98857  | 69                                   | .01157  | 69                                   | 6             | .02916  | 38                                   | .97167  | 83                                   |
| 5             | .98925  | 68                                   | .01087  | 70                                   | 7             | .02916  | 89                                   | .97167  | 84                                   |
| 6             | .98994  | 69                                   | .01016  | 71                                   | 8             | .03005  | 89                                   | .97083  | 83                                   |
| 7             | .99063  | 69                                   | .00945  | 71                                   | 9             | .03093  | 88                                   | .97000  | 84                                   |
| 8             | .99133  | 70                                   | .00875  | 70                                   | 28 0          | .03183  | 90                                   | .96916  | 84                                   |
| 9             | .99203  | 70                                   | .00803  | 72                                   | 1             | .03272  | 86                                   | .96832  | 82                                   |
| 23° 0         | .99273  | 70                                   | .00732  | 71                                   | 2             | .03363  | 86                                   | .96747  | 82                                   |
| 1             | .99344  | 71                                   | .00661  | 71                                   | 3             | .03453  | 86                                   | .96662  | 82                                   |
| 2             | .99415  | 71                                   | .00589  | 72                                   | 4             | .03544  | 87                                   | .96577  | 83                                   |
| 3             | .99486  | 71                                   | .00516  | 73                                   | 5             | .03635  | 87                                   | .96492  | 83                                   |
| 4             | .99558  | 72                                   | .00444  | 72                                   | 6             | .03727  | 88                                   | .96407  | 84                                   |
| 5             | .99630  | 72                                   | .00371  | 73                                   | 7             | .03819  | 89                                   | .96321  | 84                                   |
|               |         |                                      |         |                                      | 28° 36' 6"    | .03821  | 92                                   | .96320  | 85                                   |

TABLE XVIII. (2)—Values of  $1/f$  and  $f$  corresponding to various values of  $I$ , to be used in computing H and R for the Tides O and Q. Also used in determining  $1/f$  for the Tide  $M_1$ .

$$\text{Argument } 1/f = \frac{\sin \omega \cos^2 \frac{1}{2} \omega \cos^4 \frac{1}{2} i}{\sin I \cos^2 \frac{1}{2} I}$$

| Values of $I$ | $1/f$   | Differences for 0°.1 of $I$ | $f$     | Differences for 0°.1 of $I$ | Values of $I$ | $1/f$   | Differences for 0°.1 of $I$ | $f$     | Differences for 0°.1 of $I$ |
|---------------|---------|-----------------------------|---------|-----------------------------|---------------|---------|-----------------------------|---------|-----------------------------|
| 18° 18' 30"   | 1.24126 |                             | 0.80563 |                             | 23° 5         | 0.99434 |                             | 1.00569 |                             |
| 18.4          | .23563  | 612                         | .80932  | 401                         | .6            | .99073  | 361                         | .00936  | 367                         |
| .5            | .22951  | 601                         | .81333  | 401                         | .7            | .98715  | 358                         | .01303  | 367                         |
| .6            | .22380  | 597                         | .81734  | 401                         | .8            | .98360  | 355                         | .01667  | 364                         |
| .7            | .21753  | 590                         | .82135  | 399                         | .9            | .98009  | 351                         | .02032  | 365                         |
| .8            | .21163  | 582                         | .82534  | 399                         | 24 0          | .97660  | 349                         | .02396  | 364                         |
| .9            | .20581  | 578                         | .82933  | 398                         | .1            | .97316  | 344                         | .02758  | 362                         |
| 19 0          | .20003  | 568                         | .83331  | 398                         | .2            | .96974  | 342                         | .03121  | 363                         |
| .1            | .19435  | 565                         | .83729  | 397                         | .3            | .96635  | 339                         | .03482  | 361                         |
| .2            | .18870  | 557                         | .84126  | 397                         | .4            | .96300  | 335                         | .03843  | 361                         |
| .3            | .18313  | 551                         | .84523  | 396                         | .5            | .95966  | 334                         | .04204  | 361                         |
| .4            | .17762  | 548                         | .84919  | 395                         | .6            | .95637  | 329                         | .04563  | 359                         |
| .5            | .17214  | 538                         | .85314  | 394                         | .7            | .95310  | 327                         | .04921  | 358                         |
| .6            | .16676  | 534                         | .85708  | 394                         | .8            | .94986  | 324                         | .05280  | 359                         |
| .7            | .16142  | 528                         | .86102  | 394                         | .9            | .94666  | 320                         | .05636  | 356                         |
| .8            | .15614  | 522                         | .86496  | 393                         | 25 0          | .94347  | 319                         | .05992  | 356                         |
| .9            | .15092  | 509                         | .86889  | 392                         | .1            | .94033  | 314                         | .06347  | 355                         |
| 20 0          | .14573  | 507                         | .87281  | 391                         | .2            | .93720  | 313                         | .06702  | 355                         |
| .1            | .14064  | 501                         | .87672  | 390                         | .3            | .93410  | 310                         | .07055  | 353                         |
| .2            | .13557  | 495                         | .88062  | 391                         | .4            | .93103  | 307                         | .07408  | 353                         |
| .3            | .13056  | 492                         | .88453  | 389                         | .5            | .92798  | 305                         | .07761  | 353                         |
| .4            | .12561  | 484                         | .88842  | 389                         | .6            | .92497  | 301                         | .08112  | 351                         |
| .5            | .12069  | 480                         | .89231  | 388                         | .7            | .92198  | 299                         | .08463  | 351                         |
| .6            | .11585  | 476                         | .89619  | 387                         | .8            | .91902  | 296                         | .08812  | 349                         |
| .7            | .11105  | 470                         | .90006  | 387                         | .9            | .91608  | 294                         | .09161  | 349                         |
| .8            | .10629  | 467                         | .90393  | 386                         | 26 0          | .91316  | 292                         | .09510  | 349                         |
| .9            | .10159  | 459                         | .90779  | 385                         | .1            | .91028  | 288                         | .09857  | 347                         |
| 21 0          | .09692  | 457                         | .91164  | 385                         | .2            | .90742  | 286                         | .09857  | 346                         |
| .1            | .09233  | 451                         | .91549  | 384                         | .3            | .90458  | 284                         | .10203  | 346                         |
| .2            | .08776  | 447                         | .91933  | 384                         | .4            | .90177  | 281                         | .10549  | 345                         |
| .3            | .08325  | 444                         | .92316  | 383                         | .5            | .89897  | 280                         | .10894  | 344                         |
| .4            | .07878  | 441                         | .92698  | 382                         | .6            | .89622  | 275                         | .11238  | 344                         |
| .5            | .07434  | 437                         | .93080  | 382                         | .7            | .89347  | 275                         | .11581  | 343                         |
| .6            | .06997  | 434                         | .93461  | 381                         | .8            | .89076  | 271                         | .11924  | 343                         |
| .7            | .06563  | 430                         | .93842  | 381                         | .9            | .88806  | 271                         | .12265  | 341                         |
| .8            | .06133  | 425                         | .94222  | 380                         | 27 0          | .88538  | 270                         | .12606  | 341                         |
| .9            | .05708  | 422                         | .94601  | 379                         | .1            | .88273  | 268                         | .12946  | 340                         |
| 22 0          | .05286  | 416                         | .94979  | 378                         | .2            | .88010  | 265                         | .13285  | 339                         |
| .1            | .04870  | 413                         | .95357  | 378                         | .3            | .87750  | 263                         | .13623  | 338                         |
| .2            | .04457  | 410                         | .95734  | 377                         | .4            | .87491  | 260                         | .13961  | 338                         |
| .3            | .04047  | 404                         | .96111  | 377                         | .5            | .87234  | 259                         | .14297  | 336                         |
| .4            | .03643  | 402                         | .96486  | 375                         | .6            | .86981  | 257                         | .14634  | 337                         |
| .5            | .03241  | 396                         | .96861  | 375                         | .7            | .86729  | 253                         | .14968  | 334                         |
| .6            | .02845  | 394                         | .97235  | 374                         | .8            | .86478  | 252                         | .15302  | 334                         |
| .7            | .02451  | 390                         | .97608  | 373                         | .9            | .86231  | 251                         | .15636  | 334                         |
| .8            | .02061  | 386                         | .97981  | 373                         | 28 0          | .85985  | 247                         | .15968  | 332                         |
| .9            | .01675  | 383                         | .98353  | 372                         | .1            | .85741  | 246                         | .16299  | 331                         |
| 23 0          | .01292  | 377                         | .98725  | 372                         | .2            | .85499  | 244                         | .16630  | 331                         |
| .1            | .00915  | 376                         | .99095  | 370                         | .3            | .85260  | 242                         | .16961  | 331                         |
| .2            | .00539  | 372                         | .99464  | 369                         | .4            | .85022  | 239                         | .17289  | 328                         |
| .3            | .00167  | 368                         | .99834  | 370                         | .5            | .84785  | 238                         | .17617  | 328                         |
| .4            | .009799 | 365                         | 1.00202 | 368                         | .6            | .84551  | 234                         | .17945  | 328                         |
| .5            | .99434  |                             | .00569  | 367                         | 28 36' 6"     | .84547  |                             | .18272  | 327                         |
|               |         |                             |         |                             |               |         |                             | .18277  |                             |

COMPUTATIONS.

[PART VI.]

TABLE XVIII. (3)—Values of  $1/f$  and  $f$  corresponding to various values of  $I$ , to be used in computing  $H$  and  $R$  for the Tide  $J$ ; and for determining  $k_1$ , used in the preparation of Table XVIII. (6).

$$\text{Argument } 1/f = \frac{\sin \omega \cos \omega (1 - \frac{3}{2} \sin^2 i)}{\sin I \cos I}$$

| Values of $I$ | $1/f$   | Differences for $0^{\circ}.1$ of $I$ | $f$     | Differences for $0^{\circ}.1$ of $I$ | Values of $I$ | $1/f$   | Differences for $0^{\circ}.1$ of $I$ | $f$     | Differences for $0^{\circ}.1$ of $I$ |
|---------------|---------|--------------------------------------|---------|--------------------------------------|---------------|---------|--------------------------------------|---------|--------------------------------------|
| 18° 18' 30"   | 1.20958 |                                      | 0.82673 |                                      | 23° 5         | 0.98648 | 319                                  | 1.01371 | 329                                  |
| 18.4          | .20442  |                                      | .83028  |                                      | .6            | .98329  | 316                                  | .01700  | 328                                  |
| .5            | .19882  | 560                                  | .83416  | 388                                  | .7            | .98013  | 313                                  | .02028  | 327                                  |
| .6            | .19332  | 550                                  | .83801  | 385                                  | .8            | .97700  | 309                                  | .02355  | 325                                  |
| .7            | .18785  | 547                                  | .84186  | 385                                  | .9            | .97391  | 308                                  | .02680  | 325                                  |
| .8            | .18246  | 539                                  | .84570  | 384                                  | 24 0          | .97083  | 308                                  | .03005  | 325                                  |
| .9            | .17714  | 532                                  | .84953  | 383                                  | .1            | .96780  | 303                                  | .03328  | 323                                  |
| 19 0          | .17185  | 529                                  | .85335  | 382                                  | .2            | .96479  | 301                                  | .03650  | 322                                  |
| .1            | .16667  | 518                                  | .85715  | 380                                  | .3            | .96182  | 297                                  | .03970  | 320                                  |
| .2            | .16152  | 515                                  | .86095  | 380                                  | .4            | .95888  | 294                                  | .04289  | 319                                  |
| .3            | .15643  | 509                                  | .86474  | 379                                  | .5            | .95595  | 293                                  | .04608  | 319                                  |
| .4            | .15141  | 502                                  | .86851  | 377                                  | .6            | .95308  | 287                                  | .04924  | 316                                  |
| .5            | .14642  | 499                                  | .87228  | 377                                  | .7            | .95022  | 286                                  | .05240  | 316                                  |
| .6            | .14152  | 490                                  | .87603  | 375                                  | .8            | .94738  | 284                                  | .05554  | 314                                  |
| .7            | .13666  | 486                                  | .87978  | 375                                  | .9            | .94459  | 279                                  | .05867  | 313                                  |
| .8            | .13186  | 480                                  | .88351  | 373                                  | 25 0          | .94181  | 278                                  | .06179  | 312                                  |
| .9            | .12712  | 474                                  | .88724  | 373                                  | .1            | .93907  | 274                                  | .06489  | 310                                  |
| 20 0          | .12240  | 472                                  | .89095  | 371                                  | .2            | .93635  | 272                                  | .06798  | 309                                  |
| .1            | .11777  | 463                                  | .89465  | 370                                  | .3            | .93366  | 269                                  | .07106  | 308                                  |
| .2            | .11318  | 459                                  | .89834  | 369                                  | .4            | .93100  | 266                                  | .07412  | 306                                  |
| .3            | .10863  | 455                                  | .90201  | 367                                  | .5            | .92835  | 265                                  | .07718  | 306                                  |
| .4            | .10415  | 448                                  | .90568  | 367                                  | .6            | .92575  | 260                                  | .08021  | 303                                  |
| .5            | .09970  | 445                                  | .90934  | 366                                  | .7            | .92316  | 259                                  | .08323  | 302                                  |
| .6            | .09532  | 438                                  | .91298  | 364                                  | .8            | .92060  | 256                                  | .08625  | 302                                  |
| .7            | .09097  | 435                                  | .91662  | 364                                  | .9            | .91807  | 253                                  | .08925  | 300                                  |
| .8            | .08667  | 430                                  | .92024  | 362                                  | 26 0          | .91555  | 252                                  | .09224  | 300                                  |
| .9            | .08243  | 424                                  | .92385  | 361                                  | .1            | .91307  | 252                                  | .09520  | 299                                  |
| 21 0          | .07821  | 422                                  | .92746  | 361                                  | .2            | .91061  | 248                                  | .09816  | 296                                  |
| .1            | .07407  | 414                                  | .93105  | 359                                  | .3            | .90818  | 246                                  | .09981  | 295                                  |
| .2            | .06995  | 412                                  | .93463  | 358                                  | .4            | .90577  | 243                                  | .10111  | 293                                  |
| .3            | .06589  | 406                                  | .93819  | 356                                  | .5            | .90337  | 241                                  | .10404  | 292                                  |
| .4            | .06187  | 402                                  | .94175  | 356                                  | .6            | .90101  | 240                                  | .10696  | 292                                  |
| .5            | .05787  | 400                                  | .94530  | 355                                  | .7            | .89867  | 236                                  | .10986  | 290                                  |
| .6            | .05395  | 392                                  | .94883  | 353                                  | .8            | .89635  | 234                                  | .11276  | 290                                  |
| .7            | .05004  | 391                                  | .95235  | 352                                  | .9            | .89406  | 232                                  | .11563  | 287                                  |
| .8            | .04619  | 385                                  | .95586  | 351                                  | 27 0          | .89178  | 229                                  | .11849  | 286                                  |
| .9            | .04238  | 381                                  | .95935  | 349                                  | .1            | .88954  | 228                                  | .12135  | 286                                  |
| 22 0          | .03859  | 379                                  | .96284  | 349                                  | .2            | .88730  | 224                                  | .12419  | 284                                  |
| .1            | .03487  | 372                                  | .96631  | 347                                  | .3            | .88510  | 224                                  | .12702  | 283                                  |
| .2            | .03117  | 370                                  | .96978  | 347                                  | .4            | .88292  | 220                                  | .12982  | 280                                  |
| .3            | .02751  | 366                                  | .97323  | 345                                  | .5            | .88075  | 218                                  | .13262  | 280                                  |
| .4            | .02390  | 361                                  | .97667  | 344                                  | .6            | .88075  | 217                                  | .13540  | 278                                  |
| .5            | .02031  | 359                                  | .98010  | 344                                  | .7            | .87861  | 217                                  | .13817  | 277                                  |
| .6            | .01678  | 353                                  | .98351  | 343                                  | .8            | .87649  | 214                                  | .14092  | 275                                  |
| .7            | .01326  | 352                                  | .98691  | 341                                  | .9            | .87439  | 212                                  | .14366  | 274                                  |
| .8            | .00980  | 346                                  | .99031  | 340                                  | 28 0          | .87231  | 210                                  | .14638  | 272                                  |
| .9            | .00636  | 344                                  | .99368  | 340                                  | .1            | .87024  | 208                                  | .14910  | 272                                  |
| 23 0          | .00295  | 341                                  | .99705  | 337                                  | .2            | .86821  | 207                                  | .15179  | 269                                  |
| .1            | 0.99960 | 335                                  | 1.00040 | 337                                  | .3            | .86619  | 203                                  | .15448  | 269                                  |
| .2            | .99627  | 333                                  | .00375  | 335                                  | .4            | .86420  | 202                                  | .15715  | 267                                  |
| .3            | .99298  | 329                                  | .00708  | 333                                  | .5            | .86222  | 199                                  | .15980  | 265                                  |
| .4            | .98972  | 326                                  | .01040  | 333                                  | .6            | .86025  | 198                                  | .16245  | 265                                  |
| .5            | .98648  | 324                                  | .01371  | 331                                  | 28° 36' 6"    | .85831  | 194                                  | .16509  | 264                                  |

TABLE XVIII. (4)—Values of  $1/f$  and  $f$  corresponding to various values of  $I$ , to be used in computing H and R for the Tide Mf.

$$\text{Argument } 1/f = \frac{\text{Sin}^2 \omega \text{ Cos}^4 \frac{1}{2} i}{\text{Sin}^2 I}$$

| Values of $I$ | $1/f$   | Differences for 0°.1 of $I$ | $f$     | Differences for 0°.1 of $I$ | Values of $I$ | $1/f$   | Differences for 0°.1 of $I$ | $f$     | Differences for 0°.1 of $I$ |
|---------------|---------|-----------------------------|---------|-----------------------------|---------------|---------|-----------------------------|---------|-----------------------------|
| 18° 18' 30"   | 1.59903 |                             | 0.62538 |                             | 23° 5         | 0.99238 | 788                         | 1.00768 | 811                         |
| 18.4          | .58377  | 1657                        | .63145  | 66.3                        | 6             | .98450  | 783                         | .01579  | 813                         |
| 5             | .56720  | 1613                        | .63808  | 668                         | 7             | .97667  | 772                         | .02392  | 815                         |
| 6             | .55107  | 1599                        | .64476  | 670                         | 8             | .96895  | 761                         | .03207  | 819                         |
| 7             | .53508  | 1571                        | .65146  | 674                         | 9             | .96134  | 756                         | .04026  | 820                         |
| 8             | .51937  | 1543                        | .65820  | 676                         | 24 0          | .95378  | 740                         | .04846  | 824                         |
| 9             | .50394  | 1529                        | .66496  | 679                         | 1             | .94638  | 735                         | .05670  | 825                         |
| 19 0          | .48865  | 1490                        | .67175  | 683                         | 2             | .93903  | 724                         | .06495  | 829                         |
| 1             | .47375  | 1477                        | .67858  | 686                         | 3             | .93179  | 716                         | .07324  | 831                         |
| 2             | .45898  | 1450                        | .68544  | 688                         | 4             | .92463  | 710                         | .08155  | 833                         |
| 3             | .44448  | 1427                        | .69232  | 692                         | 5             | .91753  | 695                         | .08988  | 837                         |
| 4             | .43021  | 1414                        | .69924  | 694                         | 6             | .91058  | 691                         | .09825  | 838                         |
| 5             | .41607  | 1378                        | .70618  | 698                         | 7             | .90367  | 682                         | .10663  | 841                         |
| 6             | .40229  | 1367                        | .71316  | 701                         | 8             | .89685  | 673                         | .11504  | 844                         |
| 7             | .38862  | 1343                        | .72017  | 703                         | 9             | .89012  | 668                         | .12348  | 845                         |
| 8             | .37519  | 1321                        | .72720  | 707                         | 25 0          | .88344  | 655                         | .13193  | 850                         |
| 9             | .36198  | 1310                        | .73427  | 709                         | 1             | .87669  | 650                         | .14043  | 850                         |
| 20 0          | .34888  | 1278                        | .74136  | 713                         | 2             | .87039  | 641                         | .14893  | 854                         |
| 1             | .33610  | 1267                        | .74849  | 715                         | 3             | .86398  | 634                         | .15747  | 856                         |
| 2             | .32343  | 1247                        | .75564  | 719                         | 4             | .85764  | 629                         | .16603  | 858                         |
| 3             | .31096  | 1225                        | .76283  | 721                         | 5             | .85135  | 617                         | .17461  | 862                         |
| 4             | .29871  | 1216                        | .77004  | 723                         | 6             | .84518  | 614                         | .18323  | 863                         |
| 5             | .28655  | 1187                        | .77727  | 728                         | 7             | .83904  | 604                         | .19186  | 865                         |
| 6             | .27468  | 1177                        | .78455  | 730                         | 8             | .83300  | 598                         | .20051  | 869                         |
| 7             | .26291  | 1158                        | .79185  | 734                         | 9             | .82702  | 593                         | .20920  | 869                         |
| 8             | .25133  | 1140                        | .79919  | 736                         | 26 0          | .82109  | 582                         | .21789  | 874                         |
| 9             | .23993  | 1131                        | .80655  | 737                         | 1             | .81527  | 578                         | .22663  | 875                         |
| 21 0          | .22862  | 1104                        | .81392  | 743                         | 2             | .80949  | 571                         | .23538  | 877                         |
| 1             | .21758  | 1095                        | .82135  | 744                         | 3             | .80378  | 564                         | .24415  | 881                         |
| 2             | .20663  | 1079                        | .82879  | 747                         | 4             | .79814  | 560                         | .25296  | 881                         |
| 3             | .19584  | 1061                        | .83626  | 750                         | 5             | .79254  | 550                         | .26177  | 885                         |
| 4             | .18523  | 1053                        | .84376  | 752                         | 6             | .78704  | 546                         | .27062  | 886                         |
| 5             | .17470  | 1029                        | .85128  | 757                         | 7             | .78158  | 539                         | .27948  | 890                         |
| 6             | .16441  | 1021                        | .85885  | 759                         | 8             | .77619  | 533                         | .28838  | 891                         |
| 7             | .15420  | 1006                        | .86644  | 761                         | 9             | .77086  | 529                         | .29729  | 893                         |
| 8             | .14414  | 990                         | .87405  | 764                         | 27 0          | .76557  | 520                         | .30622  | 897                         |
| 9             | .13424  | 983                         | .88169  | 766                         | 1             | .76037  | 516                         | .31519  | 898                         |
| 22 0          | .12441  | 961                         | .88935  | 771                         | 2             | .75521  | 511                         | .32417  | 901                         |
| 1             | .11480  | 953                         | .89706  | 772                         | 3             | .75010  | 504                         | .33318  | 903                         |
| 2             | .10527  | 939                         | .90478  | 776                         | 4             | .74506  | 500                         | .34221  | 904                         |
| 3             | .09588  | 925                         | .91254  | 778                         | 5             | .74006  | 492                         | .35125  | 908                         |
| 4             | .08663  | 918                         | .92032  | 780                         | 6             | .73514  | 489                         | .36033  | 909                         |
| 5             | .07745  | 898                         | .92812  | 784                         | 7             | .73025  | 483                         | .36942  | 911                         |
| 6             | .06847  | 892                         | .93596  | 786                         | 8             | .72542  | 477                         | .37853  | 914                         |
| 7             | .05955  | 878                         | .94382  | 789                         | 9             | .72065  | 474                         | .38767  | 916                         |
| 8             | .05077  | 866                         | .95171  | 792                         | 28 0          | .71591  | 466                         | .39683  | 919                         |
| 9             | .04211  | 859                         | .95963  | 793                         | 1             | .71125  | 463                         | .40602  | 919                         |
| 23 0          | .03352  | 841                         | .96756  | 798                         | 2             | .70662  | 457                         | .41521  | 923                         |
| 1             | .02511  | 834                         | .97554  | 800                         | 3             | .70205  | 453                         | .42444  | 925                         |
| 2             | .01677  | 823                         | .98354  | 802                         | 4             | .69752  | 449                         | .43369  | 926                         |
| 3             | .00854  | 811                         | .99156  | 806                         | 5             | .69303  | 444                         | .44295  | 929                         |
| 4             | .00043  | 805                         | .99962  | 806                         | 6             | .68859  |                             | .45224  |                             |
| 5             | 0.99238 |                             | 1.00768 |                             | 28° 36' 6"    | .68852  |                             | .45239  |                             |

COMPUTATIONS.

[PART VI.]

TABLE XVIII. (5)—Values of  $1/f$  and  $f$  corresponding to various values of  $I$ , to be used in computing H and R for the Tide Mm.

$$\text{Argument } 1/f = \frac{(1 - \frac{3}{4} \sin^2 \omega) (1 - \frac{3}{4} \sin^2 i)}{1 - \frac{3}{4} \sin^2 I}$$

| Values of $I$ | $1/f$   | Differences for 0°.1 of $I$ | $f$     | Differences for 0°.1 of $I$ | Values of $I$ | $1/f$   | Differences for 0°.1 of $I$ | $f$     | Differences for 0°.1 of $I$ |
|---------------|---------|-----------------------------|---------|-----------------------------|---------------|---------|-----------------------------|---------|-----------------------------|
| 18° 18' 30"   | 0.88401 |                             | 1.13121 |                             | 23° 5         | 0.98904 | 251                         | 1.01108 | 255                         |
| 18 4          | .88550  | 163                         | .12931  | 208                         | 6             | .99155  | 252                         | .00853  | 256                         |
| 5             | .88713  | 166                         | .12723  | 211                         | 7             | .99407  | 253                         | .00597  | 256                         |
| 6             | .88879  | 167                         | .12512  | 211                         | 8             | .99660  | 257                         | .00341  | 258                         |
| 7             | .89046  | 169                         | .12301  | 211                         | 9             | .99917  | 257                         | .00083  | 258                         |
| 8             | .89215  | 169                         | .12090  | 213                         | 24 0          | 1.00174 | 261                         | 0.99826 | 257                         |
| 9             | .89384  | 170                         | .11877  | 212                         | 1             | .00435  | 262                         | .99567  | 259                         |
| 19 0          | .89554  | 173                         | .11665  | 215                         | 2             | .00697  | 265                         | .99308  | 259                         |
| 1             | .89727  | 173                         | .11450  | 216                         | 3             | .00962  | 267                         | .99048  | 260                         |
| 2             | .89900  | 176                         | .11234  | 216                         | 4             | .01229  | 269                         | .98786  | 262                         |
| 3             | .90076  | 176                         | .11018  | 218                         | 5             | .01498  | 272                         | .98524  | 262                         |
| 4             | .90252  | 178                         | .10800  | 218                         | 6             | .01770  | 274                         | .98261  | 263                         |
| 5             | .90430  | 181                         | .10582  | 220                         | 7             | .02044  | 275                         | .97998  | 263                         |
| 6             | .90611  | 181                         | .10362  | 220                         | 8             | .02319  | 279                         | .97734  | 264                         |
| 7             | .90792  | 182                         | .10142  | 220                         | 9             | .02598  | 280                         | .97469  | 265                         |
| 8             | .90974  | 185                         | .09922  | 223                         | 25 0          | .02878  | 284                         | .97203  | 266                         |
| 9             | .91159  | 185                         | .09699  | 223                         | 1             | .03162  | 285                         | .96936  | 267                         |
| 20 0          | .91344  | 187                         | .09476  | 224                         | 2             | .03447  | 288                         | .96669  | 267                         |
| 1             | .91531  | 188                         | .09252  | 224                         | 3             | .03735  | 290                         | .96400  | 269                         |
| 2             | .91719  | 191                         | .09028  | 225                         | 4             | .04025  | 292                         | .96131  | 269                         |
| 3             | .91910  | 192                         | .08803  | 227                         | 5             | .04317  | 296                         | .95862  | 271                         |
| 4             | .92102  | 193                         | .08576  | 228                         | 6             | .04613  | 297                         | .95591  | 271                         |
| 5             | .92295  | 196                         | .08348  | 229                         | 7             | .04910  | 300                         | .95320  | 272                         |
| 6             | .92491  | 196                         | .08119  | 229                         | 8             | .05210  | 303                         | .95048  | 272                         |
| 7             | .92687  | 198                         | .07890  | 230                         | 9             | .05513  | 305                         | .94776  | 274                         |
| 8             | .92885  | 200                         | .07660  | 231                         | 26 0          | .05818  | 309                         | .94502  | 275                         |
| 9             | .93085  | 201                         | .07429  | 232                         | 1             | .06127  | 310                         | .94227  | 275                         |
| 21 0          | .93286  | 204                         | .07197  | 233                         | 2             | .06437  | 314                         | .93952  | 275                         |
| 1             | .93490  | 205                         | .06964  | 234                         | 3             | .06751  | 316                         | .93677  | 277                         |
| 2             | .93695  | 207                         | .06730  | 236                         | 4             | .07067  | 318                         | .93400  | 277                         |
| 3             | .93902  | 208                         | .06494  | 235                         | 5             | .07385  | 322                         | .93123  | 278                         |
| 4             | .94110  | 209                         | .06259  | 236                         | 6             | .07707  | 324                         | .92845  | 279                         |
| 5             | .94319  | 213                         | .06023  | 238                         | 7             | .08031  | 328                         | .92566  | 279                         |
| 6             | .94532  | 214                         | .05785  | 239                         | 8             | .08359  | 330                         | .92286  | 280                         |
| 7             | .94746  | 214                         | .05546  | 238                         | 9             | .08689  | 332                         | .92006  | 280                         |
| 8             | .94960  | 218                         | .05308  | 241                         | 27 0          | .09021  | 337                         | .91726  | 282                         |
| 9             | .95178  | 219                         | .05067  | 242                         | 1             | .09358  | 338                         | .91444  | 282                         |
| 22 0          | .95397  | 221                         | .04825  | 242                         | 2             | .09696  | 341                         | .91162  | 284                         |
| 1             | .95618  | 222                         | .04583  | 242                         | 3             | .10037  | 345                         | .90878  | 284                         |
| 2             | .95840  | 224                         | .04341  | 243                         | 4             | .10382  | 347                         | .90594  | 284                         |
| 3             | .96064  | 226                         | .04098  | 245                         | 5             | .10729  | 352                         | .90310  | 285                         |
| 4             | .96290  | 228                         | .03853  | 245                         | 6             | .11081  | 353                         | .90025  | 286                         |
| 5             | .96518  | 230                         | .03608  | 246                         | 7             | .11434  | 357                         | .89739  | 286                         |
| 6             | .96748  | 232                         | .03362  | 247                         | 8             | .11791  | 361                         | .89453  | 288                         |
| 7             | .96980  | 234                         | .03115  | 249                         | 9             | .12152  | 362                         | .89165  | 287                         |
| 8             | .97214  | 236                         | .02866  | 249                         | 28 0          | .12514  | 367                         | .88878  | 289                         |
| 9             | .97450  | 237                         | .02617  | 249                         | 1             | .12881  | 369                         | .88589  | 289                         |
| 23 0          | .97687  | 239                         | .02368  | 250                         | 2             | .13250  | 374                         | .88300  | 289                         |
| 1             | .97926  | 241                         | .02118  | 251                         | 3             | .13624  | 376                         | .88011  | 291                         |
| 2             | .98167  | 245                         | .01867  | 253                         | 4             | .14000  | 378                         | .87720  | 291                         |
| 3             | .98412  | 245                         | .01614  | 253                         | 5             | .14378  | 385                         | .87429  | 291                         |
| 4             | .98657  | 247                         | .01361  | 253                         | 6             | .14763  |                             | .87137  | 292                         |
| 5             | .98904  |                             | .01108  | 253                         | 28° 36' 6"    | .14769  |                             | .87132  |                             |

TABLE XVIII. (6)—Values of  $1/f$  and  $f$  corresponding to various values of  $I$ , to be used in computing H and R for the Tide  $K_1$ .

$$\text{Argument } 1/f = \frac{1.46407 \times k_1}{\{1 + (0.46407 \times k_1)^2 + 0.92814 k_1 \cos \nu\}^{1/2}} \text{ where } k_1 = \frac{\sin \omega \cos \omega (1 - \frac{1}{2} \sin^2 \iota)}{\sin I \cos I}$$

| Values of $I$ | $1/f$   | Differences for 0°.1 of $I$ | $f$     | Differences for 0°.1 of $I$ | Values of $I$ | $1/f$   | Differences for 0°.1 of $I$ | $f$     | Differences for 0°.1 of $I$ |
|---------------|---------|-----------------------------|---------|-----------------------------|---------------|---------|-----------------------------|---------|-----------------------------|
| 18° 18' 30"   | 1.13424 |                             | 0.88165 |                             | 23° 5         | 0.99619 |                             | 1.00383 | 226                         |
| 18 4          | .13142  | 307                         | .88385  | 240                         | 6             | .99395  | 224                         | .00609  | 226                         |
| 5             | .12835  | 303                         | .88625  | 238                         | 7             | .99172  | 222                         | .00835  | 227                         |
| 6             | .12532  | 303                         | .88863  | 240                         | 8             | .98950  | 220                         | .01062  | 224                         |
| 7             | .12229  | 302                         | .89103  | 241                         | 9             | .98730  | 220                         | .01286  | 226                         |
| 8             | .11927  | 299                         | .89344  | 239                         | 24 0          | .98510  | 217                         | .01512  | 225                         |
| 9             | .11628  | 299                         | .89583  | 241                         | 1             | .98293  | 217                         | .01737  | 225                         |
| 19 0          | .11329  | 294                         | .89824  | 237                         | 2             | .98076  | 214                         | .01962  | 222                         |
| 1             | .11035  | 295                         | .90061  | 240                         | 3             | .97862  | 213                         | .02184  | 224                         |
| 2             | .10740  | 292                         | .90301  | 239                         | 4             | .97649  | 214                         | .02408  | 224                         |
| 3             | .10448  | 290                         | .90540  | 239                         | 5             | .97435  | 208                         | .02632  | 221                         |
| 4             | .10158  | 291                         | .90779  | 240                         | 6             | .97227  | 210                         | .02853  | 222                         |
| 5             | .09867  | 287                         | .91019  | 238                         | 7             | .97017  | 209                         | .03075  | 222                         |
| 6             | .09580  | 286                         | .91257  | 239                         | 8             | .96808  | 206                         | .03297  | 220                         |
| 7             | .09294  | 283                         | .91496  | 238                         | 9             | .96602  | 206                         | .03517  | 221                         |
| 8             | .09011  | 282                         | .91734  | 238                         | 25 0          | .96396  | 202                         | .03738  | 219                         |
| 9             | .08729  | 283                         | .91972  | 240                         | 1             | .96194  | 203                         | .03957  | 220                         |
| 20 0          | .08446  | 278                         | .92212  | 237                         | 2             | .95991  | 201                         | .04177  | 218                         |
| 1             | .08168  | 279                         | .92449  | 239                         | 3             | .95790  | 200                         | .04395  | 218                         |
| 2             | .07889  | 275                         | .92688  | 237                         | 4             | .95590  | 199                         | .04613  | 219                         |
| 3             | .07614  | 274                         | .92925  | 237                         | 5             | .95391  | 196                         | .04832  | 216                         |
| 4             | .07340  | 273                         | .93162  | 238                         | 6             | .95195  | 196                         | .05048  | 217                         |
| 5             | .07067  | 271                         | .93400  | 237                         | 7             | .94999  | 195                         | .05265  | 215                         |
| 6             | .06796  | 269                         | .93637  | 236                         | 8             | .94804  | 192                         | .05480  | 215                         |
| 7             | .06527  | 269                         | .93873  | 238                         | 9             | .94612  | 193                         | .05695  | 216                         |
| 8             | .06258  | 265                         | .94111  | 235                         | 26 0          | .94419  | 191                         | .05911  | 215                         |
| 9             | .05993  | 266                         | .94346  | 237                         | 1             | .94228  | 189                         | .06126  | 213                         |
| 21 0          | .05727  | 262                         | .94583  | 235                         | 2             | .94039  | 187                         | .06339  | 211                         |
| 1             | .05465  | 262                         | .94818  | 236                         | 3             | .93852  | 186                         | .06550  | 213                         |
| 2             | .05203  | 259                         | .95054  | 235                         | 4             | .93666  | 187                         | .06763  | 213                         |
| 3             | .04944  | 257                         | .95289  | 234                         | 5             | .93479  | 184                         | .06976  | 211                         |
| 4             | .04687  | 258                         | .95523  | 236                         | 6             | .93295  | 183                         | .07187  | 210                         |
| 5             | .04429  | 254                         | .95759  | 234                         | 7             | .93112  | 182                         | .07397  | 211                         |
| 6             | .04175  | 256                         | .95993  | 235                         | 8             | .92930  | 180                         | .07608  | 209                         |
| 7             | .03919  | 250                         | .96228  | 233                         | 9             | .92750  | 180                         | .07817  | 209                         |
| 8             | .03669  | 250                         | .96461  | 233                         | 27 0          | .92570  | 177                         | .08026  | 207                         |
| 9             | .03419  | 251                         | .96694  | 235                         | 1             | .92393  | 178                         | .08233  | 209                         |
| 22 0          | .03168  | 246                         | .96929  | 232                         | 2             | .92215  | 173                         | .08442  | 205                         |
| 1             | .02922  | 245                         | .97161  | 232                         | 3             | .92042  | 175                         | .08647  | 206                         |
| 2             | .02677  | 246                         | .97393  | 233                         | 4             | .91867  | 174                         | .08853  | 206                         |
| 3             | .02431  | 242                         | .97626  | 232                         | 5             | .91693  | 172                         | .09059  | 205                         |
| 4             | .02189  | 241                         | .97858  | 231                         | 6             | .91521  | 170                         | .09264  | 204                         |
| 5             | .01948  | 238                         | .98089  | 230                         | 7             | .91351  | 170                         | .09468  | 204                         |
| 6             | .01710  | 239                         | .98319  | 231                         | 8             | .91181  | 168                         | .09672  | 202                         |
| 7             | .01471  | 237                         | .98550  | 231                         | 9             | .91013  | 167                         | .09874  | 203                         |
| 8             | .01234  | 235                         | .98781  | 230                         | 28 0          | .90846  | 166                         | .10077  | 201                         |
| 9             | .00999  | 235                         | .99011  | 230                         | 1             | .90680  | 165                         | .10278  | 201                         |
| 23 0          | .00764  | 230                         | .99241  | 228                         | 2             | .90515  | 162                         | .10479  | 198                         |
| 1             | .00534  | 231                         | .99469  | 229                         | 3             | .90353  | 163                         | .10677  | 200                         |
| 2             | .00303  | 228                         | .99698  | 228                         | 4             | .90190  | 162                         | .10877  | 200                         |
| 3             | .00075  | 229                         | .99926  | 228                         | 5             | .90028  | 160                         | .11077  | 198                         |
| 4             | 0.99846 | 227                         | 1.00154 | 229                         | 6             | .89868  |                             | .11275  |                             |
| 5             | .99610  |                             | .00383  |                             | 28° 36' 6"    | .89865  |                             | .11279  |                             |

COMPUTATIONS.

[PART VI.]

TABLE XVIII. (7)—Values of  $1/f$  and  $f$  corresponding to various values of  $I$ , to be used in computing H and R for the Tide  $K_2$ .

$$\text{Argument } 1/f = \frac{1.46407 \times k_2}{\{1 + (0.46407 \times k_2)^2 + 0.92814k_2 \cos 2\nu\}^{\frac{1}{2}}} \quad \text{where } k_2 = \frac{\sin^2 \omega (1 - \frac{3}{2} \sin^2 i)}{\sin^2 I}$$

| Values of $I$ | $1/f$   | Differences for 0°.1 of $I$ | $f$     | Differences for 0°.1 of $I$ | Values of $I$ | $1/f$   | Differences for 0°.1 of $I$ | $f$     | Differences for 0°.1 of $I$ |
|---------------|---------|-----------------------------|---------|-----------------------------|---------------|---------|-----------------------------|---------|-----------------------------|
| 18° 18' 30"   | 1.33764 |                             | 0.74759 |                             | 23° 5         | 1.01137 | 571                         | 0.98876 | 561                         |
| 18.4          | .33173  | 655                         | .75090  | 372                         | .6            | .00566  | 570                         | .99437  | 567                         |
| .5            | .32518  | 647                         | .75462  | 370                         | .7            | 0.99996 | 567                         | 1.00004 | 570                         |
| .6            | .31871  | 653                         | .75832  | 377                         | .8            | .99429  | 564                         | .00574  | 574                         |
| .7            | .31218  | 654                         | .76209  | 382                         | .9            | .98865  | 561                         | .01148  | 577                         |
| .8            | .30564  | 649                         | .76591  | 383                         | 24 0          | .98304  | 556                         | .01725  | 578                         |
| .9            | .29915  | 658                         | .76974  | 391                         | .1            | .97748  | 554                         | .02303  | 584                         |
| 19 0          | .29257  | 649                         | .77365  | 391                         | .2            | .97194  | 551                         | .02887  | 587                         |
| .1            | .28608  | 655                         | .77756  | 398                         | .3            | .96643  | 548                         | .03474  | 590                         |
| .2            | .27953  | 652                         | .78154  | 400                         | .4            | .96095  | 547                         | .04064  | 595                         |
| .3            | .27301  | 651                         | .78554  | 404                         | .5            | .95548  | 540                         | .04659  | 595                         |
| .4            | .26650  | 657                         | .78958  | 411                         | .6            | .95008  | 539                         | .05254  | 601                         |
| .5            | .25993  | 648                         | .79369  | 411                         | .7            | .94469  | 537                         | .05855  | 604                         |
| .6            | .25345  | 652                         | .79780  | 417                         | .8            | .93932  | 531                         | .06459  | 606                         |
| .7            | .24693  | 651                         | .80197  | 421                         | .9            | .93401  | 530                         | .07065  | 612                         |
| .8            | .24042  | 648                         | .80618  | 423                         | 25 0          | .92871  | 530                         | .07677  | 610                         |
| .9            | .23394  | 654                         | .81041  | 432                         | .1            | .92347  | 524                         | .08287  | 618                         |
| 20 0          | .22740  | 644                         | .81473  | 430                         | .2            | .91823  | 524                         | .08905  | 621                         |
| .1            | .22096  | 648                         | .81903  | 437                         | .3            | .91303  | 520                         | .09526  | 621                         |
| .2            | .21448  | 648                         | .82340  | 442                         | .4            | .90788  | 515                         | .10147  | 628                         |
| .3            | .20800  | 643                         | .82782  | 442                         | .5            | .90273  | 515                         | .10775  | 628                         |
| .4            | .20157  | 647                         | .83224  | 451                         | .6            | .89765  | 508                         | .11403  | 633                         |
| .5            | .19510  | 638                         | .83675  | 449                         | .7            | .89257  | 508                         | .12036  | 635                         |
| .6            | .18872  | 644                         | .84124  | 458                         | .8            | .88754  | 503                         | .12671  | 638                         |
| .7            | .18228  | 638                         | .84582  | 460                         | .9            | .88254  | 500                         | .13309  | 645                         |
| .8            | .17590  | 638                         | .85042  | 463                         | 26 0          | .87755  | 499                         | .13954  | 642                         |
| .9            | .16952  | 639                         | .85505  | 470                         | .1            | .87263  | 492                         | .14596  | 649                         |
| 21 0          | .16313  | 631                         | .85975  | 469                         | .2            | .86772  | 491                         | .15245  | 651                         |
| .1            | .15682  | 633                         | .86444  | 475                         | .3            | .86284  | 488                         | .15896  | 653                         |
| .2            | .15049  | 631                         | .86919  | 480                         | .4            | .85801  | 483                         | .16549  | 661                         |
| .3            | .14418  | 628                         | .87399  | 482                         | .5            | .85317  | 484                         | .17210  | 657                         |
| .4            | .13790  | 629                         | .87881  | 489                         | .6            | .84842  | 475                         | .17867  | 664                         |
| .5            | .13161  | 622                         | .88370  | 488                         | .7            | .84366  | 476                         | .18531  | 669                         |
| .6            | .12539  | 624                         | .88858  | 496                         | .8            | .83893  | 473                         | .19200  | 668                         |
| .7            | .11915  | 619                         | .89354  | 497                         | .9            | .83425  | 468                         | .19868  | 674                         |
| .8            | .11296  | 618                         | .89851  | 501                         | 27 0          | .82959  | 466                         | .20542  | 672                         |
| .9            | .10678  | 618                         | .90352  | 508                         | .1            | .82499  | 460                         | .21214  | 680                         |
| 22 0          | .10060  | 611                         | .90860  | 506                         | .2            | .82039  | 460                         | .21894  | 681                         |
| .1            | .09449  | 611                         | .91366  | 514                         | .3            | .81583  | 456                         | .22575  | 683                         |
| .2            | .08838  | 608                         | .91880  | 516                         | .4            | .81131  | 452                         | .23258  | 690                         |
| .3            | .08230  | 606                         | .92396  | 520                         | .5            | .80679  | 452                         | .23948  | 688                         |
| .4            | .07624  | 605                         | .92916  | 526                         | .6            | .80234  | 445                         | .24636  | 694                         |
| .5            | .07019  | 598                         | .93442  | 524                         | .7            | .79789  | 445                         | .25330  | 693                         |
| .6            | .06421  | 598                         | .93966  | 532                         | .8            | .79351  | 438                         | .26023  | 700                         |
| .7            | .05823  | 597                         | .94498  | 535                         | .9            | .78912  | 439                         | .26723  | 702                         |
| .8            | .05226  | 591                         | .95033  | 537                         | 28 0          | .78477  | 435                         | .27425  | 702                         |
| .9            | .04635  | 592                         | .95570  | 544                         | .1            | .78048  | 429                         | .28127  | 707                         |
| 23 0          | .04043  | 586                         | .96114  | 544                         | .2            | .77619  | 429                         | .28834  | 712                         |
| .1            | .03457  | 584                         | .96658  | 549                         | .3            | .77193  | 426                         | .29546  | 711                         |
| .2            | .02873  | 581                         | .97207  | 552                         | .4            | .76772  | 421                         | .30257  | 717                         |
| .3            | .02292  | 578                         | .97759  | 556                         | .5            | .76351  | 421                         | .30974  | 717                         |
| .4            | .01714  | 577                         | .98315  | 561                         | .6            | .75934  | 417                         | .31693  | 719                         |
| .5            | .01137  |                             | .98876  |                             | 28° 36' 6"    | .75928  |                             | .31703  |                             |



TABLE XIX.—Values of  $\nu'$  corresponding to  $I$ , to determine initial argument of Tide  $K_1$ .

$$\tan \nu' = \frac{\sin \nu}{\cos \nu + 0.46407 \times k_1} \quad \text{where } k_1 = \frac{\sin \omega \cos \omega (1 - \frac{2}{3} \sin^2 \omega)}{\sin I \cos I}$$

| $I$      | $\nu'$ | Differences for 0°.1 of $I$ | $I$         | $\nu'$ | Differences for 0°.1 of $I$ | $I$         | $\nu'$ | Differences for 0°.1 of $I$ | $I$         | $\nu'$ | Differences for 0°.1 of $I$ | $I$        | $\nu'$ | Differences for 0°.1 of $I$ |
|----------|--------|-----------------------------|-------------|--------|-----------------------------|-------------|--------|-----------------------------|-------------|--------|-----------------------------|------------|--------|-----------------------------|
| 18 18 30 | 0° 000 |                             | 20° 47' 363 |        | 0° 124                      | 22° 58' 819 |        | 0° 021                      | 24° 68' 567 |        | 0° 049                      | 26° 7      | 6° 672 | 0° 145                      |
| 18° 4    | 1° 557 | 0° 958                      | 57' 487     |        | 112                         | 68' 840     |        | 019                         | 78' 518     |        | 053                         | 8          | 6° 527 | 155                         |
| 5        | 2° 515 | 534                         | 67' 599     |        | 109                         | 78' 859     |        | 016                         | 88' 465     |        | 057                         | 9          | 6° 372 | 158                         |
| 6        | 3° 049 | 491                         | 77' 708     |        | 103                         | 88' 875     |        | 011                         | 98' 408     |        | 059                         | 27° 0      | 6° 214 | 172                         |
| 7        | 3° 540 | 406                         | 87' 811     |        | 096                         | 98' 886     |        | 010                         | 25° 08' 349 |        | 065                         | 1          | 6° 042 | 177                         |
| 8        | 3° 946 | 358                         | 97' 907     |        | 093                         | 23° 08' 896 |        | 004                         | 18' 284     |        | 068                         | 2          | 5° 865 | 187                         |
| 9        | 4° 304 | 334                         | 21° 08' 000 |        | 085                         | 18' 900     |        | 001                         | 28' 216     |        | 071                         | 3          | 5° 678 | 200                         |
| 19° 0    | 4° 638 | 286                         | 18' 085     |        | 081                         | 28' 901     |        | 001                         | 38' 145     |        | 076                         | 4          | 5° 478 | 206                         |
| 1        | 4° 924 | 274                         | 28' 166     |        | 077                         | 38' 900     |        | 006                         | 48' 069     |        | 078                         | 5          | 5° 272 | 228                         |
| 2        | 5° 198 | 251                         | 38' 243     |        | 070                         | 48' 894     |        | 008                         | 57' 991     |        | 085                         | 6          | 5° 044 | 237                         |
| 3        | 5° 449 | 231                         | 48' 313     |        | 068                         | 58' 886     |        | 013                         | 67' 906     |        | 088                         | 7          | 4° 807 | 256                         |
| 4        | 5° 680 | 223                         | 58' 381     |        | 061                         | 68' 873     |        | 015                         | 77' 818     |        | 092                         | 8          | 4° 551 | 280                         |
| 5        | 5° 903 | 199                         | 68' 442     |        | 059                         | 78' 858     |        | 019                         | 87' 726     |        | 097                         | 9          | 4° 271 | 292                         |
| 6        | 6° 102 | 193                         | 78' 501     |        | 053                         | 88' 839     |        | 022                         | 97' 629     |        | 100                         | 28° 0      | 3° 979 | 344                         |
| 7        | 6° 295 | 180                         | 88' 554     |        | 049                         | 98' 817     |        | 025                         | 26° 07' 529 |        | 107                         | 1          | 3° 635 | 372                         |
| 8        | 6° 475 | 169                         | 98' 603     |        | 047                         | 24° 08' 792 |        | 029                         | 17' 422     |        | 111                         | 2          | 3° 263 | 427                         |
| 9        | 6° 644 | 163                         | 22° 08' 650 |        | 040                         | 18' 763     |        | 032                         | 27' 311     |        | 116                         | 3          | 2° 836 | 553                         |
| 20° 0    | 6° 807 | 149                         | 18' 690     |        | 038                         | 28' 731     |        | 036                         | 37' 195     |        | 123                         | 4          | 2° 283 | 614                         |
| 1        | 6° 956 | 144                         | 28' 728     |        | 034                         | 38' 695     |        | 040                         | 47' 072     |        | 125                         | 5          | 1° 669 | 642                         |
| 2        | 7° 100 | 136                         | 38' 762     |        | 029                         | 48' 655     |        | 041                         | 56' 947     |        | 136                         | 6          | 0° 027 |                             |
| 3        | 7° 236 | 127                         | 48' 791     |        | 028                         | 58' 614     |        | 047                         | 66' 811     |        | 139                         | 28° 36' 6" | 0° 000 |                             |
| 4        | 7° 363 |                             | 58' 819     |        |                             | 68' 567     |        |                             | 76' 672     |        |                             |            |        |                             |

N.B.—In the above table  $\nu'$  is positive when  $N$  is between 0° and 180°, and negative when  $N$  is between 180° and 360°; thus it is necessary to observe what is the value of  $N$ , because  $I$  is always positive.

COMPUTATIONS.

[PART VI.]

TABLE XX.—Values of  $2\nu''$  corresponding to  $I$ , to determine initial argument of Tide  $K_2$ .

$$\tan 2\nu'' = \frac{\sin 2\nu}{\cos 2\nu + 0.46407 \times k_2} \quad \text{where } k_2 = \frac{\sin^2 \omega (1 - \frac{1}{2} \sin^2 \epsilon)}{\sin^2 I}$$

| $I$      | $2\nu''$ | Differences for $0.1$ of $I$ | $I$         | $2\nu''$ | Differences for $0.1$ of $I$ | $I$         | $2\nu''$ | Differences for $0.1$ of $I$ | $I$         | $2\nu''$ | Differences for $0.1$ of $I$ | $I$         | $2\nu''$ | Differences for $0.1$ of $I$ |
|----------|----------|------------------------------|-------------|----------|------------------------------|-------------|----------|------------------------------|-------------|----------|------------------------------|-------------|----------|------------------------------|
| 18 18 30 | 0'000    |                              | 20 4 13 963 |          | 0 265                        | 22 5 17 422 |          | 0 072                        | 24 6 17 463 |          | 0 078                        | 26 7        | 13 931   | 0 291                        |
| 18 4     | 2 810    | 1 738                        | 5 14 228    |          | 246                          | 6 17 494    |          | 0 068                        | 7 17 385    |          | 0 086                        | 8           | 13 640   | 3 09                         |
| 5        | 4 548    | 0 983                        | 6 14 474    |          | 240                          | 7 17 562    |          | 0 059                        | 8 17 299    |          | 0 095                        | 9           | 13 331   | 3 19                         |
| 6        | 5 531    | 907                          | 7 14 714    |          | 227                          | 8 17 621    |          | 0 052                        | 9 17 204    |          | 0 099                        | 27 0        | 13 012   | 3 48                         |
| 7        | 6 438    | 758                          | 8 14 941    |          | 216                          | 9 17 673    |          | 0 047                        | 25 0 17 105 |          | 1 113                        | 1           | 12 664   | 3 59                         |
| 8        | 7 196    | 674                          | 9 15 157    |          | 210                          | 23 0 17 720 |          | 0 035                        | 1 16 992    |          | 1 117                        | 2           | 12 305   | 3 83                         |
| 9        | 7 870    | 632                          | 21 0 15 367 |          | 194                          | 1 17 755    |          | 0 032                        | 2 16 875    |          | 1 128                        | 3           | 11 922   | 4 09                         |
| 19 0     | 8 502    | 549                          | 1 15 561    |          | 189                          | 2 17 787    |          | 0 023                        | 3 16 747    |          | 1 136                        | 4           | 11 513   | 4 23                         |
| 1        | 9 051    | 528                          | 2 15 750    |          | 179                          | 3 17 810    |          | 0 016                        | 4 16 611    |          | 1 142                        | 5           | 11 090   | 4 70                         |
| 2        | 9 579    | 488                          | 3 15 929    |          | 168                          | 4 17 826    |          | 0 012                        | 5 16 469    |          | 1 157                        | 6           | 10 620   | 4 91                         |
| 3        | 10 067   | 454                          | 4 16 097    |          | 164                          | 5 17 838    |          | 0 000                        | 6 16 312    |          | 1 162                        | 7           | 10 129   | 5 29                         |
| 4        | 10 521   | 438                          | 5 16 261    |          | 149                          | 6 17 838    |          | 0 004                        | 7 16 150    |          | 1 172                        | 8           | 9 600    | 5 83                         |
| 5        | 10 959   | 399                          | 6 16 410    |          | 145                          | 7 17 834    |          | 0 012                        | 8 15 978    |          | 1 184                        | 9           | 9 017    | 6 09                         |
| 6        | 11 358   | 388                          | 7 16 555    |          | 136                          | 8 17 822    |          | 0 020                        | 9 15 794    |          | 1 189                        | 28 0        | 8 408    | 7 20                         |
| 7        | 11 746   | 364                          | 8 16 691    |          | 126                          | 9 17 802    |          | 0 024                        | 26 0 15 605 |          | 2 07                         | 1           | 7 688    | 7 80                         |
| 8        | 12 110   | 346                          | 9 16 817    |          | 122                          | 24 0 17 778 |          | 0 036                        | 1 15 398    |          | 2 13                         | 2           | 6 908    | 9 01                         |
| 9        | 12 456   | 336                          | 22 0 16 939 |          | 109                          | 1 17 742    |          | 0 040                        | 2 15 185    |          | 2 26                         | 3           | 6 007    | 1 166                        |
| 20 0     | 12 792   | 310                          | 1 17 048    |          | 105                          | 2 17 702    |          | 0 049                        | 3 14 959    |          | 2 39                         | 4           | 4 841    | 1 300                        |
| 1        | 13 102   | 303                          | 2 17 153    |          | 97                           | 3 17 653    |          | 0 056                        | 4 14 720    |          | 2 45                         | 5           | 3 541    | 3 483                        |
| 2        | 13 405   | 286                          | 3 17 250    |          | 88                           | 4 17 597    |          | 0 061                        | 5 14 475    |          | 2 68                         | 6           | 0 058    |                              |
| 3        | 13 691   | 272                          | 4 17 338    |          | 84                           | 5 17 536    |          | 0 073                        | 6 14 207    |          | 2 76                         | 28 0 36 6'' | 0 00     |                              |
| 4        | 13 963   |                              | 5 17 422    |          |                              | 6 17 463    |          |                              | 7 13 931    |          |                              |             |          |                              |

N.B.—In the above table  $2\nu''$  is positive when  $N$  is between  $0^\circ$  and  $180^\circ$ , and negative when  $N$  is between  $180^\circ$  and  $360^\circ$ ; thus it is necessary to observe what is the value of  $N$ , because  $I$  is always positive.

PART VII.

Levelling Operations.

CHAPTER I. INSTRUMENTS AND THEIR ADJUSTMENTS.

1. The lines of levels executed by the Trigonometrical Branch of the Survey of India Department are generally of very great length, and the only independent means of verifying their accuracy is by comparison with the sea-level at their extremities: consequently, it is of the utmost importance that errors of every description be eliminated, continually, during the progress of the levelling, a matter which cannot be too strongly impressed on any one undertaking the work.

Introduction.

2. The levels which have hitherto been used on the main lines are of three distinct patterns, *viz* :—

Levels.

A Rectangular level by Troughton and Simms.

Three Cylindrical levels by Troughton and Simms.

A Cushing's Reversible level by Cooke and Sons.

3. The telescope in the rectangular level has a total length of 24 inches. The object-glass has an effective diameter of  $2\frac{1}{4}$  inches and a focal length of 21 inches. There are two eye-pieces of powers 39 and 50 respectively.

Rectangular level.

In order to secure rigidity the telescope is enclosed in a prism, the transverse section of which is a square of 3-inch side.

There are two levels mounted on the prism, each having an effective length of 9 inches: the value of one division of the level-scale of A is  $0''\cdot960663$ , and of B  $1''\cdot04347$ : from the latter a subtense table has been constructed on the basis that 1 division =  $\cdot00033388$  feet at a distance of one chain. Level A has never been used, as it is considered sufficient to depend on the readings of B. To enable the observer to read A, if required, without shifting his position, a couple of upright sockets are fixed to the top of the prism between the two levels; on each of these a swinging mirror can be placed, capable of adjustment to any angle required for giving, by reflection, the readings of the ends of the

bubble. This instrument has a much higher magnifying power than any of the others and, under exceptional circumstances, it may be used to read the ordinary staff at a distance of 20 chains.

4. The cylindrical levels are numbered 1, 3, and 4, and are all alike in general construction, each having a clamp and tangent-screw fixed to the boss of the tribrach. The telescope has a total length of 24 inches. The object-glass has an effective diameter of  $2\frac{1}{4}$  inches and a focal length of 21 inches. There are two eye-pieces with each instrument, giving a power of about 35. The effective length of the level tube which in Nos. 1 and 4 is protected by an outside glass case, is  $8\frac{3}{4}$  inches.

The value of one division of the level-scale is for

| No. 1.          | No. 3.           | No. 4.           |
|-----------------|------------------|------------------|
| $3''\cdot0534:$ | $1''\cdot70915:$ | $1''\cdot01728,$ |

and from these values, tables of subtenses have been prepared on the basis that 1 division at 1 chain distance =  $0\cdot0009770$  feet for No. 1;  $0\cdot000546889$  for No. 3; and  $0\cdot0003255$  for No. 4.

5. The telescope in Cushing's reversible level has a total length of 23 inches. The object-glass has an effective diameter of 2 inches and a focal length of 21 inches. There are two eye-pieces with an arrangement for pushing forward a dark glass when required: the magnifying power of these is however low, being only about 25. The effective length of the level tube is  $6\frac{3}{4}$  inches, and the value of 1 division of the level-scale is  $5''\cdot0525$ , from which a subtense table has been prepared on the basis that 1 division =  $0''\cdot001617$  feet at 1 chain distance. The object and eye-ends of the telescope are readily interchangeable by means of the following arrangement:—To the internal tube of the telescope is fixed a gun-metal socket, turned and ground with a short conical fitting and wide flange to receive the eye-end with its eye-piece and diaphragm. On the opposite end of the outer tube a precisely similar fitting receives the cell containing the object-glass, both of the ends being identical as regards the fitting. The eye-piece is attached to the telescope by two screws placed  $180^\circ$  apart in the flange of the socket: these screws are

\* See Mathematical Instrument Office Descriptive Catalogue, fig. 131.

† The description is taken from a paper "On Cushing's Reversible Level" by Major E. H. Courtney, R.E.

not intended to be taken out, but corresponding holes in the flange of the eye-end allow the latter to be inserted in the socket, when a short rotatory motion from left to right will bring it into its proper position against a stop. The object-glass has precisely the same kind of attachment, and will, like the eye-end, fit the socket at either end of the telescope.

Instead of threads, lines are finely engraved on a glass disc which fits into a sliding diaphragm. The horizontal plate and tribrach are both cast hollow underneath, securing maximum strength and rigidity with minimum weight. A new feature is introduced in the attachment of the supports of the telescope: the support nearest the object-end is in contact with the plate, but is capable of a slight rocking motion in the direction of the axis of the telescope, so as to admit of the second adjustment (hereafter described), whilst the other support is provided with two large nuts for clamping and permanently securing the telescope to the plate when this adjustment has been performed.

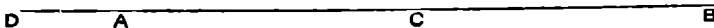
6. The adjustments of these levels before commencing work are performed as follows:—The instrument is levelled very carefully by means of the foot-screws, so that the one end of the level reads the same while the instrument makes a complete rotation round the vertical axis; then by means of the adjusting screws of the level itself, the bubble is brought as nearly as possible into the centre of its scale, care being taken that the edge of the scale corresponds with the longitudinal axis of the bubble.

7. If the staves are set up at exactly equal distances from the instrument, all errors arising from imperfect adjustment of the level to the optical axis of the telescope, are cancelled. But although this is the case, it is still very important that the telescope should be, as nearly as possible, truly adjusted. These adjustments are two-fold:—

*Firstly.* To bring the horizontal wire of the diaphragm into the visual axis of the telescope.

The instrument is set up at C,

Fig. 1.



exactly mid-way and in a line with two staves at A and B, distant two or three chains from each other, carefully levelled and the difference of readings,  $x$ , of the staves at A and B found: this is the true value, notwithstanding the errors of adjustment, since C is the middle point of AB. The instrument is then shifted to a point D near A, say  $\frac{1}{4}$  chain off, and so placed that both staves are simultaneously in the field of the telescope. It is carefully levelled and the difference of readings,  $y$ , of the staves at A and B found. If  $x$  is equal to  $y$ , the horizontal wire is in the visual axis of the telescope, but if not the difference  $y - x$  will be equal to  $bg$  in Fig. 2,

Fig. 2.



where  $Db$  is the apparent line of sight and  $DB$  the true line. The point B on the staff is determined from the formula

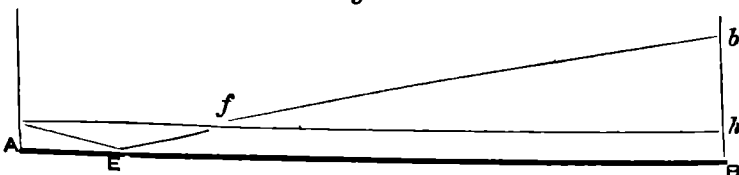
$$b B = (y - x) \frac{DB}{AB}$$

and the horizontal wire brought opposite the point B by means of the diaphragm screws, the instrument being kept level. The horizontal wire will then be in the visual axis of the telescope.

*Secondly.* To make the visual axis of the telescope perpendicular to the axis of rotation:—

The instrument is set up at E, Fig. 3, between the staves, but much nearer to A than to B. It is carefully levelled and the difference of readings,  $z$ , of the staves at A and B found. If  $z$  is equal to  $x$ , the visual axis of the telescope is perpendicular to the axis of rotation, but if not  $z - x$  is equal to  $bh$  in Fig 3.

Fig. 3.



The point B is determined from the equation

$$bB = (z - x) \frac{EB}{EB - EA}$$

and the horizontal wire is made to intersect B by means of the adjusting screws under and near each extremity of the horizontal plate. This, of course, disturbs the bubble of the level: it must now be corrected by its own screws, and the whole operation repeated.

The rectangular level has no horizontal plate with adjusting screws: its visual axis must be set at right angles to the axis of rotation by moving the screws of the diaphragm; as the first adjustment cannot be made, the greatest care must be taken to have the instrument at equal distances from the staves during field work.

8. The adjustment of the bubble is the same as described in para. 6,

and there are three other adjustments for this instrument:—*firstly*, to eliminate vertical collimation; *secondly*, to make the line of collimation perpendicular to the vertical axis; and *thirdly*, to make the bubble tube parallel to the line of collimation. These are effected as follows:—

*Firstly.* The instrument is set up on its stand, with one foot-screw under the telescope. The small fixing-screw at the top of the object-glass cell is taken out and the cross lines focused; the telescope is then directed on any convenient object, *e.g.* a small circular dot on a sheet of paper placed about 20 feet from the instrument, the object focused and bisected with the horizontal wire. The eye-end is then carefully turned in its socket, from right to left, until the holes in the flange of the eye-end are opposite the heads of the screws in the socket and removed. It is replaced again but in an inverted position, care being taken to turn it from left to right until it comes to a stop, when the diaphragm will be in its proper position. If the point is still bisected the collimation is perfect, but if not half the deviation from horizontal line is corrected by the foot-screw under the telescope, and half by the two screws that give vertical motion to the diaphragm. This process is repeated till the adjustment is perfect.

*Secondly.* The object being now bisected and all parallax eliminated, the eye-end and the object-glass cells are removed from their respective sockets and placed in the opposite ends of the telescope. If the

object is still bisected on turning the telescope half round, the line of collimation is perpendicular to the vertical axis; but if not, half the error is corrected by the large clamping nuts at one end of the horizontal limb, and the other half by the foot-screw under the telescope. As soon as it is found that the object and eye-ends can be reversed without any apparent change in the position of the object intersected, the small fixing screw should be returned and the object cell made secure. It is important in changing the object-glass from end to end to keep that part of the cell which has the small screw hole in it, always uppermost.

*Thirdly.* The instrument stand is levelled approximately by the legs, the telescope turned so that its axis is parallel to a line joining two foot-screws, and the bubble brought by motion of the latter to the centre of its run. If it remains so on turning the telescope half round, the level is correct; but if not, the bubble is brought half way back by the foot-screws over which it stands, and the other half by the two opposing nuts at the eye-end of the bubble-tube. The levelling is completed by turning the telescope a quarter round, so that one end of the level is over the third foot-screw by which the bubble must be brought to the centre of its run. The bubble should now remain in the centre during a complete revolution, and the small cross-level can then be adjusted.

9. The staves are graduated on both sides, one face being painted white with black divisions and graduated to hundredths of a foot from 0 to 10·00 feet, the other face being black, with white divisions, similarly divided from 5·55 to 15·55 feet. They are both shod and capped with brass, the extreme graduations being laid off on the brass. They are supplied with plummets let into their sides and visible through glass doors, so that they may be adjusted to within an inch or two of the true perpendicular, and they are held in position by ropes attached to a swivel on their summits.

The units of the levelling staves must be determined in terms of the 10-foot standard of the G. T. Survey. For this purpose a portable metal bar, on which the length of the standard bar has been laid off, is taken with each party of levellers. The staves should be compared with it at least twice during the course of the season, *viz.*, at the beginning and end of the season's work. It is also advisable that additional comparisons



be made every six weeks or two months, especially if the rise or fall is great, and the value of the work likely to be affected by the slight variations in length to which the staves are liable. As the wood of the staves has a tendency to shrink, a partial separation from the brass at the ends may be caused. In this case an exact foot should be taken off, with a beam compass, from any of the intermediate feet (1 to 9) all of which are defined by dots on brass pins let into the staff and the spot where one end of the compass falls on the brass should be marked, the other end being on the nearest foot. The mark will indicate the true position of the zero before separation, and if it is referred to when the staff is compared with the portable standard, no corrections for the separation will be required.

In choosing staves, at the commencement of the field operations, the observer must be careful to examine the differences between the zeros of the black and white faces of each staff. In most staves it is 5.55 but in some it is 5.60 feet; both of a pair should have the same difference in common, otherwise the observer will be perplexed and delayed by troublesome discrepancies between the results from black and white faces.

10. The comparison of staves is generally done by two observers who make independent measures of the difference in length between the standard bar and 10 feet of the staff by means of a finely graduated scale, the temperature being noted at the time of observation.

The following table shows how the comparison is conducted and needs no remark except that it is the mean of the two observations which is entered in the fifth column:—

INSTRUMENTS AND THEIR ADJUSTMENTS.

[PART VII.]

COMPARISON OF STAVES.

Value of the 10-foot Portable Standard Bar No. 3 (Edge A) = 9.999424 feet at 62° Fahrenheit  
 Factor of expansion = 0.000064 per degree per foot.

| Date of Comparison | Temperature                     | STAVES         |               | Value of Staves in terms of Portable Standard Bar when observed | Correction for Temperature to Portable Standard Bar | Corrected length of Portable Standard Bar when observations were made | Corrected Value of Staves | Corrected Mean of both Faces | Difference of Length of Staves from 10 feet | Mean Difference of Length pairs of Staves from 10 feet | Mean Correction for 10 feet for pairs of Staves used in different Sections | Remarks                                              |           |           |                                                     |
|--------------------|---------------------------------|----------------|---------------|-----------------------------------------------------------------|-----------------------------------------------------|-----------------------------------------------------------------------|---------------------------|------------------------------|---------------------------------------------|--------------------------------------------------------|----------------------------------------------------------------------------|------------------------------------------------------|-----------|-----------|-----------------------------------------------------|
|                    |                                 | No. of Staff   | Face of Staff |                                                                 |                                                     |                                                                       |                           |                              |                                             |                                                        |                                                                            |                                                      |           |           |                                                     |
| 15th January       | 79.25<br>80.0<br>80.0<br>80.0   | B <sub>1</sub> | W             | 10.000700                                                       | + .001104                                           | 10.008528                                                             | 10.001228                 | 10.001952                    | + .001952                                   | + .002064                                              | + .001174                                                                  | For Section from Navánár to Lakhpát G. T. S. Station |           |           |                                                     |
|                    |                                 |                | B             | 10.002100                                                       | .001152                                             | 576                                                                   | 2676                      | 10.001952                    | + .001952                                   | + .002064                                              |                                                                            |                                                      |           |           |                                                     |
|                    |                                 | B <sub>2</sub> | W             | 10.001600                                                       | .001152                                             | 576                                                                   | 2176                      | .002176                      | .002176                                     | .002176                                                |                                                                            |                                                      | + .000283 | + .000291 | For Section from Lakhpát G. T. S. Station to Tatta. |
|                    |                                 |                | B             | 10.001600                                                       | .001152                                             | 576                                                                   | 2176                      | .002176                      | .002176                                     | .002176                                                |                                                                            |                                                      | + .000299 |           |                                                     |
| 14th March         | 81.25<br>81.65<br>81.8<br>81.95 | B <sub>1</sub> | W             | 9.999600                                                        | .001232                                             | 10.000656                                                             | 10.000256                 | .000294                      | .000294                                     | + .000283                                              | + .000291                                                                  | For Section from Lakhpát G. T. S. Station to Tatta.  |           |           |                                                     |
|                    |                                 |                | B             | 9.999650                                                        | .001258                                             | 682                                                                   | 0332                      | .000294                      | .000294                                     | .000294                                                |                                                                            |                                                      | + .000299 |           |                                                     |
|                    |                                 | B <sub>2</sub> | W             | 10.000350                                                       | .001267                                             | 691                                                                   | 0041                      | .000271                      | .000271                                     | .000271                                                |                                                                            |                                                      | + .000299 |           |                                                     |
|                    |                                 |                | B             | 9.999800                                                        | .001277                                             | 701                                                                   | 0501                      | .000271                      | .000271                                     | .000271                                                |                                                                            |                                                      | + .000299 |           |                                                     |
| 22nd May           | 87.0<br>87.0<br>87.0<br>87.0    | B <sub>1</sub> | W             | 9.999450                                                        | .001600                                             | 10.001024                                                             | 10.000474                 | .000299                      | .000299                                     | + .000299                                              | + .000299                                                                  | For Section from Lakhpát G. T. S. Station to Tatta.  |           |           |                                                     |
|                    |                                 |                | B             | 9.999100                                                        | .001600                                             | 1024                                                                  | 0124                      | .000299                      | .000299                                     | .000299                                                |                                                                            |                                                      | + .000299 |           |                                                     |
|                    |                                 | B <sub>2</sub> | W             | 9.999450                                                        | .001600                                             | 1024                                                                  | 0474                      | .000299                      | .000299                                     | .000299                                                |                                                                            |                                                      | + .000299 |           |                                                     |
|                    |                                 |                | B             | 9.999100                                                        | .001600                                             | 1024                                                                  | 0124                      | .000299                      | .000299                                     | .000299                                                |                                                                            |                                                      | + .000299 |           |                                                     |

11. The best way of determining the run of a level is by means of an instrument, called a 'bubble-tester,' specially constructed for the purpose. One of these is kept in the Mathematical Instrument Office, Calcutta, and another in the Trigonometrical Branch Office, Dehra Dún.

Run of a level.

Another simple way is by attaching the level to the vertical circle of an alt-azimuth instrument or a theodolite; the readings of the ends of the bubble and of the micrometers or verniers are then taken in different positions of the bubble along its scale, and the number of divisions of the level-scale compared with angle given by the circle, in each of the different positions, are used to obtain a series of values of one division of the level-scale in terms of seconds of arc. The mean of these will give the run of the level.

12. When the run of a level is known, it is easy to ascertain the correction to a staff reading for any amount of dislevelment within the range of the bubble of the level. Suppose, for example, that the value of 1 division of the scale is equal to  $1'' \cdot 709$ , then the amount subtended by an angular deflection measured by the movement of the bubble through 1 division will be equal to  $1 \cdot 709$  times the amount subtended by an angle of  $1''$  at the given distance. At a distance of 10 chains or 660 feet from the instrument, the maximum distance at which the staff is usually set up, the subtense for 1 division of the level will be

Subtense table.

$$\begin{aligned} &= 1 \cdot 709 \times \sin 1'' \times 660 \text{ feet} \\ &= 1 \cdot 709 \times \cdot 0000048481 \times 660 \text{ feet} \\ &= \cdot 0054684 \text{ feet.} \end{aligned}$$

With such a datum, it is necessary to construct a table of subtenses for varying amounts of dislevelment and ranges of distances. As the staves are divided into tenths and hundredths and the readings are estimated to the thousandths of a foot, the subtenses must also be expressed in thousandths of a foot as follows :—

INSTRUMENTS AND THEIR ADJUSTMENTS.

[PART VII.]

TABLE OF SUBTENSES.

| Divisions | Distance in chains |      |      |      |      |      |      |      |      |      | Divisions |
|-----------|--------------------|------|------|------|------|------|------|------|------|------|-----------|
|           | 1                  | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   |           |
| 0' 1      | .000               | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .000 | .001 | 0' 1      |
| 0' 2      | .000               | .000 | .000 | .000 | .001 | .001 | .001 | .001 | .001 | .001 | 0' 2      |
| 0' 3      | .000               | .000 | .000 | .001 | .001 | .001 | .001 | .001 | .001 | .002 | 0' 3      |
| 0' 4      | .000               | .000 | .001 | .001 | .001 | .001 | .002 | .002 | .002 | .002 | 0' 4      |
| 0' 5      | .000               | .001 | .001 | .001 | .001 | .002 | .002 | .002 | .002 | .003 | 0' 5      |
| 0' 6      | .000               | .001 | .001 | .001 | .002 | .002 | .002 | .003 | .003 | .003 | 0' 6      |
| 0' 7      | .000               | .001 | .001 | .002 | .002 | .002 | .003 | .003 | .003 | .004 | 0' 7      |
| 0' 8      | .000               | .001 | .001 | .002 | .002 | .003 | .003 | .003 | .004 | .004 | 0' 8      |
| 0' 9      | .000               | .001 | .001 | .002 | .002 | .003 | .003 | .004 | .004 | .005 | 0' 9      |
| 1' 0      | .001               | .001 | .002 | .002 | .003 | .003 | .004 | .004 | .005 | .005 | 1' 0      |
| 1' 1      | .001               | .001 | .002 | .002 | .003 | .003 | .004 | .004 | .005 | .006 | 1' 1      |
| 1' 2      | .001               | .001 | .002 | .002 | .004 | .004 | .005 | .005 | .006 | .006 | 1' 2      |
| 1' 3      | .001               | .001 | .002 | .003 | .004 | .004 | .005 | .005 | .006 | .007 | 1' 3      |
| 1' 4      | .001               | .001 | .003 | .003 | .004 | .004 | .006 | .006 | .007 | .007 | 1' 4      |
| 1' 5      | .001               | .002 | .003 | .003 | .004 | .005 | .006 | .006 | .007 | .008 | 1' 5      |
| 1' 6      | .001               | .002 | .003 | .003 | .005 | .005 | .006 | .007 | .007 | .008 | 1' 6      |
| 1' 7      | .001               | .002 | .003 | .004 | .005 | .005 | .007 | .007 | .008 | .009 | 1' 7      |
| 1' 8      | .001               | .002 | .003 | .004 | .005 | .006 | .007 | .007 | .009 | .009 | 1' 8      |
| 1' 9      | .001               | .002 | .003 | .004 | .005 | .006 | .007 | .008 | .009 | .010 | 1' 9      |
| 2' 0      | .001               | .002 | .003 | .004 | .005 | .007 | .008 | .009 | .010 | .011 | 2' 0      |
| 2' 1      | .001               | .002 | .003 | .004 | .005 | .007 | .008 | .009 | .010 | .012 | 2' 1      |
| 2' 2      | .001               | .002 | .003 | .004 | .006 | .008 | .009 | .010 | .011 | .012 | 2' 2      |
| 2' 3      | .001               | .002 | .003 | .005 | .006 | .008 | .009 | .010 | .011 | .013 | 2' 3      |
| 2' 4      | .001               | .002 | .004 | .005 | .006 | .008 | .010 | .011 | .012 | .013 | 2' 4      |
| 2' 5      | .001               | .003 | .004 | .005 | .006 | .009 | .010 | .011 | .012 | .014 | 2' 5      |
| 2' 6      | .001               | .003 | .004 | .005 | .007 | .009 | .010 | .012 | .013 | .014 | 2' 6      |
| 2' 7      | .001               | .003 | .004 | .006 | .007 | .009 | .011 | .012 | .013 | .015 | 2' 7      |
| 2' 8      | .001               | .003 | .004 | .006 | .007 | .010 | .011 | .012 | .014 | .015 | 2' 8      |
| 2' 9      | .001               | .003 | .004 | .006 | .007 | .010 | .011 | .013 | .014 | .016 | 2' 9      |
| 3' 0      | .002               | .003 | .005 | .006 | .008 | .010 | .011 | .013 | .014 | .016 | 3' 0      |
| 3' 1      | .002               | .003 | .005 | .006 | .008 | .010 | .011 | .013 | .014 | .017 | 3' 1      |
| 3' 2      | .002               | .003 | .005 | .006 | .009 | .011 | .012 | .014 | .015 | .017 | 3' 2      |
| 3' 3      | .002               | .003 | .005 | .007 | .009 | .011 | .012 | .014 | .015 | .018 | 3' 3      |
| 3' 4      | .002               | .003 | .006 | .007 | .009 | .011 | .013 | .015 | .016 | .018 | 3' 4      |
| 3' 5      | .002               | .004 | .006 | .007 | .209 | .012 | .013 | .015 | .016 | .019 | 3' 5      |
| 3' 6      | .002               | .004 | .006 | .007 | .010 | .012 | .013 | .016 | .017 | .019 | 3' 6      |
| 3' 7      | .002               | .004 | .006 | .008 | .010 | .012 | .014 | .016 | .017 | .020 | 3' 7      |
| 3' 8      | .002               | .004 | .006 | .008 | .010 | .013 | .014 | .016 | .018 | .020 | 3' 8      |
| 3' 9      | .002               | .004 | .006 | .008 | .010 | .013 | .014 | .017 | .018 | .021 | 3' 9      |
| 4' 0      | .002               | .004 | .007 | .009 | .011 | .013 | .015 | .018 | .020 | .022 | 4' 0      |
| 4' 1      | .002               | .004 | .007 | .009 | .011 | .013 | .015 | .018 | .020 | .023 | 4' 1      |
| 4' 2      | .002               | .004 | .007 | .009 | .012 | .014 | .016 | .019 | .021 | .023 | 4' 2      |
| 4' 3      | .002               | .004 | .007 | .010 | .012 | .014 | .016 | .019 | .021 | .024 | 4' 3      |
| 4' 4      | .002               | .004 | .008 | .010 | .012 | .014 | .017 | .020 | .022 | .024 | 4' 4      |
| 4' 5      | .002               | .005 | .008 | .010 | .012 | .015 | .017 | .020 | .022 | .025 | 4' 5      |
| 4' 6      | .002               | .005 | .008 | .010 | .013 | .015 | .017 | .021 | .023 | .025 | 4' 6      |
| 4' 7      | .002               | .005 | .008 | .011 | .013 | .015 | .018 | .021 | .023 | .026 | 4' 7      |
| 4' 8      | .002               | .005 | .008 | .011 | .013 | .016 | .018 | .021 | .024 | .026 | 4' 8      |
| 4' 9      | .002               | .005 | .008 | .011 | .013 | .016 | .018 | .022 | .024 | .027 | 4' 9      |
| 5' 0      | .003               | .005 | .008 | .011 | .014 | .016 | .019 | .022 | .024 | .027 | 5' 0      |

If, however, neither bubble-tester, alt-azimuth instrument nor a well graduated theodolite is available, a subtense table may be made by directing the levelling instrument on a staff set up at any convenient distance at which it can be read with accuracy, taking a series of staff readings and comparing their difference with the differences of the corresponding readings of the bubble. The observations should be taken in the morning before the atmosphere is agitated by the sun's rays, or in the afternoon a couple of hours before sunset, and a cloudy day is preferable to one of bright sunshine for the purpose. Suppose that the staff is set up at 10, 8 and 5 chains from the instrument, and that the means of 20 observations at each distance are for 1 division of the level-scale equal respectively to .0057 feet, .0042 feet and .0028 feet. Then the value to be adopted for the subtense of 1 division at a distance of 10 chains

$$= \frac{1}{3} \left\{ .0057 + \frac{10}{8} \times .0042 + \frac{10}{5} \times .0028 \right\} = .0055 \text{ feet,}$$

and the subtense table will be computed in the same form as before by simple proportion.

13. The dislevelment must be deduced from the readings of the bubble and may be expressed in terms of the divisions of the scale. As the numbering of the divisions is carried from the centre outwards, the readings of the ends are identical when the bubble is exactly in the middle of its tube; hence it is evident, that if the level were truly adjusted to the visual axis of the telescope, the instrument would be in a truly horizontal position when the readings of the two ends of the bubble were identical, and that when the readings were not identical, half their differences would indicate the amount of deflection of the instrument from horizontality. As however there is no certainty that the level is in exact adjustment to the visual axis of the telescope, it is necessary after having taken a pair of readings of the bubble to turn the instrument through an exact semi-revolution and take a fresh pair of readings: the four readings must then be combined together in such a manner as to eliminate the effect of the error of adjustment, in deducing the amount of dislevelment. In practice it is not necessary to reverse the instrument and take two pairs of readings for the mere elimination of the errors of the adjustment of the level to the visual axis of the telescope,

Dislevelment.

as this purpose is answered by the readings which are taken when the telescope is pointed in succession to the back and forward staves.

If the telescope has been pointed to the back staff in the first instance, and  $d_b$  and  $d_f$  be the readings of the ends of the level towards the back and forward ends respectively, and if when the telescope is pointed to the forward staff  $d'_b$  and  $d'_f$  be the readings towards the back and forward ends respectively, then the amount of dislevelment expressed in divisions of the scale will be

$$\begin{aligned} &= \frac{1}{4} \{ (d_b + d'_b) - (d_f + d'_f) \}, \\ &= \frac{1}{4} \{ (d_b - d_f) + (d'_b - d'_f) \}; \end{aligned}$$

if this is positive, the end of the instrument towards the back staff will be above the horizontal plane passing through the centre of the instrument.

The effect of this instrumental dislevelment on the rise or fall deduced from the staff readings will evidently be equal to the sum of the subtense on the back and forward staves, or twice the value given by the subtense table. Hence in practice half the expression within the brackets is taken, instead of one quarter, as the quantity with which to enter the subtense table.

The above argument is only strictly true when the instrument is in a line with the staves. If the instrument is on either side of this line, one end of the level is towards the staff on which the telescope is pointed, and the other is towards the opposite direction and not towards the other staff. In this case twice the difference between the respective amounts of dislevelment in two vertical planes passing through the instrument and the staves is determined, and this is all that is required for the correction of the observations, provided that the essential condition of setting up the instrument at equal distances from the staves has been fulfilled.

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#### CHAPTER II. PRECAUTIONS AGAINST ERRORS.

1. The chief cause of the inaccuracies so frequently found at the close of a line of levels is due to errors of staff readings. Even when an observer is careful to repeat the readings at each station, before proceeding to the next, he

Errors in staff readings.

may occasionally make a mistake without finding it out. It is to guard against this class of error that the staves are graduated differently on the two faces, so that if an observer make a mistake in one reading he is not likely to make a similar mistake in the reading on the opposite face, and the error will be shown up immediately by the different results from the black and white faces, if the computation has been correctly performed.

2. Errors of reading are, however, of such an uncommon occurrence, and the results from black and white faces so constantly coincide, that the recorder in taking out the second result immediately under the first is liable to be biassed by it and to fancy that they coincide, when in reality there is a difference. If only one observer was working, such a mistake would probably not be found out until the preparation of the duplicate records some months afterwards, and it could not then be rectified. To remove this source of error, a second observer follows the first over the line, station by station, with an independent set of instruments.

*Errors in writing down results.*

3. Errors of permanent instrumental adjustments, as well as those which might be caused by normal atmospheric refraction and the earth's curvature, are wholly eliminated when the staves are at equal distances from the instrument. Errors caused by instrumental deflection from horizontality are obviated by reading the ends of the level, and applying a correction for dislevelment to the difference of the staff readings.

*Instrumental errors.*

4. The instrument generally stands at such a height that it is impossible for the observer to look down on it. From above, the bubble is seen to be sharply defined and the scale can be read with great accuracy, but the observer usually stands with his eye nearly on a level with the instrument, in which position the rim round the bubble, caused by the adhesion of the liquid to the sides of the glass tube, becomes so prominent that, in certain lights, its extremities may be taken for the true ends of the bubble. When light falls obliquely on the instrument, the outer edge of the rim, being towards the light, is more clearly defined than the inner edge, while at the opposite end of the bubble, the inner edge or true end is most clearly defined. Hence there is a liability to read

*Errors in reading the level.*

one end erroneously, and thus introduce one or two divisions of level error. This must be most carefully guarded against, and the observer should take as many opportunities as he can of reading the bubble with his eye above the instrument, and again with his eye on the same level, until he is satisfied that his readings are correct.

5. Cumulative errors are caused by the constant recurrence of small errors, either personal, instrumental or atmospheric; and, though too small to attract attention at any one station, become manifest when the results of different observers on the same line of levels are compared, or those of a single observer returning station by station over a line from terminus to origin.

Small errors recurring in a constant order—such as might be caused by a uniformly rising or sinking refraction, or by a tendency in the instrument to settle on its axis more one way than another on being set up for observation—may be cancelled by alternating the order of observation to the back and forward staves, at the successive stations, *i.e.*, by observing the back staff first at one station, and the forward staff first at the next. This method might be still further amplified by the following circuit system, by which an observer working only in one direction has a means of finding out whether there are such errors in his work, by comparing the differences between the respective reductions to origin from black and white faces, one pair of which may be treated so as to give the results of an ‘up line’ and the other of a ‘down line’:—

$$\begin{array}{l}
 \text{Odd stations} \left\{ \begin{array}{l} 1 \left\{ \begin{array}{l} \text{Forward Staff} \\ \text{Back} \quad \quad \quad \text{,,} \end{array} \right\} \text{White Face Down Line.} \\ 2 \left\{ \begin{array}{l} \text{Back} \quad \quad \quad \text{,,} \\ \text{Forward} \quad \quad \text{,,} \end{array} \right\} \text{Black} \quad \quad \text{Up} \quad \quad \text{,,} \end{array} \right. \\
 \\
 \text{Even stations} \left\{ \begin{array}{l} 1 \left\{ \begin{array}{l} \text{Back} \quad \quad \quad \text{,,} \\ \text{Forward} \quad \quad \text{,,} \end{array} \right\} \text{Black} \quad \quad \text{Up} \quad \quad \text{,,} \\ 2 \left\{ \begin{array}{l} \text{Forward} \quad \quad \text{,,} \\ \text{Back} \quad \quad \quad \text{,,} \end{array} \right\} \text{White} \quad \quad \text{Down} \quad \quad \text{,,} \end{array} \right.
 \end{array}$$

This amplification is however seldom carried out in practice.



Errors recurring in a constant order might be caused by an irregularity in the socket of the axis, if the instrument be invariably set up in the same direction with regard to the line followed. This can be obviated by marking one of the legs of the tripod stand, or of the instrument, and directing it back and forward at alternate stations.

6. The circuit system which requires an observer to close on his origin is theoretically the best; but a circuit, when levelled by a single observer, is exposed to a greater chance of accidental error than a single line executed by two levellers consecutively and independently. For both observers to work twice over the same line would more than double the time and cost of the undertaking, as they would never be able to increase the distance between the staves, when favoured by clear weather, except at the cost of losing all intermediate comparisons, but would be compelled to adopt short unvarying distances. If, however, an amount of extra trouble, equivalent to both observers marching over the whole line once, be accepted, all the advantages of closing on an origin may be obtained by the simple expedient of dividing the line into as nearly as possible equal sections and both observers working alternate sections in opposite directions. The sections should be so arranged that the sum of the forward ones is equal to that of the back ones. If, as is usual, the sections are each a day's work—in good ground 4 or 5 miles—and circumstances will permit of encamping near their extremities, it is easy to walk the 4 or 5 miles to the commencement of a section, without fatigue, in the early morning before sunrise and then level back to camp; in this way the delay of extra marching is avoided and steady progress made at a rapid rate. This section system has apparently contributed, more than anything else, to reduce cumulative errors in the operations, and should invariably be adopted.

7. A tendency is sometimes observed, in a long line of levels, for the negative level corrections to exceed the positive ones or *vice versa*. It is of importance that they should cancel each other, as any error in the determination of the runs of the levels is thus eliminated. The observer must therefore be prepared to prevent a preponderance in any direction by setting up his instrument with a slight tilt in the opposite direction.

8. It has also been found that the sun exercises a constant dis-  
 levelling effect on instruments, tending to  
 Cumulative errors caused by the sun. raise the end towards itself and lower the  
 opposite end. This is shown by the excess of negative corrections  
 for level errors in lines worked from south to north and of positive  
 corrections in the opposite lines. Thus an error in the adopted value  
 of the run of the level would affect the results by the same fraction  
 of the accumulated correction that it is of the run. In practice this  
 may be cancelled by watching the signs of the correction and occasion-  
 ally tilting the instrument to counteract the disposition to rise towards  
 the sun.

There is however another order of error which no *modus operandi*  
 will wholly cancel; for, however carefully the instrument may be  
 shaded, the action of the sun tends to cause a constantly recurring  
 displacement in the level in the interim between the reading of the  
 staff and that of the bubble ends: these readings should be strictly  
 simultaneous in order to compare with each other, but they are neces-  
 sarily consecutive; and short as the interval between them may be,  
 it is enough to cause slight error, of a cumulative nature, which may  
 amount to a large quantity at the end of a long line of levels. It  
 would be maximum on a meridional line, but would not affect a line  
 carried from east to west or *vice versâ*. This error is of a class which  
 cannot be eliminated by working in a circuit: so long as the cause  
 remains constant, the effect is the same in both up and down opera-  
 tions; thus the opposite points of a circuit, which closes without  
 apparent error, may yet be considerably erroneous.

Leaving this class of error out of consideration, atmospheric in-  
 fluences must tend to cancel each other, on a long line of operations,  
 except under the following circumstances:—

9. *Firstly*—when more stations are observed before than after-  
 noon: during the forenoon the refraction will  
 Error due to refraction. generally be sinking and rising in the after-  
 noon, consequently the first of a pair of staff readings will have a  
 tendency to be more refracted than the second, thus introducing cumu-  
 lative error unless the precaution is taken to alternate the order of  
 observation by taking the back staff first at one station and the forward  
 staff first at the next.

10. *Secondly*—when operations are carried over a line of country which slopes uniformly in one direction, as in proceeding from the sea to the foot of a range of mountains, the rays of light from the up-staff to the observer are generally nearer to the ground than those from the down-staff, and they must therefore be more subject to extremes of refraction. The influence of this source of error will vary with the seasons, and it is evidently beyond the control of the observer.

11. The sinking of a peg that supports a staff, in the interval between its being the forward end of one section and the back end of the next, has often been suggested as a cause of error. But there appears to be as much reason for a firmly driven peg to be raised by the reaction of the ground as to be lowered by the weight of the staff, and there are nearly as many instances in which the signs of the errors indicate the possibility of the one event having happened as the other.

12. On the whole, then, when the precautions which are taken to guard against accidental gross errors, such as misreadings the feet and tenths, mistaking the identity of the pegs on the line, or confounding one bench-mark with another, are such as to preclude the possibility of any error arising from these causes, certain other precautions must still be taken against the accumulation of minute errors which may arise (1) from instrumental defects, (2) from the rising of the pegs on hard ground or their sinking on soft ground, (3) from rising or sinking refraction, (4) from personal bias, (5) from the effect of the sun in disturbing the horizontality of the instrument in the interval between the staff and level readings, (6) from unequal illumination of the ends of the bubble, (7) from working continuously over a long slope, on which the average height above the surface of the ground will be sensibly different for up and down rays.

The alternation of the order of observation of the back and forward staves should eliminate all error arising from the first three sources. Those which cannot be eliminated by this method may be met to some extent by multiplying the number of observers, in order to diminish the effect of individual bias, and by working under every possible variation of climate.

## CHAPTER III. PROCEDURE IN THE FIELD.

1. Every main line of levels is executed by two observers, each working with his own instrument quite independently of the other, one being a station in advance of the other.

Instructions to observers.

Before beginning work, each observer must be careful to see that the instruments are in perfect adjustment.

2. These adjustments are as follows:—The stand must be rigid, and there must be no play between it and the telescope. In soft or sandy ground, the feet of the stand should rest on wooden pegs firmly driven.

Adjustments. Stand.

3. The telescope must be adjusted so that, *firstly*, the horizontal wire of the diaphragm is in the visual axis of the telescope, and, *secondly*, the visual axis of the telescope is perpendicular to the axis of rotation. For the methods of effecting these adjustments, see page 283 for the cylindrical levels and the rectangular level, and page 285 for Cushing's reversible level.

Telescope.

4. When the instrument is level, the bubble should be adjusted so as to be in the centre of its run and should be bisected crosswise by the edge of the scale; it should remain thus in any position of the instrument.

Bubble.

5. The staves must be compared with a standard 10-foot bar at the beginning and end of the season's work and also during the field season as often as is convenient, see page 286.

Staves.

In choosing a pair of staves the observer must carefully examine that the difference between the zeros of the black and white faces of each staff is the same, see page 287.

6. The chains should also be compared occasionally with the 10-foot bar.

Chains.

7. Before beginning work, the observer should determine the value of the level-scale and prepare a subtense table, giving the value in feet of one division of the level-scale at various distances, see page 289.

Level-scale.

8. The staves must invariably rest on wooden pins driven very firmly into the ground. A hemispherical brass brad should also be let into the head of each pin after it is driven, to offer a point instead of an uncertain surface, for the staves to stand on, and so that they may be rotated freely to present each face in succession to the observer. The brad also affords a common point of reference for the successive observers, whose results may thus be compared rigorously station by station.

Support for staves.

9. The staves must be set up at equal distances from the instrument; if the ground will not permit of the latter being set up midway on the line between the staves, then by shifting the positions of the instrument and of the forward staff some point can always be found where the instrument will occupy the vertex of an isosceles triangle of which the line between the staves is the base. Unequal distances should always be avoided: there is no excuse for them excepting on unusually rough ground when the observer is pressed for time and then the distances should be made as short as possible. On a good day the maximum distance between the instrument and the staves is from 16 to 20 chains in the early morning and about 8 chains up to 10 o'clock. The distances should be carefully measured with a chain.

Distance between staves.

10. When the difference of level is small and a staff has once been set up and observed, it should not be re-adjusted if it has to be observed at the next station, because then any error caused at the first station by deflection of the staff will in a great measure be cancelled at the next. But the staff should be carefully re-adjusted whenever, having been read near the bottom at one station, it is liable to be read near the top at the next.

Re-adjustment of staves.

11. The instrument being set up at the same distance from the two staves, must be carefully levelled. The white face of the back staff should then be intersected and the reading taken to three decimal places: when the staves are very

Readings.

close a fourth place of decimals might be estimated, but this is so seldom the case that it may be treated as exceptional. The ends of the bubble must then be read with great care, as quickly as possible after the staff reading, see page 293. The white face of the forward staff must next be intersected and the level readings taken: after this the process must be repeated on the black faces of the staves. The observations of the back and forward staves should follow each other as quickly as possible, consistently with accuracy, in order that they may be strictly differential. If the differences of height deduced from the white and black faces agree within  $\cdot 005$ , and the observer is otherwise satisfied with them, he should proceed to the next station. Should they differ by a greater amount, they must be repeated once or twice according to discretion. If the differences are very large and obviously due to grazing rays, the station should be rejected and a better one selected if possible.

After finishing the first station, the observer must leave his forward staff as it was, so that it may become the back staff at the second station and its zero error be eliminated. In observing at the second station the forward staff should first be read and then the back one, and so on alternately, see page 294: this is not apparent from the field books which are arranged to facilitate computation, but it must be attended to.

12. The observer in setting up the instrument ought to mark one of the legs of the tripod, or of the instrument, and always point the marked leg to the same staff-holder throughout a section, see page 295.

13. Proceeding in this way, the observer should finish the first day's work of say 4 miles, if in good ground, between day light and 10 o'clock, and his camp ought to be waiting for him at the end of his work.

14. As the day's work will necessarily close more frequently on temporary pins, over which the line of levels is carried, than on permanent bench-marks, the closing station of the previous day must be relevelled next morning to test the permanence of the pins, before carrying on operations beyond them.

15. The next morning before day break, the observer should march out to the end of his second section and level back into camp, see page 295, afterward relevel-  
 Second day's work. ling his closing station of the previous day.

16. In consequence of alternating the direction of operations, the verification is made in the opposite direction to the first measurement, consequently the two results have opposite signs: they cannot be conveniently combined together and the mean employed, as when the work is carried only in one direction, and there is a liability to confusion at the time, and subsequent misapprehension if the results are brought up by any one but the leveller himself. This should be carefully guarded against by the method of recording the repetition in the Field Book. A clear space of two inches should be left between the record of the extreme station of the section in progress and that of the repeated station of the adjacent section, and the reduction to the origin of the section in progress should be entered above the record of the repetition. Opposite the repetition a remark should be made as follows:—

|                   |     |     |     |       |
|-------------------|-----|-----|-----|-------|
| By up line fall   | ... | ... | ... | 2·327 |
| By down line rise | ... | ... | ... | 2·331 |

The terminal pins should be large and strong, firmly driven and covered over with thorns for protection. A trench should be cut across the line at the extremity of each section to guard against the possibility of introducing into one section the stations of another.

17. The observer should proceed thus levelling in opposite directions on alternate days till the line is finished, and during the whole work he must be careful to prevent a preponderance of level error as described on page 295.  
 Level error to be guarded against.

18. The results as deduced by each observer from the white and black faces of the staves must agree within ·005, and the mean of these as determined by each observer must also agree within ·005, otherwise the observations at the particular station must be repeated.  
 Agreement of results.

19. At intervals of every 10 or 12 miles an 'embedded' bench-mark should be laid down. A stone 18 × 12 × 12 inches is generally built into a block of masonry  
 Bench-marks.

from about 3 to 4 feet cube, with the upper surface of the stone flush with the upper surface of the masonry which is about 6 inches below the ground level, and the whole block covered with earth. A small hollow square, a quarter of an inch deep and 5 inches square, is cut in the upper surface of the stone for the foot of the levelling staff to rest in. The letters G.T.S. B.M. and the date A.D. 18 . are also cut on the bench-mark. These bench-marks are as a rule laid down in the vicinity of masonry buildings, and to mark the spot the letters B.M. are cut somewhere on the building close by so that identification is unmistakeable. Occasionally, bench-marks are placed in the verandah floors of travellers' bungalows and in other such places; in these cases the stone is placed flush with the floor level.

'Inscribed' bench-marks should be placed on such points as the copings of railway platforms, parapets of bridges, &c. The letters G.T.S. B.M. and a small circle should be cut deep into the masonry, the circle showing the position which the bottom of the staff occupied. In the neighbourhood of tidal observatories, pairs of test bench-marks should be laid down for the detection of secular changes of relative level between the land and the sea. They should be placed some 8 or 10 miles inland, and connected very carefully with the tide-gauge: suitable embedded bench-marks on the line of levels will of course answer this purpose.

20. Each embedded bench-mark must be transferred for safe custody to the care of responsible railway or civil authorities. The transfer papers are in triplicate: one copy is kept for record in the office of the Tidal and Levelling Party, the second by the responsible authority and the third by the individual entrusted by the responsible authority with the actual care of the bench-mark. The papers are signed by the last mentioned individual and by the officer in charge of the Levelling Operations.

21. If possible, all principal stations of the Great Trigonometrical Survey, in the neighbourhood of a line of levels, should be connected by branch lines.

22. There are a few cases which require exceptional treatment. Thus supposing the height of a tower station has to be determined, *firstly* when the foot can be

Protection of bench-mark.  
 Connection with triangulation.  
 Exceptional cases. Levelling to a tower station.



levelled up to, and *secondly* when it cannot be reached. In the first case the levelling should be carried up to a peg or mark at the foot of the tower and a tape suspended from a staff laid horizontally on the summit of the tower and levelled by means of a spirit-level placed upon it. By this means a direct measurement of the difference of height between the summit of the tower and the mark levelled to, is obtained. In the second case a levelling staff should be erected on the summit of the tower and the angular elevation of two divisions, near the top and bottom of the staff, measured with a small theodolite centered over the last point levelled. The height of the summit of the tower above the eye of the observer and consequently above the levelled point can then be readily deduced.

23. Again supposing a great length of steep ground intervenes between the summit of a hill station and the last point levelled to. In this case a staff should be erected over the last point and a second staff laid transversely with one end resting on a convenient spot on the side of the slope while the other is supported by a man and raised or lowered till the indications of a small bubble placed on it show that it is horizontal: the reading on the vertical staff corresponding to the lower edge of the transverse staff should then be taken several times and the difference of level deduced. In this way by a succession of measurements, the top of the hill will be reached and the difference of level found.

Levelling up steep ground.

24. Again supposing the levels have to be carried across a river or creek whose width is too great to admit of the ordinary staves being read. In this case staves of Strange's pattern graduated to tenths and subdivided to two-hundredths of a foot are used: they can be read pretty fairly with the rectangular level at 50 or 60 chains, and several measurements should be taken on two or more days at the most favourable time so as to reduce the chances of error to a minimum.

Levelling across a river or creek.

If however the river or creek is too wide to admit of even this alternative, two staves should be embedded on opposite sides of the water and as nearly as possible at right angles to the direction of the current and a large number, say 2 or 300, of simultaneous readings of the height of the water on the staves taken at intervals of 5 minutes or so, and the difference of height on the two sides deduced from these.

25. The following is a specimen of the Field Book used in the leveling operations of the Trigonometrical Branch of the Survey of India Department. The 1st column gives the number of the station and indicates the horizontal lines on which the observations to the respective staves are entered; six of these lines are required to each station, three for observations to the black faces and their reduction, and three for the white faces. The 2nd column gives the distances and bearings of the staves from the instrument, the latter being necessary to enable the line of levels to be protracted. The 3rd and 4th columns give the back and forward end readings of the bubble, the 5th and 6th the calculations of dislevelment and the 7th the corresponding correction obtained from the subtense table, expressed in thousandths of a foot, omitting decimal points and cyphers. The 8th column gives the staff readings; the 9th and 10th the approximate results deduced by subtracting the reading of the forward from that of the back staff, one column being allotted to rises and the other to falls. The 11th and 12th columns give the true differences of level, *i.e.*, the approximate values of the 9th and 10th columns corrected for dislevelment; in these columns the mean of the results from black and white faces is also entered. The 13th column gives the reduction of the levels to the origin of the line, which should be deduced station by station and checked by summing up the rises and falls at the foot of the page as indicated. The 14th column is reserved for remarks and descriptions of the bench-marks: the latter should be sufficiently lucid to enable any person who may have occasion to visit a bench-mark, either for the purpose of commencing or closing a new line of levels, to ascertain the exact position of the mark without the slightest doubt; drawings of the bench-marks should also be given with their distances and bearings from any prominent buildings in the neighbourhood. The remaining columns on the right hand side are required for the entry of details regarding the small cumulative errors described in pages 295 and 296. The level and staff readings are recorded by a native assistant who calculates the results before handing the Field Book over to the observer, by whom they are checked before proceeding to the next station. The headings, remarks and descriptions of stations should invariably be written by the observer.

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**SPECIMEN OF FIELD BOOK.**

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# SURVEY OF INDIA DEPARTMENT.

## FORWARD SECTION KALIANPUR TO DEORI VILLAGE.

With No. 3 Standard—value of 1 Division of Scale = 1''·709. Length of Chain = 66 feet. Magnetic North by  
Compass East of true North by  $2\frac{1}{2}^\circ$ .

*RULE FOR CORRECTING DISLEVELMENT.—Consider Back End level readings to be −, and Forward End to be +. Find their Difference and enter it with sign of whichever is greater. Half the Algebraical sum of the Differences is the quantity for which a correction is to be taken from Subtense Tables. The correction to have the same sign as the half sum.*

| Station No. and Staff Positions | Distances and Bearings of Staves from Instrument | Level Readings—Dislevelment and Corrections |               |         |                   |              | Staff Readings | Approximate Differences of Level |        | Corrected Differences of Level |        | Level reduced to Origin | Remarks                                                                                                                                                      | Level Correction | Cumulative level Correction |
|---------------------------------|--------------------------------------------------|---------------------------------------------|---------------|---------|-------------------|--------------|----------------|----------------------------------|--------|--------------------------------|--------|-------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------|-----------------------------|
|                                 |                                                  | Back End −                                  | Forward End + | Diffce. | $\frac{1}{2}$ Sum | Correc-tions |                | Rise +                           | Fall − | Rise +                         | Fall − |                         |                                                                                                                                                              |                  |                             |
| Back ...                        | 10·00                                            | 73·9                                        | 73·1          | − 0·8   | ...               | ...          | 11·234         |                                  |        |                                |        |                         | Commenced forward section on yesterday's initial picket, having first tested the stability of it by re-observing station 1 of that day, 11th February, 1862. |                  |                             |
| Forward                         | 10·00                                            | 74·6                                        | 72·3          | − 2·3   | ...               | ...          | 11·755         | ...                              | 0·521  | ...                            | 0·529  |                         |                                                                                                                                                              |                  |                             |
| 1                               |                                                  |                                             | Sum           | − 3·1   | 1·55              | − 8          |                |                                  |        |                                |        |                         |                                                                                                                                                              |                  |                             |
| Back ...                        | 34°                                              | 73·9                                        | 73·1          | − 0·8   | ...               | ...          | 5·686          | ...                              | 0·539  | ...                            | 0·528  |                         |                                                                                                                                                              |                  |                             |
| Forward                         | 158                                              | 71·0                                        | 76·0          | + 5·0   | ...               | ...          | 6·225          |                                  |        |                                |        |                         |                                                                                                                                                              |                  |                             |
|                                 |                                                  |                                             | Sum           | + 4·2   | 2·10              | + 11         | ...            | ...                              | ...    | ...                            | 0·529  | − 0·529                 |                                                                                                                                                              |                  | + 2                         |

PROCEDURE IN THE FIELD.

[PART VII.]

CHAP. III.]

PROCEDURE IN THE FIELD.

|             |       |      |      |       |      |      |             |       |       |             |                                                                                                                                                                                                                                                                                                                                                                                             |         |         |
|-------------|-------|------|------|-------|------|------|-------------|-------|-------|-------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------|---------|
| Back ...    | 11.24 | 76.1 | 70.4 | - 5.7 | ...  | ...  | 9.334       | 1.138 | ...   | 1.138       | <p>Forward staff on stone Bench-mark imbedded at Pipalwala Chauki in lands of Jorah or Jori village. The stone is about 100 feet west of the high road, and 50 feet west of a Pipal tree, under which a Road Chauki has been placed.</p> <p>Repeated on account of discrepancy between first and second results. The general mean of all the observations is the value finally adopted.</p> | - 2     | 0       |
| Forward     | 11.24 | 70.3 | 76.0 | + 5.7 | ...  | ...  | 8.196       | 1.138 | ...   | 1.138       |                                                                                                                                                                                                                                                                                                                                                                                             |         |         |
| 2           |       |      | Sum  | 0.0   | 0    | ...  |             |       |       |             |                                                                                                                                                                                                                                                                                                                                                                                             |         |         |
| Back ...    | 335°  | 76.4 | 69.4 | - 7.0 | ...  | ...  | 3.785       | 1.141 | ...   | 1.130       |                                                                                                                                                                                                                                                                                                                                                                                             |         |         |
| Forward     | 154   | 71.4 | 74.9 | + 3.5 | ...  | ...  | 2.644       |       |       |             |                                                                                                                                                                                                                                                                                                                                                                                             |         |         |
|             |       |      | Sum  | - 3.5 | 1.75 | - 11 |             |       |       |             |                                                                                                                                                                                                                                                                                                                                                                                             |         |         |
| Back ...    | ...   | 76.7 | 69.0 | - 7.7 | ...  | ...  | 9.336       |       |       |             |                                                                                                                                                                                                                                                                                                                                                                                             |         |         |
| Forward     | ...   | 70.4 | 75.1 | + 4.7 | ...  | ...  | 8.195       | 1.141 | ...   | 1.132       |                                                                                                                                                                                                                                                                                                                                                                                             |         |         |
| Repetition  |       |      | Sum  | - 3.0 | 1.50 | - 9  |             |       |       |             |                                                                                                                                                                                                                                                                                                                                                                                             |         |         |
| Back ...    | ...   | 73.0 | 72.5 | - 0.5 | ...  | ...  | 3.766       | 1.121 | ...   | 1.133       |                                                                                                                                                                                                                                                                                                                                                                                             |         |         |
| Forward     | ...   | 70.5 | 75.0 | + 4.5 | ...  | ...  | 2.645       |       |       |             |                                                                                                                                                                                                                                                                                                                                                                                             |         |         |
|             |       |      | Sum  | + 4.0 | 2.00 | + 12 | ...         | ...   | ...   | 1.133       |                                                                                                                                                                                                                                                                                                                                                                                             | ...     | + 0.604 |
| Back ...    | 10.50 | 72.0 | 71.5 | - 0.5 | ...  | ...  | 9.075       |       |       |             |                                                                                                                                                                                                                                                                                                                                                                                             |         |         |
| Forward     | 10.50 | 69.7 | 73.9 | + 4.2 | ...  | ...  | 10.414      | ...   | 1.339 | ...         |                                                                                                                                                                                                                                                                                                                                                                                             | 1.328   |         |
| 3           |       |      | Sum  | + 3.7 | 1.85 | + 11 |             |       |       |             |                                                                                                                                                                                                                                                                                                                                                                                             |         |         |
| Back ...    | 348°  | 71.4 | 72.1 | + 0.7 | ...  | ...  | 3.524       | ...   | 1.341 | ...         | 1.326                                                                                                                                                                                                                                                                                                                                                                                       |         |         |
| Forward     | 155   | 69.9 | 74.1 | + 4.2 | ...  | ...  | 4.865       |       |       |             |                                                                                                                                                                                                                                                                                                                                                                                             |         |         |
|             |       |      | Sum  | + 4.9 | 2.45 | + 15 | ...         | ...   | ...   | ...         | 1.327                                                                                                                                                                                                                                                                                                                                                                                       | - 0.723 |         |
| Recorded by |       |      |      |       |      |      | Examined by |       |       | Levelled by |                                                                                                                                                                                                                                                                                                                                                                                             |         |         |
|             |       |      |      |       |      |      | Totals ...  |       |       | 1.133       | 1.856                                                                                                                                                                                                                                                                                                                                                                                       | - 0.723 |         |

## CHAPTER IV. DUTIES IN RECESS.

1. It is a practice in the Levelling Party to prepare duplicates of the field books. This is done by transcribing from the original, the observations only, that is the entries in columns 1 to 4, 8 and 14, see pages 306 and 307. The remaining columns of the form are next filled in from computation made quite independently, and compared with the original. The advantage of this method is that errors of calculation are often brought to light and corrected in the original.

Duplicate field books.

2. The next step is the preparation of the abstract of which a sample is here given. As all the necessary information is contained in the headings of the form, it needs no further explanation. The abstract should first be made out for the sectional points of the main line, next for the intermediate points, and lastly for points on the several branch lines. The abstract is also prepared in duplicate by two computers working independently.

Preparation of abstract.

3. The press copy of pamphlets of heights is also prepared in the office of the Levelling Party, a copy of which is given in the original field book. In addition to the descriptions and heights above sea level, &c., of the bench-marks, exhibited in a tabular form, section by section, there is added an introduction giving a brief account of the operations, mentioning the names of the observers and the instruments employed, also the error generated in the course of the work.

Press copy of spirit-levelled heights.

4. A skeleton chart is also prepared by the field party showing the season's operations.

Chart.

7.]

DUTIES IN RECESS.

| Points determined | By Rectangular Level               |                               |                             | By Cylindrical Level No. 3 |                               |                             | REMARKS                                                       |         |
|-------------------|------------------------------------|-------------------------------|-----------------------------|----------------------------|-------------------------------|-----------------------------|---------------------------------------------------------------|---------|
|                   | Sectional Values                   |                               | Height above Mean Sea Level | Sectional Values           |                               | Height above Mean Sea Level |                                                               |         |
|                   | Observed                           | Corrected to Standard to Unit |                             | Observed                   | Corrected to Standard to Unit |                             |                                                               |         |
| G.T.S.            | ...                                | ...                           | 10'409                      | ...                        | ...                           | 10'409                      | * From Spirit-levelled Heights No. 1 Bombay Presidency, 1879. |         |
| 1 B.M.<br>C.      | Navánár Tidal Station              | ...                           | ...                         | ...                        | ...                           | ...                         |                                                               |         |
| G.T.S.            | ...                                | ...                           | 11'683                      | 1'312 +                    | 1'312                         | 11'721                      |                                                               |         |
| 2 B.M.<br>G.T.S.  | Tapaska Dhoi                       | ...                           | ...                         | 73'003 +                   | 72'987                        | 84'708                      | Devji Khoja's House.                                          |         |
| 6 B.M.<br>G.T.S.  | Kapáia Mota village at entrance to | ...                           | 84'687                      | 60'330 +                   | 60'317                        | 190'683                     |                                                               |         |
| 9 B.M.<br>G.T.S.  | Kálághogha, Stone W. of Village    | ...                           | 60'297 +                    | 60'304                     | 190'684                       |                             |                                                               |         |
| 6 B.M.<br>G.T.S.  | Kapáia Mota village entrance to    | ...                           | ...                         | ...                        | ...                           | 84'708                      | Devji Khoja's House.                                          |         |
| 3 B.M.<br>G.T.S.  | Mundra embedded B.M.               | ...                           | 49'324 -                    | 49'330                     | 35'357                        | 49'349 -                    | 49'338                                                        | 35'364  |
| 4 B.M.<br>G.T.S.  | Mundra. Jairám Shivji's            | ...                           | 43'365 -                    | 43'370                     | 41'317                        | 43'387 -                    | 43'378                                                        | 41'324  |
| 9 B.M.<br>G.T.S.  | Kálághogha, Stone W. of Village    | ...                           | ...                         | ...                        | ...                           | ...                         | ...                                                           | ...     |
| 1 B.M.<br>G.T.S.  | Khára Nála Well                    | ...                           | 55'261 -                    | 55'267                     | 135'420                       | 55'292 -                    | 55'280                                                        | 135'395 |
| 9 B.M.            |                                    | ...                           |                             |                            |                               |                             |                                                               | 135'408 |

INTERMEDIATE POINTS.  
BRANCH LINE POINTS.

## PENDULUMS.

[PART VIII.]

## PART VIII.

**Pendulums.**

It is altogether beyond the province of this Handbook to enter upon the details of such an extensive and difficult subject as the swinging of pendulums to determine variations of the force of gravity; more especially as there is no immediate probability of any such work being again undertaken in India. It is, however, such an important factor in a geodetic survey that it cannot be entirely passed over without reference; but it will be sufficient for the purposes of this Handbook if a list of the standard works on the subject is brought within the reach of the reader to enable him to master the details, should he ever be called upon to take part in such operations.

It fortunately happens that Colonel Herschel, R.E., the officer to whom the writing of the Volume of the G. T. Survey of India (Vol. V) on Pendulums, was entrusted, has made a particular study of the bibliography of Pendulums, and has added to that volume an appendix containing a list of nearly three hundred books and papers on this subject. In addition to the list the volume gives a detailed account of the instruments used, and the whole procedure adopted in India, and the reader is referred thereto as a trustworthy guide, if at any time observations of the same kind should be again required.

## PART IX.

**Miscellaneous Subjects.**

## CHAPTER I. ANNUAL REPORTS, &amp;c.

I. The operations of the Trigonometrical Branch are so varied in their character, that it is almost impossible to lay down exact rules by which all Executive Officers may be guided in the preparation of their annual reports.

Executive Officers must always keep in mind that the object of these reports is to enable the authorities to know exactly what is done



from year to year; they must therefore so draw them up, that all who are interested in such matters will have sufficient details available to enable them to see and judge for themselves of the progress and quality of the work turned out.

2. In all cases the dates of going to, and returning from, the field must be shown, the strength of the establishment, and the duties allotted to each member of it. The amount of work done and the programme for the ensuing field season, as well as a report on the conduct and efficiency of the subordinates attached to the party, must be given. To this extent the circumstances of all the parties are nearly similar. For details executive officers may with advantage consult the published annual reports of the Survey of India Department which will give them a clear idea of what it is advisable to insert.

General remarks on annual reports.

3. For instance, an officer engaged upon Triangulation may with propriety describe the country over which his work extends; and if it is a more or less unknown district, he should make the best use he can of his opportunities for collecting notes about the inhabitants, their manners, customs, language, &c., for insertion in his report. He should also discuss any peculiarities of his instrument, noting any weak points in it, and describing any precautions taken to guard against any evil effects therefrom; he should state his average triangular error, the number of azimuths observed, giving also a general description of the stations, and the precautions taken for their protection. Any political difficulties met with in the past, or anticipated in the future, should also be touched upon.

Annual report of a triangulation party.

4. An officer employed upon Latitude observations should describe in detail the programme of star observations at each station, the instrument he used, and any peculiarities noticed in it; also any remedies he devised to cancel them. He should also draw up a tabular statement giving the values of the latitudes observed, with their probable errors, the number of stars from which they were deduced and the differences between the astronomic and geodetic latitudes of the stations occupied. Any hints for future observers deduced from his own experience may properly find a place in his report.

Annual report of a latitude party.

5. A report on Electro-Telegraphic Longitude Operations should contain a list of the arcs measured with the name of the observer at each station, in the order of measurement. A discussion of any peculiarities or abnormal results met with should follow, with suggestions for future avoidance. The method adopted for ascertaining the observer's personal equations is important, and must be described, and a statement of the results at each determination must be prepared.

The values of the several arcs, as determined astronomically, and geodetically, must be compared, and the circuit errors presented in a convenient form. The form of battery used, and the number of cells in circuit, may be added with advantage.

6. The reports of the Tidal and Levelling party are somewhat voluminous. They should contain a brief account of the working of all the instruments at each tidal observatory, specifying the date of inspection, and the steps taken to remedy any defects found. Statements must also be furnished showing the tidal constants both for short-period, and long-period tides at all the observatories, also tabular statements showing:—

- (1) Percentage and amount of errors in predicted times of high water
- (2)                   "                   "                   "                   low                   "
- (3)                   "                   "                   heights of high                   "
- (4)                   "                   "                   "                   low                   "
- (5) Table of average errors in predicted times and heights of high and low water. The progress and present state of the tide tables, as worked out by the Tide Predictor, must also be noticed.

The lines of levels completed should be specified, and the errors in closing on bench-marks noted, as well as any discrepancies found between the heights of principal stations as determined by triangulating, and by levelling, respectively. New bench-marks must also be noticed, when they are in connection with tidal stations.

7. In addition to the annual reports explained above it is necessary for every Executive Officer in charge of a party to submit a precis of his report. This precis must be a brief account of the operations of the party, so written as to be as far as possible intelligible to a non-professional reader, and

free from technicalities. Its object is to enable a Secretary to Government, or other persons not conversant with scientific details, to gain a fair idea of the progress and state of the work: it should be written in the third person, and the services of the individual members of the party should not be alluded to, unless they are particularly praiseworthy, when they may be mentioned in a footnote.

As this precis is intended to be inserted *verbatim* in Part II of the Annual Reports of the Survey of India, it saves a great deal of trouble in compiling these reports if attention is paid to making the form and wording of the precis suitable for this purpose, so as to avoid as far as possible the necessity of subsequent alteration in the Deputy Surveyor General's Office. Specimens may be found in the Annual Reports of each year, which may be taken as guides to the Executive Officer. When Indian terms are used a translation of them should be given.

8. All reports on survey operations or on any other kindred subjects, beyond the limits of British India, and of the feudatory Native states, are to be considered in the first instance as strictly confidential.

Confidential reports.

9. Officers of the senior division must consider it part of their duty during recess to visit the offices of any other survey parties that may be recessing in the same station. It frequently happens that the procedure of different parties varies in some particulars, and mutual visits between the officers concerned tends to utilize to the utmost their varied experiences.

Visits to other survey offices in the same station.

10. Officers of both divisions visiting the presidency should, unless incapacitated by ill-health, be officially conducted through the offices of the Survey of India; and be made acquainted as far as possible with the procedure of the Drawing, Engraving, Mathematical Instrument, Photographic and Lithographic offices. Advantage should be taken by officers passing through Dehra of the opportunity to pay an official visit to the offices of the Trigonometrical Branch, after having obtained permission from the Deputy Surveyor General.

Visits to Head-Quarters offices.

11. Officers and surveyors of all grades will in future not only report in person their arrival at the Head-Quarters of the Survey of India at Calcutta and

Arrivals to be reported to senior survey officer.

Dehra, but at all stations where an office of the Survey of India is established, they will visit that office, and intimate their addresses to the officer in charge. In the case of a station where there are several parties quartered together, the visit will be made to the senior officer present.

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CHAPTER II. MANAGEMENT OF THE HEALTH OF A PARTY.\*

1. Although in general the occupation of a surveyor may be reckoned more healthy than any other mode of life in India, on account of the fresh air, constant exercise, and interesting nature of the employment, still there are some tracts of country in which trigonometrical survey parties are exposed to considerable risk from jungle fever and other diseases which prevail with greater or less virulence for several months in the year.

2. As these parts of the country must, however, be surveyed it becomes an important matter to consider the best sanitary arrangements for the protection of survey parties, from jungle fever in the first instance ; and secondly, the proper treatment and line of conduct to be pursued in the event of the disease breaking out as an epidemic in camps.

3. That it is practicable, by judicious sanitary arrangements, to carry on operations with comparative safety for several months in such tracts, has been proved by the experience of the Survey Department in all parts of India ; while it has also been demonstrated that by proper medical treatment, disaster may be prevented, and the establishment restored to efficiency after being attacked by jungle fever.

4. Although it may, at first, appear to be beyond the province of a non-medical person to enter on a subject of this nature, still surveyors are of necessity thrown upon their own resources, and compelled to qualify themselves, as well as they can, to perform this work of humanity, in addition to their own all-absorbing duties. Fortunately, the treatment of jungle fever, if taken in hand on the first appearance of premonitory symptoms, and the patient be not unnecessarily exposed to aggravating circumstances, is sufficiently simple to be learned by any

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\* This section is reprinted from an Extract from Remarks on the Sanitary arrangements of Survey parties in the jungly tracts of India, by Colonel Sir Andrew Scott Waugh, Surveyor General of India ; Dehra Dún, October 1861.

person of common sagacity, and will prove successful in nineteen cases out of twenty ; though, if once the disease be allowed to gain head, it will require the utmost skill and refinement of the physician's science to reduce it.

5. Whatever rules are here given have been derived, in the first instance, from the advice of medical friends ; they have been tested by experience, and are believed to be consistent with modern practice, as far as they go. When once a moderate degree of skill is acquired by experience, combined with the advice which medical officers are always willing to afford, it will be found that a degree of confidence will be inspired among all parties in camp,—a feeling indispensable to success in all arduous undertakings.

6. Although several cases of failure have occurred in the progress of the Indian surveys, owing to the disastrous effects of jungle fever, these have generally happened from the want of experience ; and instances are numerous where the work has been carried out with success, notwithstanding the prevalence of this disease. There are even instances where one portion of a party has escaped on the same tract in which another portion has suffered, merely owing to the difference of habit and treatment.

7. The benefit of breakfasting the whole party before starting has been proved by the practice of a number of officers ; and to this, as well as to the superior stamina derived from good living, may be attributed the circumstance, that Mussalmans are less subject to the influence of miasma than Hindus, who cannot conveniently cook oftener than once a day. Nevertheless, the latter class of people should be induced to eat parched grain or something similar to break their fast, or be compelled to cook a regular meal in the early morning, if employed in very insalubrious places.

8. Early in the season, when marching to the field of survey, and afterwards on returning to quarters, when the weather is very hot, it is usual to move at night, to save the men from exposure to sun. It is impossible so far to break up established habits as to breakfast in the middle of the night ; but it is usual for officers to take a cup of coffee and a biscuit ; and as a further precaution, if likely to be detained

out long, it is a good plan to fill the pockets with biscuits, ginger-bread-nuts, or something of that sort, and carry a bottle of cold tea or cold weak coffee without milk.\*

9. In the survey season, surveyors are liable to be out all day, from morning till night. In such circumstances it is desirable to be accompanied with a basket containing sandwiches, &c. ; for the great object is to avoid having to labour under a feeling of exhaustion arising from mere want of food—a state in which the human frame is peculiarly susceptible to malarious influences. For this reason the men should have a lunch of parched grain and sugar, when out at work through the whole day.

10. Now, it may be remarked that most officers take these precautions for themselves: they also sleep under canvas, protected from the dew, and upon beds raised above the exhalations of the ground. But the experience of the Department shows that a large proportion of the native establishment and servants may be laid up with fever and brought to the verge of death, without the officer in charge or assistants being affected in the smallest degree. This will always be the case, unless the Europeans are of weakly constitution, peculiarly susceptible to fever, or neglectful of their comforts.

11. It is difficult in the case of Hindus to get them to cook their meals at proper times, but as far as possible it should be done ; and all surveyors who are careful of their men are sedulous to protect them from dew at night, and to furnish them with straw to sleep on. The use of camel carriage in Upper India renders it easy to carry charpoys for some of the upper servants ; and when men are detached to show signals from hill stations, they are always directed to hut themselves, which can easily be effected in the jungles at the expense of two or three rupees. In the case of a party recently employed in malarious tracts in the Central Provinces, the executive officer has furnished his men with tea, blankets and filters. The effects have been so to increase their efficiency that the cost of these comforts has been abundantly repaid by increased outturn of work.

\* Make coffee as usual and pour into a bottle till half full, fill up with hot water and add sugar according to taste: a man can work all day on this beverage.

12. In addition to the ordinary precautions above directed, it is usual in the case of persons very susceptible of fever to fortify themselves by taking quinine while they are employed in unhealthy localities; and this certainly has a good preventive effect.

Quinine as a preventive.

13. In order to avoid the particular spots which are unhealthy, all due enquiry should be made of the indigenous inhabitants, with whom relations of amity should be maintained. The cleared and inhabited parts should be preferred for camping on, or high open spots, and the vicinity of rivers avoided, as well as closely wooded places, where there is no circulation of air or sunshine on the ground. Every enquiry should also be made for good drinking water; when it is likely to be scarce, it should always be carried on from the last stage, in order that time may be given to find out the best water at the new encampment: many persons use boiled water—a precaution which even the native inhabitants practise in some parts. In some parts of India well water is considered the most wholesome, while that from streams is reckoned deleterious. In other places well water is considered to occasion fluxes, but this is only where the soil is impregnated with saline substances. As a general rule, in all tracts much covered with vegetation, river water is dangerous, and resort should be had to wells. But it must be recollected that the water in old wells not in daily use, and filled with rubbish, is always stagnant, putrid, and more dangerous than any other water.

Importance of open camping grounds and good water.

14. Survey parties are usually attended by hospital assistants, who compound the medicines and act according to the orders of the officer in charge. These men are very useful in attendance on the sick, in general arrangement and hospital duties, and some are pretty skilful in the use of medicines; but, as a general rule, they are wanting in a strong mind to fall back on for advice in times of difficulty and danger, being either disposed to act with timidity or rashness. Be this as it may, the attendance of the officer on the sick prevents neglect, and cheers the poor men in their misfortunes. It sometimes happens, however, that the hospital assistant requires to be left at some distance, while observations are being taken on hill tops, moreover it usually happens that some of the subordinates being detached are unable to avail themselves of the.

Hospital Assistants.

hospital assistant's services. Under such conditions in order to avoid the weighing and mixing of medicines which is not a very pleasant business at any time, and particularly irksome to a man fagged by a hard day's survey work, it is a good plan to furnish each person with a set of medicines prepared and weighed out beforehand, whereby all trouble will be saved.

15. It may be said that the sedulous attention here recommended to ensure the comfort of every individual in camp would substitute, for the ancient hardy habits of the Indian surveyors, a sybaritic regard for luxury, inconsistent with the nature of the duties. It may look much more manly to be able to brave malaria on an empty stomach: but what is the use of hardy habits and contempt for comfort if no work is done? It is a very good thing to be able to brave an Indian sun with impunity; but it will be found that an umbrella will enable any man to accomplish more survey work than he could without that protection. The quantity of work is the only criterion of good habit and arrangement, and all other experience shows that, unless a party take the field well equipped with tents and other comforts, health is endangered, and the interests of Government are sacrificed.

16. The number of diseases to which a party under canvas are exposed are comparatively few, provided the health of the persons composing the party was good at starting, that the coolies &c., are well provided with warm blankets during the cold season, and that every care is taken to keep the camp clean, dry, and within easy reach of a supply of good drinking water.

17. Most of the diseases to which survey parties are liable are treated of at considerable length in Moore's Family Medicine for India, a book which all European members of a Survey party should carry with them. The price of this work is Rs. 3 to all government officials receiving less than Rs. 500 per mensem, and Rs. 4 to all others, and to the public at large. A certificate declaring that the book is to be purchased only for the personal use of the officer is necessary for the lower rate. It may be convenient however to give here a brief description of the symptoms and treatment of the commoner forms of sickness prevalent in survey camps.



18. Malarious fevers are beyond question the diseases which most tend to interfere with the progress of Survey parties in India; and a clear conception as to how they should be treated in a general way in cases of emergency is of very great value to every person liable to exposure to malarial influences. The fevers are referred to under various designations in this country, such as *Fever and Ague*, *Terai fever*, *Marsh fever*, *Bengal fever*, *Jungle fever*, &c. The affections commonly referred to by these names correspond to what medical men describe as *Intermittent* and *Remittent* forms of malarious fevers. By intermittent fever is meant that form of fever which, whilst recurring daily, on alternate days, or every third day, leaves the patient fever-free in the intervals; there is an *intermission* of feverish symptoms—in the first case of one day, in the second of two, and in the last of three, more or less, clear days. In the case of remittent fever, however, the feverishness does not completely disappear between paroxysms, there is simply a diminution—a *remission* of the attack. It is the severer attacks of this latter form of malarious disease that is usually meant when the terms *jungle* and *terai* fever are used, whilst the former is commonly referred to as *ague*.

It is, however, frequently difficult, even for the experienced physician, to decide definitely whether a person is suffering from the intermittent or the remittent form of the malady, as the symptoms often 'shade off' one into another—the fever being intermittent for a few days, then remittent, and again, perhaps, intermittent.

Fortunately the general line of treatment to be adopted is the same in both forms, and it will probably be found that when working in some malarious localities, the two forms of the disease may prevail simultaneously.

19. When it has been decided to visit a locality known to be productive of fever, it should be laid down as a general rule that every member of the party should receive small doses of some cinchona preparation during a few days previously, and twice daily during the stay in the locality. Two to three grains of quinine should be taken morning and evening, or, what is equally efficacious and very much cheaper, the same quantity of the *Cinchona febrifuge*, prepared at the Government cinchona

Prophylactic measures.

plantation. The expense of a prophylactic measure of this kind would be very trifling, especially if the *febrifuge* be resorted to.

20. An attack of ague consists of three more or less distinctly marked stages: a *cold* stage, often not well marked, but it may be severe and last from one to four hours; a *hot* stage, which may last from three to six hours—seldom more than twelve; and a *sweating* stage, which terminates in a couple of hours, and is followed by complete relief. The average duration of the attack is from five to six hours.

Intermittent fever.

21. When the slightest symptoms exist of an impending attack of fever, the condition of the bowels should be attended to, and if any tendency to constipation exist, a mild purgative may be administered—a small dose of castor oil being the safest. Should it be considered advisable to administer a purgative, wait for an hour after its administration, and then give 10 to 15 grains of quinine [or an equivalent dose of the cinchona febrifuge], and repeat the dose in about six hours. Early and vigorous treatment of this kind, if not always successful in warding off the attack altogether, tends greatly to diminish its severity.

Treatment Intermittent fever.

Should the symptoms continue, and shivering set in, marking the advent of the *cold* stage, the patient should be well wrapped in blankets; hot bottles, or hot bricks wrapped in flannel, should be applied to the feet, and warm tea, toast and water, or some such simple fluid given him to drink. Should constipation have existed which the castor oil had not relieved, a mild purgative may again be administered during this stage. Say a drachm of compound jalap powder in a wine-glassful of water, or a podophyllin pill.

Should, however, there be only slight constipation, await the advent of the *hot* stage, and administer two teaspoonfuls of granular citrate of magnesia or a laxative dose of pyretic saline in half a tumbler of water. Half a teaspoonful of this granular preparation, added, as required, to a little sweetened water, and administered every hour or so, will form a pleasant effervescent febrifuge. If, however, the bowels are relaxed, it will be better to keep to cold tea or toast and water, according to the patient's inclination. The bed clothes may be gradually lessened as the hot stage progresses, and attempts should be made to relieve headache by applying wet cloths to the head.

As soon as the *sweating* stage sets in, 10 grains of quinine [or an equivalent quantity of the *Cinchona febrifuge*] should be given every eight hours or so during the *intermission*. The quinine (or the *cinchona febrifuge*) may either be taken in powder, or dissolved by means of half a teaspoonful of the juice of a lime, and taken in a wine-glassful of water. Care should be taken to avoid a chill during the hot and sweating stages. Should diarrhœa be present, a dose of chlorodyne may be administered during the intermission, and each dose of quinine combined with 3 grains of Dover's powder.

22. The general rules thus laid down for the treatment of intermittent fever are equally applicable to the treatment of the generally more serious form of malarious disease called jungle fever. The premonitory symptoms should be treated on precisely similar principles, with the addition that it is more advisable to abstain from partaking solid food shortly before an attack is anticipated, as vomiting, which is a prominent symptom here, is apt to be more aggravated than when the stomach is empty.

Should the mild purgative and preliminary doses of quinine not have sufficed to ward off an attack, the cold stage will be ushered in by more or less distinctly marked shivers; but it will be found that, as a rule, this stage is less marked here than in an ordinary attack of ague; the succeeding or hot stage, however, is more prolonged, and generally more severe. The temperature runs high, the pulse is very quick, often bilious vomiting of a very obstinate character occurs; the tongue is furred, and sometimes jaundice sets in. The patient is very restless, and his remarks often incoherent.

This stage may last from six to eight hours or longer, and, instead of terminating in severe sweating, followed by complete relief from all painful sensations, as in intermittent fever, there is only a subsidence in the severity of the symptoms—a *remission*, not a cessation, of the fever. The duration of the paroxysm is uncertain—may last from 6 to 24 hours, and an attack may recur daily for a week, 10 days, or even longer.

When the cold stage has passed away, and the pulse becomes full and the face flushed, administer frequent small doses of the granular citrate of magnesia as above directed, and apply cold to the head: avoid stimulants of all kinds, but attempts should be constantly made

to get the patient to partake of beef tea, Liebig's extractum carnis, and such like nutritious food in a liquid form.

If the patient becomes delirious, the lower part of the back of the head should be shaved, and a blister (about 2 inches  $\times$  3 inches) raised, by means of blistering fluid, across the nape of the neck. If this be of no avail, and the pulse continues full and bounding, and the patient be a strong man, four to six leeches may be applied to the temples—the bleeding being arrested when the leeches have dropped off. It is advisable to defer giving quinine until the remission sets in. This will be known by the diminution in the temperature, and the appearance of a gentle perspiration, together with, probably, the disappearance of the head symptoms. Five grains of quinine may now be given every three hours in the form of a mixture, or dissolved in the juice of a lime with water. Nourishing food should be given, with a very little stimulant, if desired. Should the patient suffer from diarrhœa also, the 5 grain quinine doses given during the remission may be combined with 3 grains of Dover's powder. If the head symptoms have been severe, it will perhaps be advisable to keep to quinine, but otherwise an equivalent dose of the cinchona febrifuge may be safely substituted for it.

If the attack has been severe, have the patient removed, if practicable, as quietly and as expeditiously as possible, to the nearest station where medical aid can be procured. This is especially desirable, if the disease has recurred two or three times without any appreciable diminution in the aggravation of the successive attacks.

23. Diarrhœa and dysentery sometimes appear suddenly in a camp, and in aggravated forms, and require prompt attention, especially as the former is very apt to pass into the latter in malarious districts.

The first precaution to be taken on the advent of diarrhœa is abstinence from solid food; nutritious food in a liquid, or semi-solid form should be substituted, and cold drinks should be avoided. Should the diarrhœa have been immediately preceded by constipation, a small dose of castor oil should be taken, and possibly after this has acted, the bowels may return to their ordinary condition.

If, however, the diarrhœa has not been preceded by constipation, 30 drops of chlorodyne may be taken in half a wine-glassful of lukewarm

water. Should there be pain in the bowels, warm water fomentations should be applied to the stomach, and a flannel belt tightly drawn round. Three or four hours after taking the chlorodyne, 5 grains of quinine, or 5 grains of the cinchona febrifuge should be taken in combination with 5 grains of Dover's powder, and the patient sent to bed. It is probable that after prompt action of this kind, the patient may find himself perfectly well by the next morning: if not, let the chlorodyne be repeated, followed by the quinine and Dover's powder, twice daily, for a day or two, or until perfect recovery.

Should the diarrhœa pass on to dysentery, which may be inferred if irregular attacks of griping pain in the abdomen persist with increasing tendency to strain at stool, the condition of the stools themselves should be carefully noted, and if traces of blood or fleshy shreds be observed, it is absolutely necessary that complete rest should be enjoined. The patient should be given a grain of opium, or 30 drops of chlorodyne in a little water, to be followed in about quarter of an hour by 20 grains of ipecacuanha powder in half a wine-glassful of water, or the powder may be given as a bolus. This medicine will probably be succeeded by nausea, but every attempt should be made to keep it down. This is best done by the patient remaining on his back; indeed throughout the attack the patient should be made to lie down. Repeat the ipecacuanha powder in about three hours, and continue the dose twice a day for two or three days, or until the disease stops. Hot bran or linseed meal poultices should be applied constantly to the stomach; and should there be severe pain, apply a turpentine stupe (made by pouring about a tablespoonful of turpentine on a piece of flannel wrung out of hot water) over the painful part.

If the patient does not recover in about a week after a fair trial of this treatment, he should be carried to the nearest station where medical aid may be procured.

24. When cholera breaks out in a camp, the first step to be taken is, to shift the tents to some considerable distance; and if this does not suffice, shift again, across a river if practicable: of course the greatest possible attention should be paid to the sanitary state of the camp, and especial care taken during periods of this kind in particular to avoid unwholesome food and impure water.

Cholera.

25. Unfortunately no very certain remedy is known for this disease, but much may be done by early treatment. The premonitory diarrhœa, if present, should be treated with 30 drops of chlorodyne every three or four hours, and complete rest enjoined. Should, however, the stools become watery in appearance and mixed with bran-like flocculi, the pulse become small and thready, the skin blue and pinched, and the voice husky, 10 drops, of the '*Cholera drops*,' supplied by the Medical Stores Department, should be given every half an hour in a tablespoonful of water. Or, if this medicine be not at hand, 15 drops of sal volatile should be given instead, in half a wine-glassful of water.

Treatment of cholera.

When severe cramps of the limbs or of the muscles of the chest come on, the painful parts should be sedulously rubbed with a mixture of ginger (or mustard) powder and linseed meal, easily obtainable in any bazaar, and cold drinks (soda water and the like) given when desired to quench the great thirst, from which the patient will in all probability suffer.

As soon as reaction sets in, the pulse regain its strength, and the unexposed parts of the body become warmer to the applied hand, the stimulating medicine (whether '*cholera drops*' or ammonia) should be discontinued. Attempts should be made to get the patient to swallow a little beef tea, Liebig's extractum carnis, or such other nutritious food as may be available of an allied character. Occasionally, small effervescing draughts, made by adding half a teaspoonful of citrate of magnesia to a little sweetened water may be given. Should the urinary functions not have become restored towards the end of the second day, the loins should be well fomented with warm water, followed by a turpentine stupe applied over the kidneys, with a linseed or bran poultice over the bladder. Should indications of stupor be observed, and the tongue become dry, the back of the head should be shaved, and the nape of the neck painted two or three times with blistering fluid. As a rule, when the urinary secretion becomes re-established, convalescence is rapid.

26. The Europeans of a party are more liable to suffer from sun-stroke than the natives, and it not uncommonly happens that when one attack has occurred, it is

Sun-stroke.

followed during the next few days by others. It is therefore especially necessary that the tents should be pitched in shaded positions, and that great care should be taken to avoid unnecessary exposure to the sun's influence for some days after a case of this kind has occurred.

No very clear description can be given of the symptoms which precede an attack, and very frequently none whatever are observed: a person may become suddenly faint, and fall without having given any previous intimation of his state.

27. The *treatment* for such a case is, to lay the patient on his back in the shade, have his limbs well rubbed, and a dose of sal volatile (40 drops) in water administered, or, if ammonia be not at hand, a little brandy and water may be substituted.

Treatment of sun-stroke.

Should, however, the face get flushed, the pulse full, and the heat of the body to the applied hand very great, with stertorous respiration, the *bhisties* should be called and directed to pour a continuous stream of cold water from their *mussucks* upon the patient's head for several minutes, the head being, meanwhile, kept well elevated. Should this procedure not prove successful in improving the patient's condition, the back of his head should be shaved, and blistering fluid painted over the part, so as to raise a blister about the size of a man's hand.

Two drachms of compound jalap powder should be administered in a wine-glassful of water, followed in the course of half an hour by 20 grains of quinine, in solution if possible. Attempts should also be made to clear the bowels by means of warm water enemata, if practicable. During recovery absolute rest must be insisted on, nourishing food should be given, with only a minimum of stimulants, and the patient not allowed to resume work in the sun for some weeks.

28. A short list is appended of the medicines which would be found useful in cases of emergency of the kind referred to above. Only the simpler kind of remedies have been mentioned, as it would be useless to do more than suggest the general line of treatment which meets with the approval of the majority of the profession. Were more than this done, it would tend to confuse non-professional persons, and an elaborate list of remedies would imply a great addition to the weight to be carried.

Supply of medicines for detached parties.

**A LIST OF MEDICINES, &c., WHICH, IT IS SUGGESTED, SHOULD BE TAKEN BY SMALL DETACHED SURVEY PARTIES OF ABOUT 25 MEN.**

- $\frac{1}{2}$  oz. *Sulphate of Quinine.*  
 2 oz. 'Cinchona Febrifuge' or 'Mixed Cinchona Alkaloids.'  
 1 oz. *Chlorodyne.*  
 2 oz. 'Cholera Drops,' as issued by Medical Stores Department.  
     Ol. Anisi, Ol. Cajeput, Ol. Juniper, each  $\frac{1}{2}$  ounce,  
     Æther  $\frac{1}{2}$  ounce; Liquor Acid. Halleri  $\frac{1}{2}$  ounce; Tinct.  
     Cinnam. 2 ounces; mix. Useful as a stimulant in  
     the cold stage of cholera, and in other cases where a  
     stimulant is desirable: 10 to 15 drops in water for a dose.  
 2 oz. *Sal Volatile* (Sp. Ammon. Aromat).  
 8 oz. Granular *Citrate of Magnesia.*  
 8 oz. *Epsom Salts.*  
 8 oz. *Castor Oil.*  
 2 oz. *Compound Jalap Powder*: a brisk purgative: dose 1 or 2  
     drachms.  
 1 oz. *Dover's Powder.*  
 25   One-grain opium pills [in a well-corked phial].  
 50   *Podophyllin pills* [Recipe, Podophylli Resinæ gr.  $\frac{1}{4}$ ; Extr.  
     Hyoscyam. gr. 1; Pil. Colocynth Co. gr. 3. Make a  
     pill]. Preserve in a well-corked phial.  
 50   Two-and-a-half grain *Quinine pills.* In a well-corked phial.  
 25   *Gallic acid* (three grains) and *opium* (half a grain) pills.  
     In a well-corked phial. Useful in mild cases of  
     diarrhœa: one or two for a dose.  
 1 oz. *Ipecacuanha Powder.* For treatment of dysentery.  
 1 oz. *Blistering Fluid.* [In order to produce a blister, paint  
     the part three or four times with a feather].  
 2 oz. *Solution of Acetate of Lead* [Liq. Plumbi Subacetas].  
     For making Goulard's Lotion, add about a tablespoonful  
     to a pint of water. A useful application to sprains and  
     to indolent sores.  
 1 oz. *Tincture of Steel.* Useful for stopping hæmorrhage. Dip  
     a cotton rag into the solution, press it into the wound,  
     and apply a bandage.  
 1 oz. *Tincture of Iodine.* Useful in reducing swollen glands  
     (buboes) in the armpit, groin, &c. Paint the part twice  
     daily with the tincture.



- 4 oz. *Zinc Ointment*, in a wide-mouthed stoppered bottle. Useful for sores, burns, scalds, &c.
- 3 Rolls of 1½-inch calico *bandages*.
- 1 Stick of *Caustic* (Nitrate of Silver,) mounted in a quill and put into a phial. Useful for sores, bites, &c.
- 1 Small roll of *Isinglass adhesive plaster* (*i. e.*, Court plaster). This is preferable to ordinary sticking-plaster, as the latter, owing to the heat, often becomes useless. To be moistened before application.

29. As several of the powder-medicines deteriorate greatly when kept in paper packages, and frequently, owing to damp, &c., become useless when thus previously weighed out, it is recommended that phials should be adopted in all cases and the doses weighed as required; a pair of small scales and a 2-oz. graduated glass measure being packed with the medicines. A small tin box, divided into compartments, could readily be devised; and the whole need not weigh more than 8 or 10 pounds.

#### APOTHECARIES' WEIGHTS AND MEASURES.

##### *Solids.*

|            |              |
|------------|--------------|
| 20 grains  | = 1 scruple. |
| 3 scruples | = 1 drachm.  |
| 8 drachms  | = 1 ounce.   |
| 12 ounces  | = 1 pound.   |

##### *Fluids.*

|           |                   |
|-----------|-------------------|
| 60 minims | = 1 fluid drachm. |
| 8 drachms | = 1 fluid ounce.  |
| 20 ounces | = 1 pint.         |

#### APPROXIMATE FLUID MEASURES.

|                |                      |
|----------------|----------------------|
| 1 minim        | = about 1 drop.      |
| 1 fluid drachm | = 1 teaspoonful.     |
| 2 „ drachms    | = 1 dessertspoonful. |
| 4 „ drachms    | = 1 tablespoonful.   |
| 1 „ ounce      | = 2 tablespoonfuls.  |
| 2 „ ounces     | = 1 wine-glassful.   |

## CHAPTER III. CARE AND TREATMENT OF ELEPHANTS.

1. In many parts of India and Burma elephants are almost indispensable for Trigonometrical survey parties, when employed on Triangulation, and as both the purchase and keep of these animals form a somewhat heavy charge, it is very desirable that executive officers should themselves exercise a careful supervision over their treatment in order to ensure their efficiency.

*Necessity of supervision.*

2. It would be impossible to enter here into all the diseases to which elephants are subject, but there are two which are the most frequent cause of temporary disablement, and which are generally preventible by reasonable care. These two are, (1) injuries to the feet, and (2) sore backs.

*Two principal causes of disablement.*

Injuries of the feet are very liable to occur in marching over ground from which bushes and small jungle have been cut down, with short sharp stumps left standing in the ground. An elephant treading on these, or even on small sharp pieces of stone, may cripple itself more or less: when such a road has to be traversed it is a good plan to harden the soles of the feet by the application of a paste, of which the ingredients are known to mahouts, and generally used by them for this purpose.

3. There are two kinds of disease peculiar to the feet of elephants, named 'Tawakh' and 'Sarjan'. They are both contracted through neglect, and from allowing the animal to stand for many consecutive days on the same 'tahan' without cleansing it. The best treatment is by nitric acid, applied with thin strips of copper, which should be dipped into the bottle containing the acid, and rubbed over the wound. The feet must be well pared and washed first; the acid should be applied morning and evening.

*Injuries to the feet, and their treatment.*

4. One of the best native remedies for 'Sarjan' is the following:—  
 'Tua' or 'Chowdara', one fruit, cut up, powdered, and mixed with one seer of mustard oil; this must be kept on the fire until it burns, and becomes thick and black. The mixture is spread over the wounded surface whilst it is quite hot. An ordinary paint-brush is the best means of applying it.

*Continued.*

Carbolic lotion is useful for dressing wounds on the feet of the elephant, and should be applied on plugs of tow inserted in the wounds after they have been carefully cleaned out, and all proud flesh cut away.

5. Sore backs are generally caused by the padding being out of order, in consequence of which the loads press directly on the back, and thus occasion inflammation, and very frequently suppuration to an extent that sometimes require months to cure. If on the first symptom of a swelling being observed, the place is well fomented and rubbed, a sore back may probably be avoided. But mahouts, as a class, are proverbially unobservant and heedless in the performance of their duties; and the chances are that the swelling will not be noticed by them, until it becomes so large that it is impossible to avoid seeing it. The mischief is then done, and suppuration, which must be allowed to take its course, has most probably set in. These suppurating abscesses usually take place a little behind the tips of the shoulder blades. When they are quite ripe they should be opened with a long lancet-shaped knife, four incisions in the lower surface being made—two on either side. It is better to open them thoroughly at first, otherwise the pus is liable to find its way into the surrounding cellular tissue, and an enormous diffused abscess is the result. After the matter has been well pressed out, the cavity should be thoroughly syringed with lukewarm water, twice a day, morning and evening—the cavity being filled between the syringings with tow steeped in carbolic acid lotion of the following strength, *viz.*, 1 oz. of carbolic acid to 10 oz. of water. The wound should be constantly protected by a broad pad of cloth, otherwise its cure will be considerably retarded by the habit elephants have of blowing dust or dirt over their bodies—more particularly when they have sores on any part of them.

6. In the case of sore backs prevention is easier than cure, and as the “gadhela” or pads are generally the cause, too much attention cannot be given to ensure their being kept in a proper condition. They are very liable to become rotten when exposed to rain, and should therefore be always kept under cover when not in actual use; a single month of monsoon weather is sufficient to destroy them, if recklessly exposed to it. The pads and other furniture at present in use for the elephant are as follows:—

Elephant gear described.

1st. The "namda" which consists of hair well felted together, is about an inch thick, and two yards square, and has a covering of gunny on the upper side, and one of coarse cloth on the under. The latter comes in immediate contact with the animal's back.

2nd. The "gadhela" placed on the top of the namda, consists of two bags of gunny filled with bulrushes, a foot thick, and two feet broad, the length being four to five feet, according to the size of the animal. These bags are joined edge to edge at either extremity, the open space between them being intended to receive the spinal ridge of the animal's backbone.

3rd. The "Nímgadhi" which is of similar construction to the gadhela, but smaller in its dimensions.

4th. The "jhúl" a coarse cloth consisting of gunny which is thrown over the whole.

5th. A rope about one inch in diameter by which the above are fixed on the back—one end of this rope is converted into a loop, sufficiently long nearly to surround the body of the animal and the pads. It is tied on the top, then passed singly round the neck, then along the upper sides of the pads, then below the tail, and finally tied again at the place of the first knot on the top. The parts which pass under the belly and tail are sheathed with leather, to prevent abrasion of the elephant's skin.

7. It may be assumed as a principle that sore backs and injuries to the feet of elephants are the result of negligence on the part of the attendants, and it is therefore a good rule to enforce, that so long as the animals are under treatment for such injuries, half of the attendants' pay should be stopped; unless good cause can be shown, in individual cases, why the rule should not be put in operation.

8. Elephants are occasionally very useful in pushing down trees; when so occupied pads should be provided to defend their heads. If an elephant bruise or injure itself when so employed, and this it is liable to do, it will probably refuse to give assistance again in this way.

Head pads.

Stoppage of attendants' pay.

9. Elephants should not be picketed out in a burning sun; inflammation of the brain, and other similar diseases may be caused by such exposure. If shade be not available a white padded covering should be fastened over the head.

10. On coming off a march, an elephant should be allowed to cool previously to being washed; negligence in this particular is apt to occasion serious illness. The animal may be allowed to drink water when in a heated state without any untoward effects resulting, but it should not be allowed to throw water over itself.

11. Elephants should be watered twice daily; at each time they take about 15 or 16 gallons of water. They prefer river water to all others, and willingly drink that procured by digging holes in the sandy beds of rivers, after it has stood for a few minutes to allow the sediment to sink. Elephants cannot be kept with impunity for more than 24 hours without water.

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CHAPTER IV. EQUIPMENT, NATIVE ESTABLISHMENT, &c.

1. The entertainment of low caste men for the greater portion of the native establishment such as "kahárs", "korís", "chamárs" &c., is advisable, though it is useful to have a small proportion of Brahmins and Muhammadans. Young hands should be entertained for survey work, and executive officers should be careful not to employ old men, unless of superior attainments and merit. Every khalási should be provided with a service book on joining. All inefficient men should be discharged at the close of the field season. Others not absolutely required in recess-quarters, or at the field depôt, may be allowed leave of absence during the recess on such portion of their salary, not exceeding half, as may be deemed advisable by the executive officer, to induce them to return to the party at the commencement of the next field season; a certificate to this effect being furnished to each man, showing the date and place ordered for his return. This leave does not constitute a break in service.

2. Stores and public property should not be kept in a closed tent, with a single sentry outside; for such an arrangement is unsafe, and unfair to the man on guard. The most secure method of guarding property is to collect it in an open spot, from which the sentry can have a clear view on all sides; the articles should be raised above the influence of damp ground and of white ants by means of stones and bricks. Delicate instruments liable to injury from exposure can be guarded securely if placed in a "shuldári" or open "pál." When the strength of the guard is insufficient for furnishing a double night sentry, the khalásis of the establishment must take their turn of such duty.

3. Assistants are not exempted from all office work for the days on which they may have marched. The daily office hours will be regulated with due consideration to the length of the day's march, and to the exigency of the work in hand. Every detail concerning the arrangement of camps and order of marching is under the control of the officer in charge of the party.

4. The field season generally commences about the 15th of October, and lasts about six months, but there are few districts in India or Burma, where this whole period can be utilized. In malarious tracts little benefit is gained by commencing field work before the middle of December, and in others, such as Rajputana, out-of-door work becomes almost impossible for Europeans after April. In Burma little in the way of triangulation or reconnaissance can be done after the middle of March, owing to the dense haze which obscures all distant points. Executive officers must apply for orders from the Surveyor General, or the Deputy Surveyor General of their branch, as to the times of taking and leaving the field. They are on no account, unless by special sanction, to return to recess-quarters until all the instruments and stores have been properly stowed away in the field depôt, and all accounts with the field establishment properly adjusted.

